Value of Information analyses and decision analysis types

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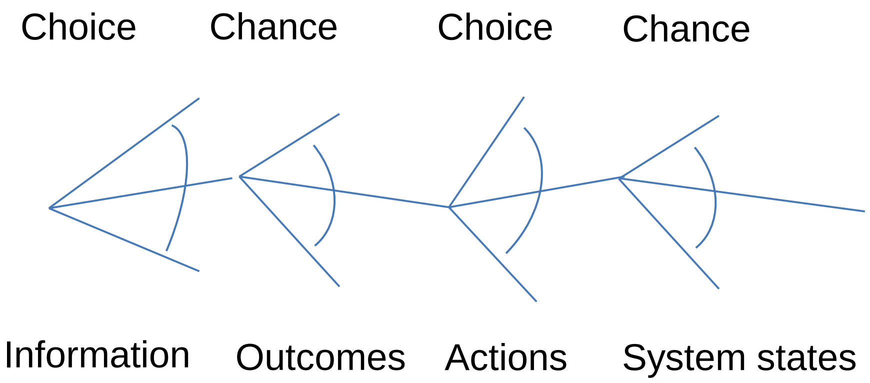
Day 3: Decision analyses

Value of Information analyses and decision analysis types

- Types of Value of Information
- Analyses types
- Derivation of decision rules

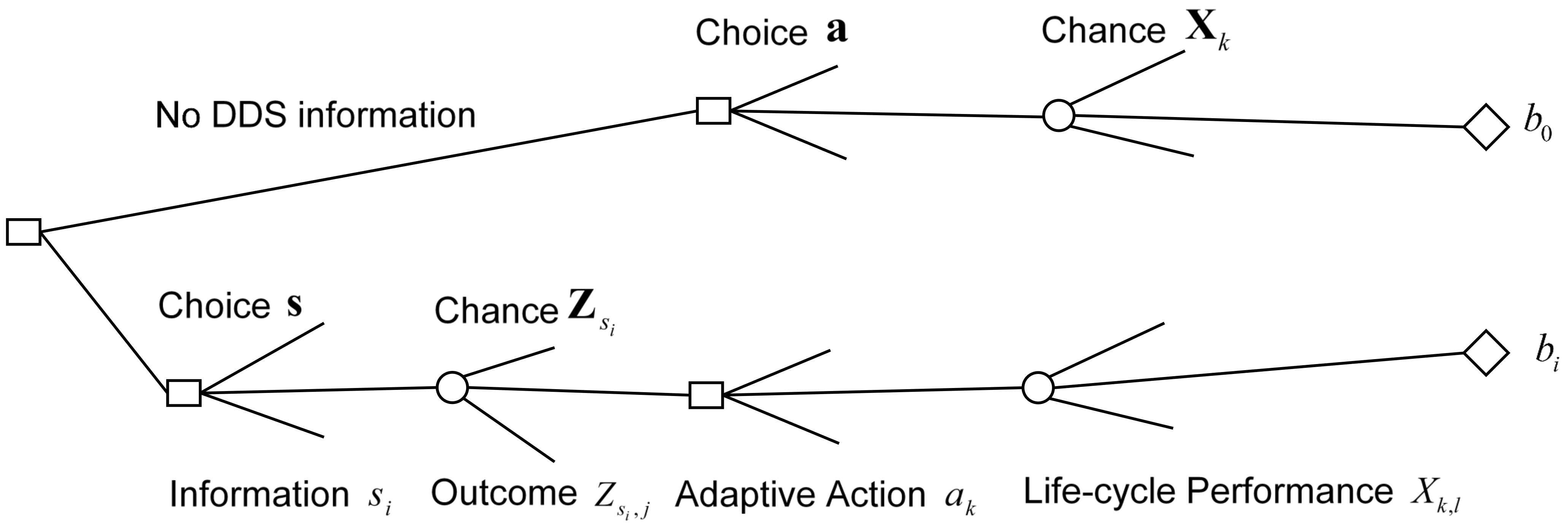
Value of Information



A Value of information analysis is the quantification of the utility or benefit gain due to additional or unknown information.

- The Value of Information theory developed by Raiffa and Schlaifer in 1961.
- An information is characterised by its content, its precision and its costs.

Value of SHM Information



Value of SHM Information

$$B_0 = E_{X_{k,l}}\left[b_0\left(a_k^{*,0}, X_{k,l}\right)\right]$$

$$a_k^{*,0} = \arg\max_{a_k}\left(E_{X_{k,l}}\left[b_0\left(a_k, X_{k,l}\right)\right]\right)$$

$$B_1 = E_{Z_{s_i,j}}\left[E''_{X_{k,l}}\left[b_i\left(s_i^*, Z_{s_i,j}, a_k^{*,i}, X_{k,l}\right)\right]\right]$$

$$\left(s_i^*, a_k^{*,i}\right)$$

$$= \arg\max_{s_i} E_{Z_{s_i,j}}\left[\arg\max_{a_k} E''_{X_{k,l}}\left[b_i\left(s_i, Z_{s_i,j}, a_k, X_{k,l}\right)\right]\right]$$

The Value of SHM Information is defined as the difference between the maximized expected benefits with and without additional information.

$$V = B_1 - B_0$$

The relative Value of Information is defined as the expected benefit gain in relation to the maximized expected benefits without additional information.

$$V = \frac{B_1 - B_0}{B_0}$$

Types of Value of Information

Four types of value information analyses are distinguished (Raiffa and Schlaifer (1961)):

- EVSI/CVSI: Expected/Conditional Value of Sample Information
  - EVSI: Difference of utilities in pre-posterior and prior decision analysis
  - CVSI: Difference of utilities in posterior and prior decision analysis
  - Sample information refers to information with a finite precision (uncertain)

- EVPI/CVPI : Expected/Conditional Value of Perfect Information
  - In analogy to EVSI/CVSI but with infintely precise information, i.e. without uncertainties

Types of Value of Information

The Value of Expected Information facilitates to optimise a decision before any action is performed.

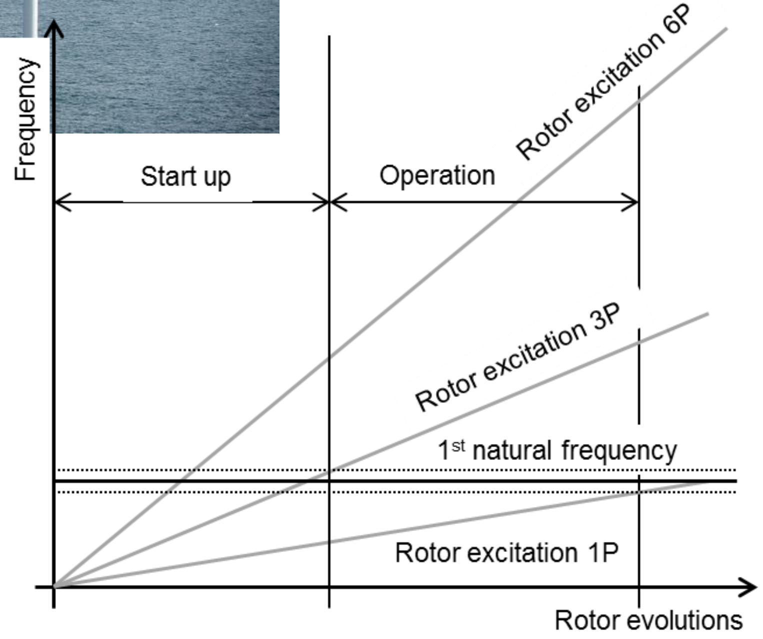
Value of Expected Information

- Will the information acquirement be cost efficient?
- Pre-posterior decision analysis

Value of Conditional Information

- Has the spent money for acquiring additional information cost efficient?
- Posterior decision analysis

Value of SHM Information: Example



A wind turbine is operating. The control data reveal that there maybe a problem with resonance of the support structure and the rotor excitations. It is estimated that with a probability of 20% there is a resonance problem (system state x_2).

You have two action options:

Do nothing (action a_0)

Modify the operational range (action a_1). This costs 20 as the control has to be modified and certified.

Benefits and costs:

		a_0	a_1
X_1	No resonance	100	70
X_2	Resonance	-200	70

Value of SHM Information: Example

Is it worth to perform an experimental modal analysis?

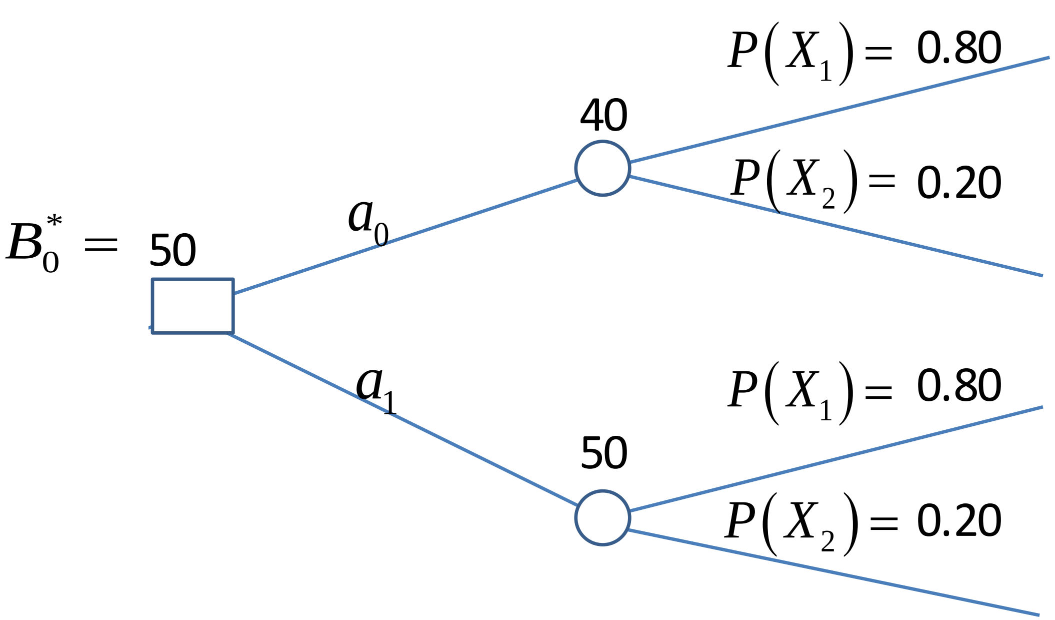
Based on prior experience and studies you know the probabilities of indication (e.g. $P(Z_1 \mid X_1)$ = 90%) according to the table below. The cost of the analysis is 10.

	X_1	X_2
Z_1	0.9	0.15
Z_2	0.1	0.85

Denotation: e_1 denotes performing the modal analysis, e_0 denotes not performing the modal analysis

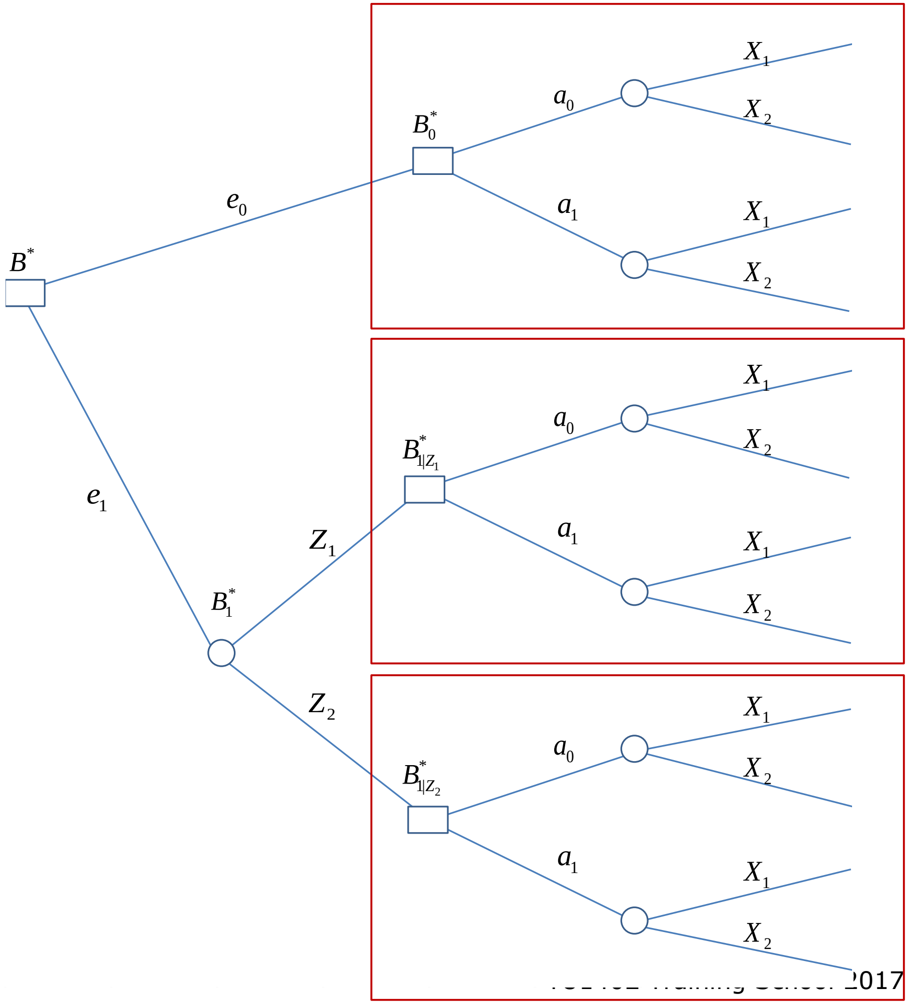
Prior decision analysis (known information)

Result: The operational range should be modified (a_1) as doing nothing (a_0) leads to lower benefits.



Pre-posterior decision analysis (unknown information)

Decision tree for example



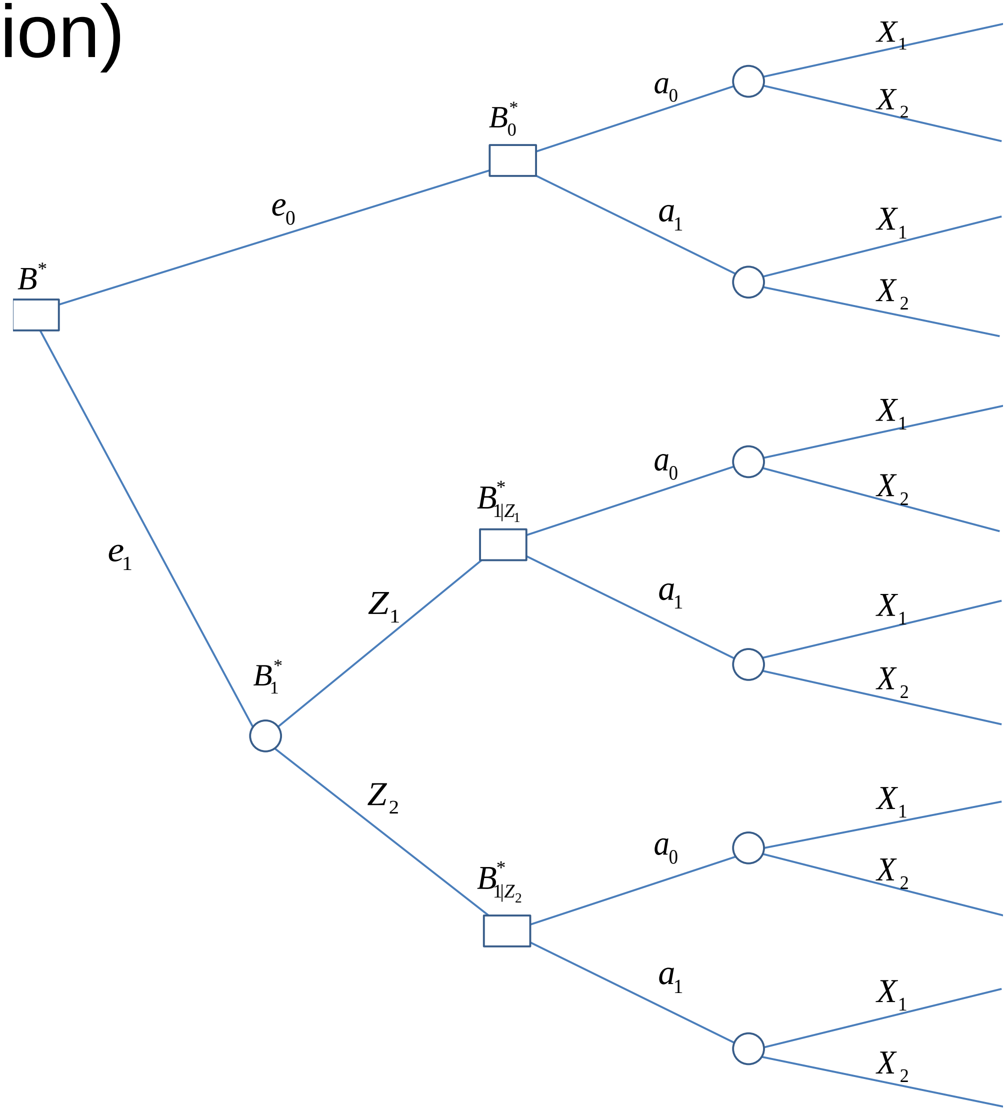
Prior analysis

Posterior analysis with additional information Z_1

Posterior analysis with additional information Z_2

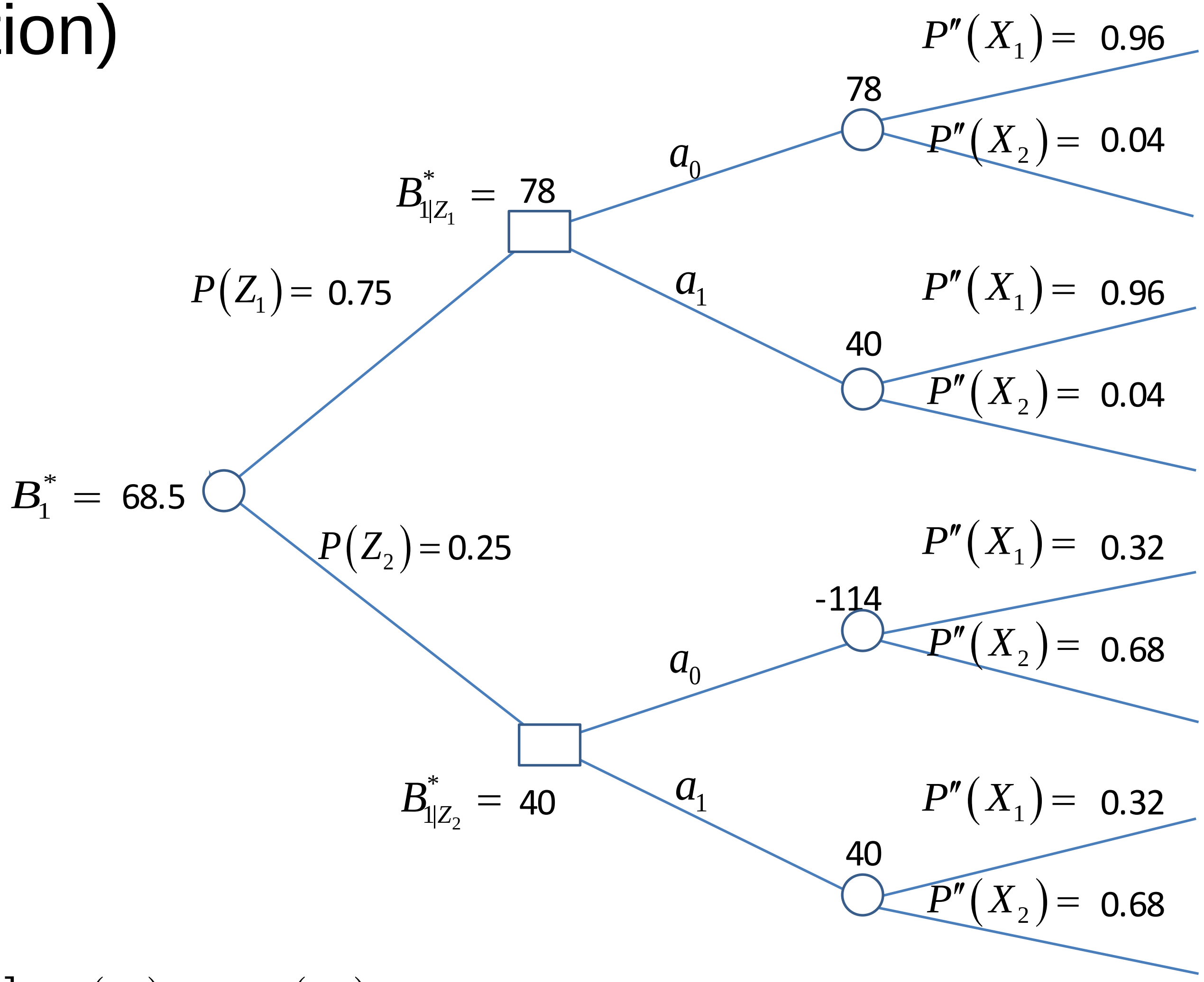
Pre-posterior decision analysis (unknown information)

Decision tree



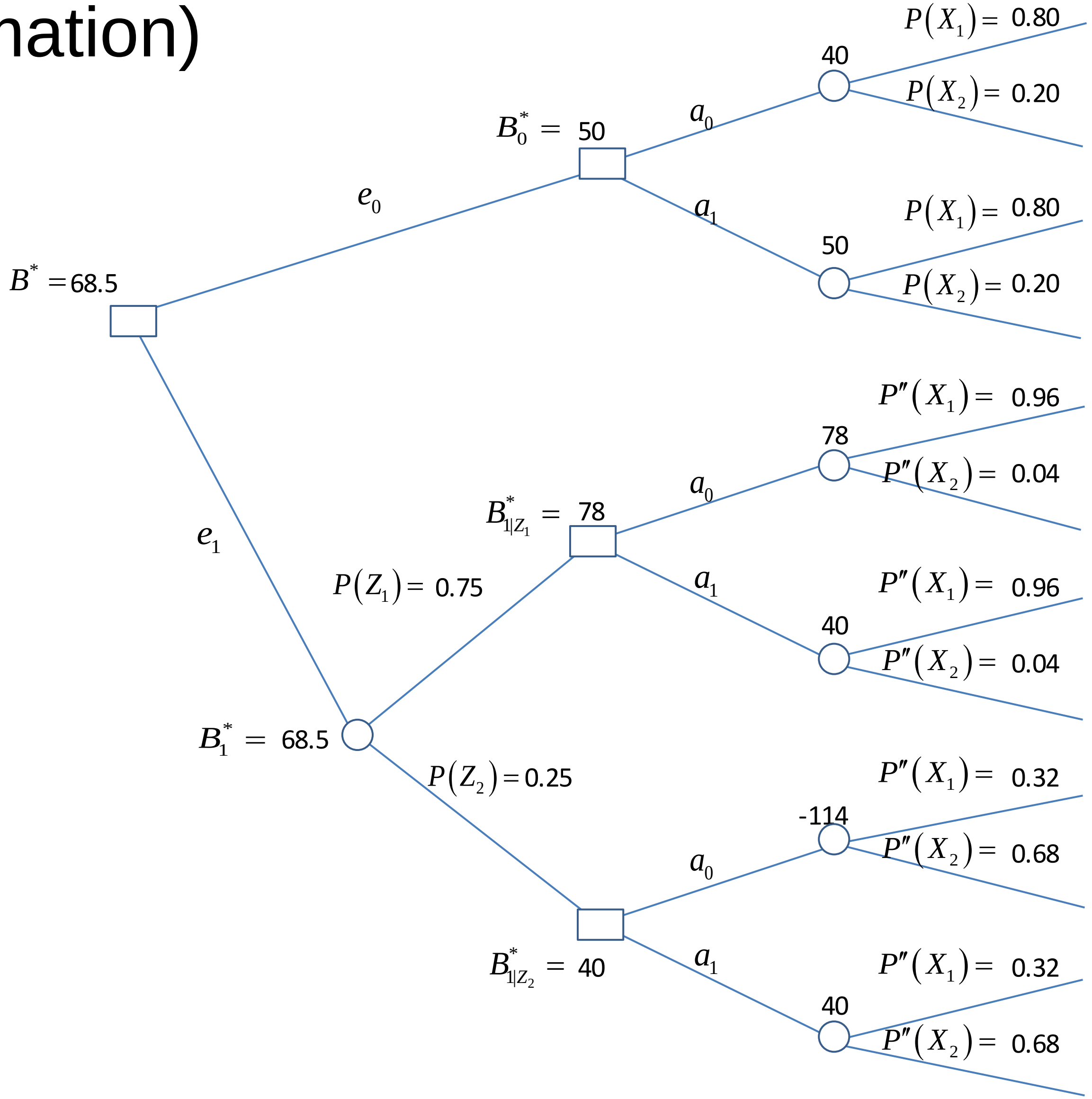
Benefit	Costs	Total
100	0	100
-200	0	-200
70	20	50
70	20	50
100	10	90
-200	10	-210
70	30	40
70	30	40
100	10	90
-200	10	-210
70	30	40
70	30	40

Pre-posterior decision analysis (unknown information)

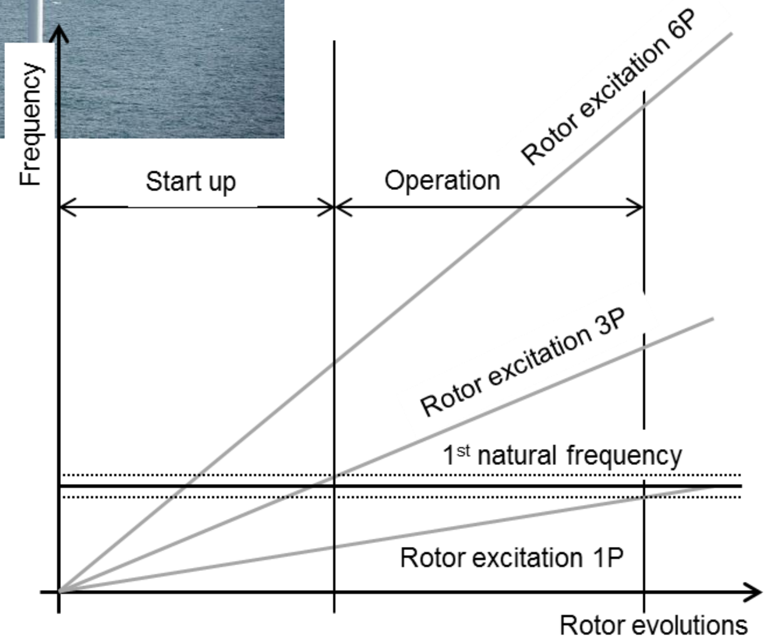


$$B_1^* = \max_{a_0, a_1} E\left[B \mid e_1\right] = P\left(Z_1\right) \cdot 78 + P\left(Z_2\right) \cdot 40 = 0.75 \cdot 78 + 0.25 \cdot 40 = 68.5$$

Pre-posterior decision analysis (unknown information)

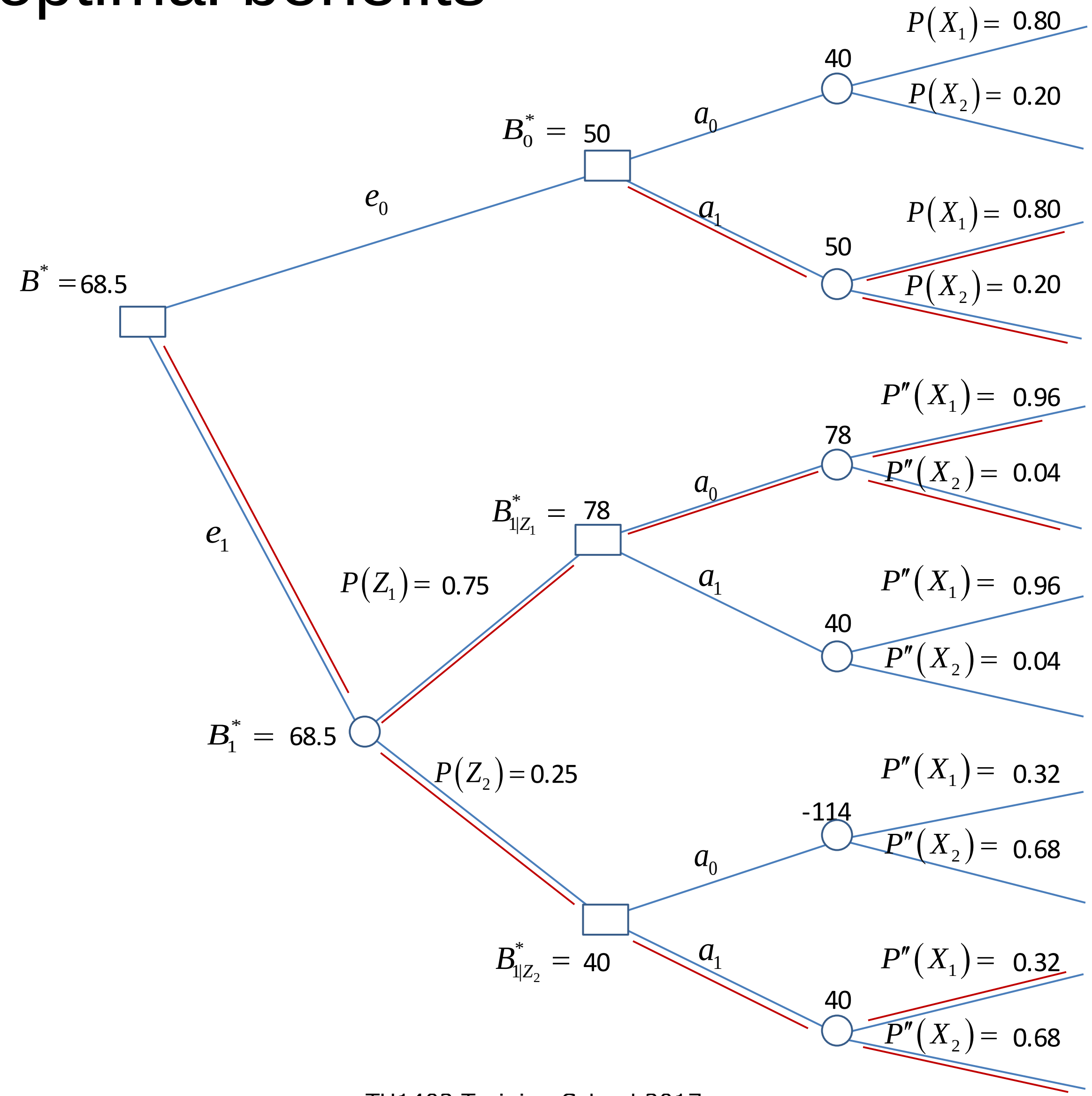


Pre-posterior decision analysis (unknown information)

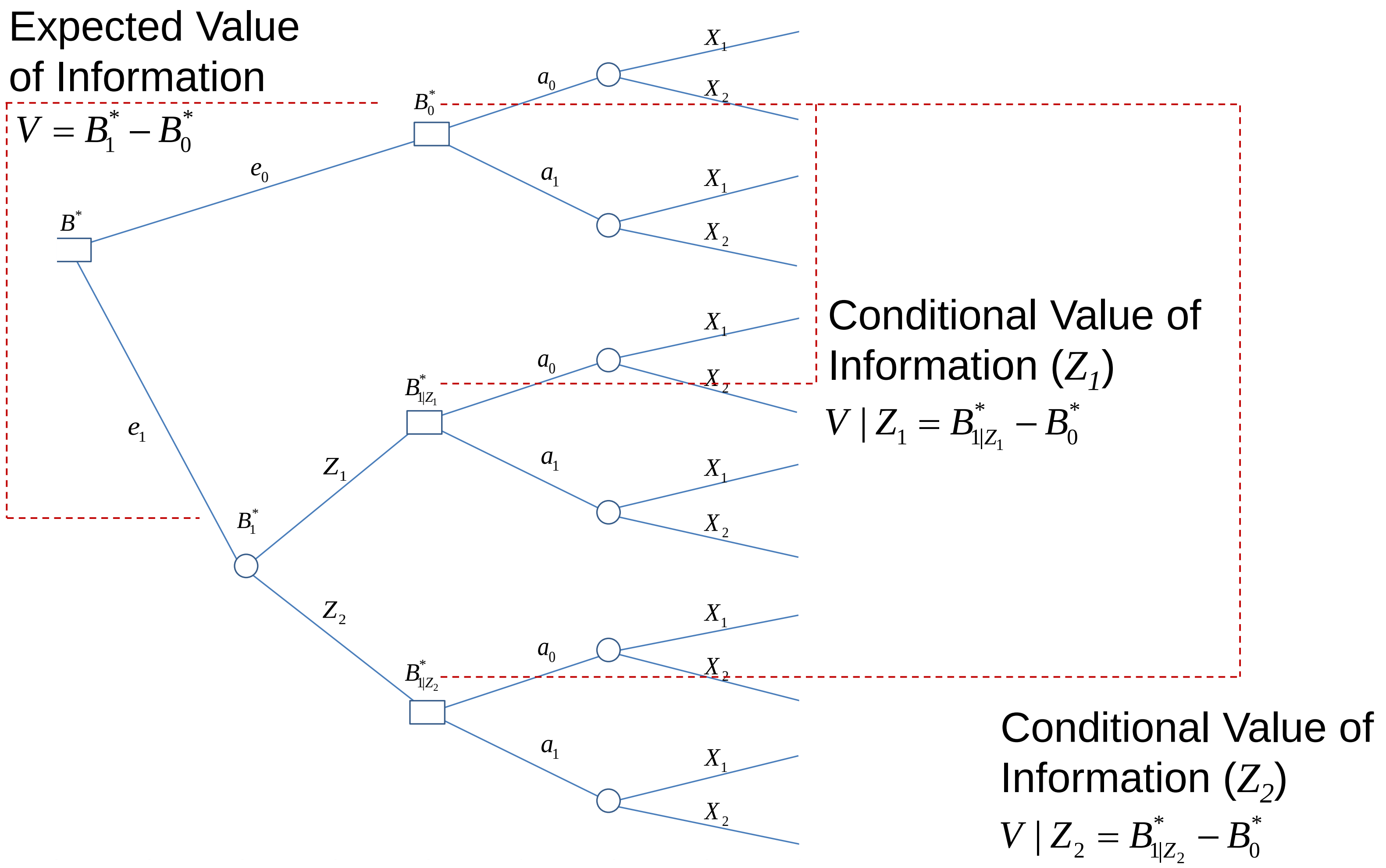


Result: The experimental modal analysis should be performed (e_1) as it is associated with higher benefits.

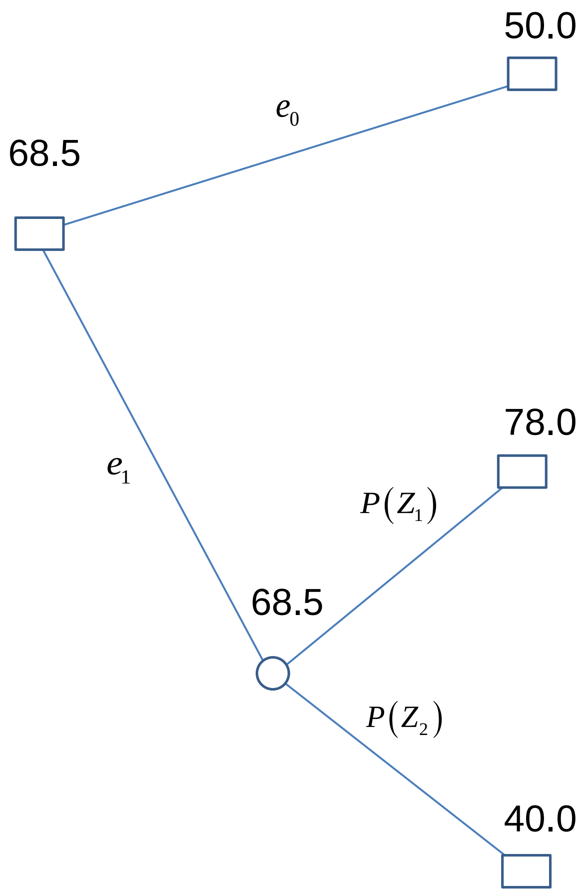
Branches leading to optimal benefits



Types of Value of Information analyses: Example



Types of Value of Information analyses: Example



CVSI: Conditional value of sample information:

$$V \mid Z_1 = \max_{a_0, a_1} E\left[B \mid Z_1\right] - \max_{a_0, a_1} E\left[B\right]$$

$$= 78.0 - 50.0 = 28.0$$

$$V \mid Z_2 = \max_{a_0, a_1} E\left[B \mid Z_2\right] - \max_{a_0, a_1} E\left[B\right]$$

$$= 40.0 - 50.0 = -10.0$$

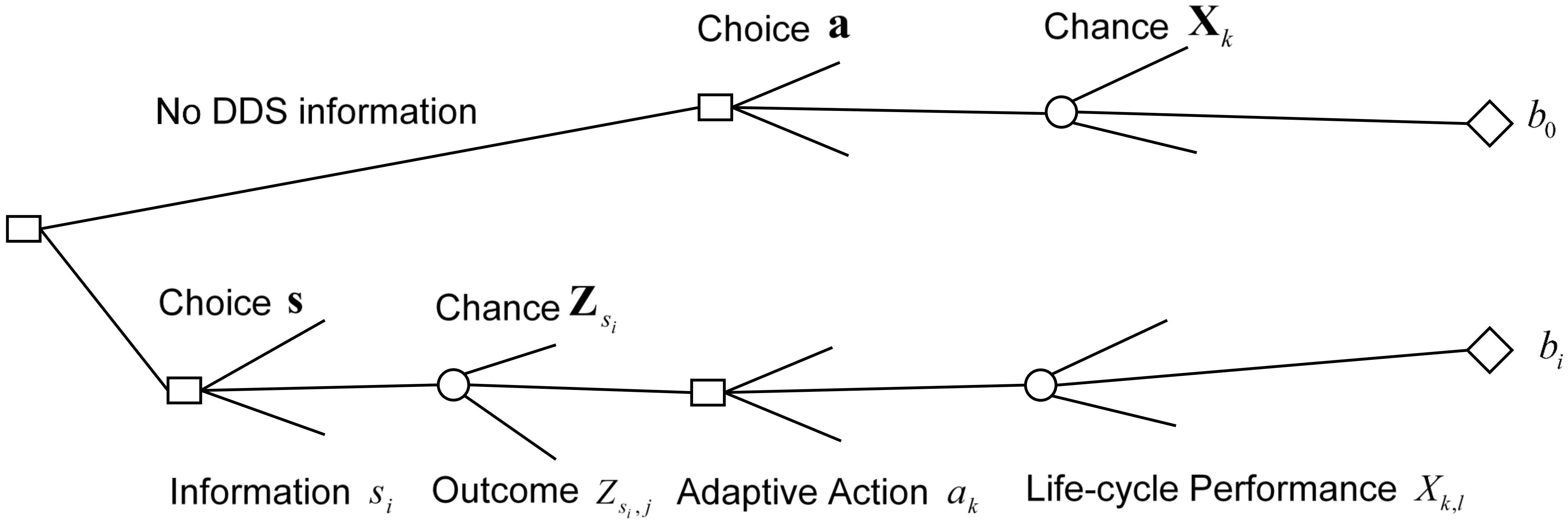
EVSI: Expected value of sample information

$$V = \max_{a_0, a_1} E\left[B \mid \mathrm{e}_1\right] - \max_{a_0, a_1} E\left[B \mid \mathrm{e}_0\right]$$

$$= 68.5 - 50.0 = 18.5$$

$$V_{e_2} = \max_{a_0, a_1} E\left[B \mid \mathrm{e}_2\right] - \max_{a_0, a_1} E\left[B \mid \mathrm{e}_0\right] = \ldots.$$

Value of SHM Information: decision analysis types



Value of SHM Information: Extensive form

$$B_0 = E_{X_{k,l}}\left[b_0\left(a_k^{*,0}, X_{k,l}\right)\right]$$

$$a_k^{*,0} = \underset{a_k}{arg\,max}\left(E_{X_{k,l}}\left[b_0\left(a_k, X_{k,l}\right)\right]\right)$$

$$B_1 = E_{Z_{s_i,j}}\left[E''_{X_{k,l}}\left[b_i\left(s_i^*, Z_{s_i,j}, a_k^{*,i}, X_{k,l}\right)\right]\right]$$

$$\left(s_i^*, a_k^{*,i}\right) =$$

$$\underset{s_i}{arg\,max}\, E_{Z_{s_i,j}}\left[\underset{a_k}{arg\,max}\, E''_{X_{k,l}}\left[b_i\left(s_i, Z_{s_i,j}, a_k, X_{k,l}\right)\right]\right]$$

The Value of SHM Information is defined as the difference between the maximized expected benefits with and without additional information.

$$V = B_1 - B_0$$

The relative Value of Information is defined as the expected benefit gain in relation to the maximized expected benefits without additional information.

$$V = \frac{B_1 - B_0}{B_0}$$

Value of SHM Information: Normal form

$$B_0 = E_{X_{k,l}}\left[b_0\left(a_k^{*,0}, X_{k,l}\right)\right]$$

$$a_k^{*,0} = \underset{a_k}{arg\,max}\left(E_{X_{k,l}}\left[b_0\left(a_k, X_{k,l}\right)\right]\right)$$

$$B_1 = E_{X_{k,l}}\left[E_{Z_{s_i,j}|X_{k,l}}\left[b_i\left(s_i^*, Z_{s_i,j}, d_m^*\left(Z_{s_i,j}\right), X_{k,l}\right)\right]\right]$$

$$\left(s_i^*, a_k^{*,i}\right) =$$

$$\underset{s_i,d_m}{arg\,max}\, E_{X_{k,l}}\left[E_{Z_{s_i,j}|X_{k,l}}\left[b_i\left(s_i, Z_{s_i,j}, d_m\left(Z_{s_i,j}\right), X_{k,l}\right)\right]\right]$$

The Value of SHM Information is defined as the difference between the maximized expected benefits with and without additional information.

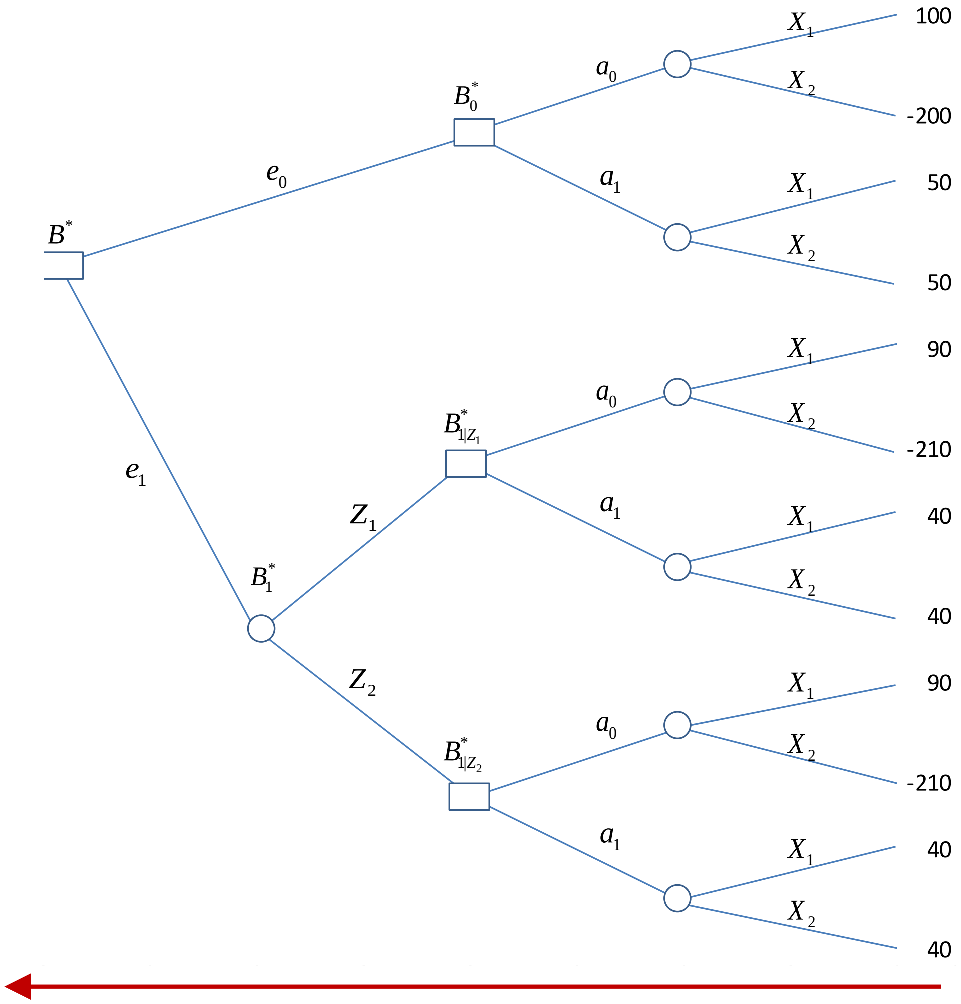
$$V = B_1 - B_0$$

The relative Value of Information is defined as the expected benefit gain in relation to the maximized expected benefits without additional information.

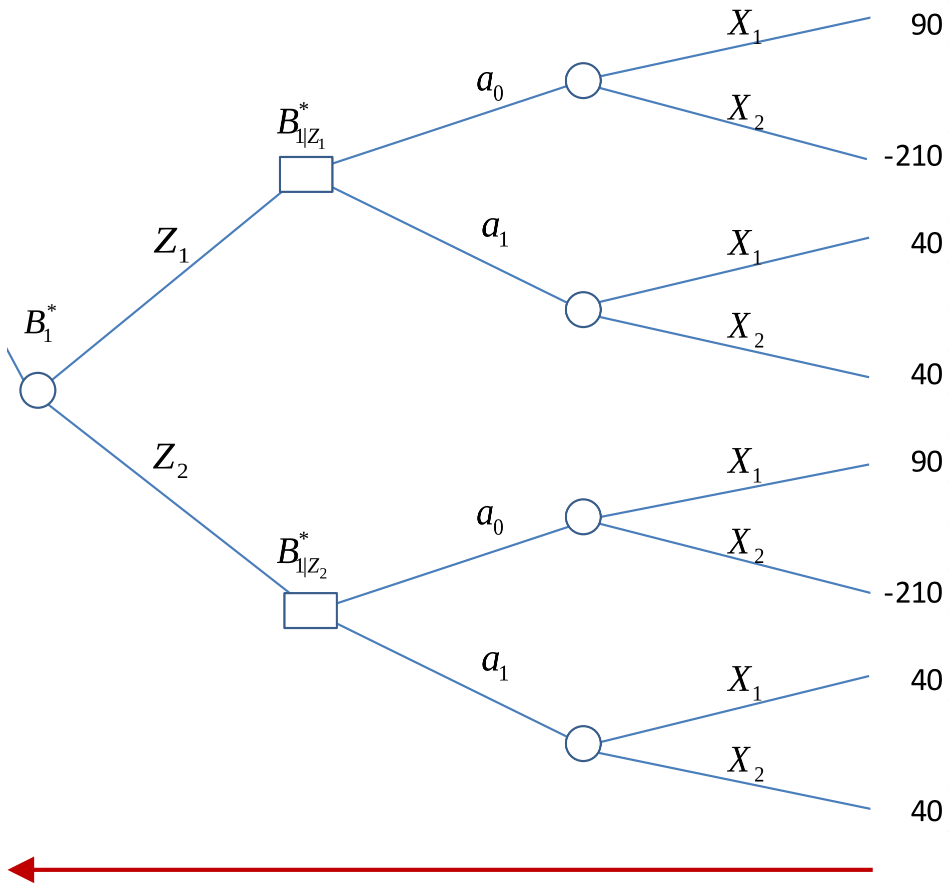
$$V = \frac{B_1 - B_0}{B_0}$$

Normal and extensive form decision analyses

Extensive form analysis.



Normal and extensive form decision analyses

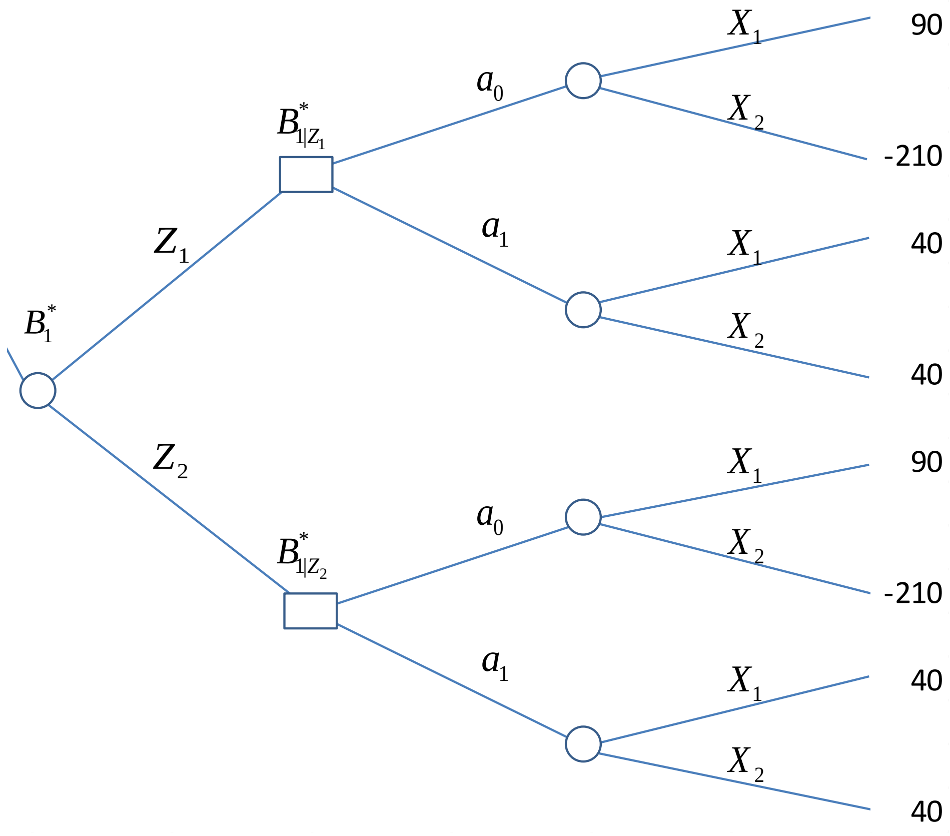


Extensive form analysis

$$B_1^* = P(Z_1)\cdot\max\begin{bmatrix}\begin{pmatrix}P''(X_1)\cdot b(e_1,Z_1,a_0,X_1)\\ +P''(X_2)\cdot b(e_1,Z_1,a_0,X_2)\end{pmatrix}\\ \begin{pmatrix}P''(X_1)\cdot b(e_1,Z_1,a_1,X_1)\\ +P''(X_2)\cdot b(e_1,Z_1,a_1,X_2)\end{pmatrix}\end{bmatrix}$$

$$+P(Z_2)\cdot\max\begin{bmatrix}\begin{pmatrix}P''(X_1)\cdot b(e_1,Z_2,a_0,X_1)\\ +P''(X_2)\cdot b(e_1,Z_2,a_0,X_2)\end{pmatrix}\\ \begin{pmatrix}P''(X_1)\cdot b(e_1,Z_2,a_1,X_1)\\ +P''(X_2)\cdot b(e_1,Z_2,a_1,X_2)\end{pmatrix}\end{bmatrix}$$

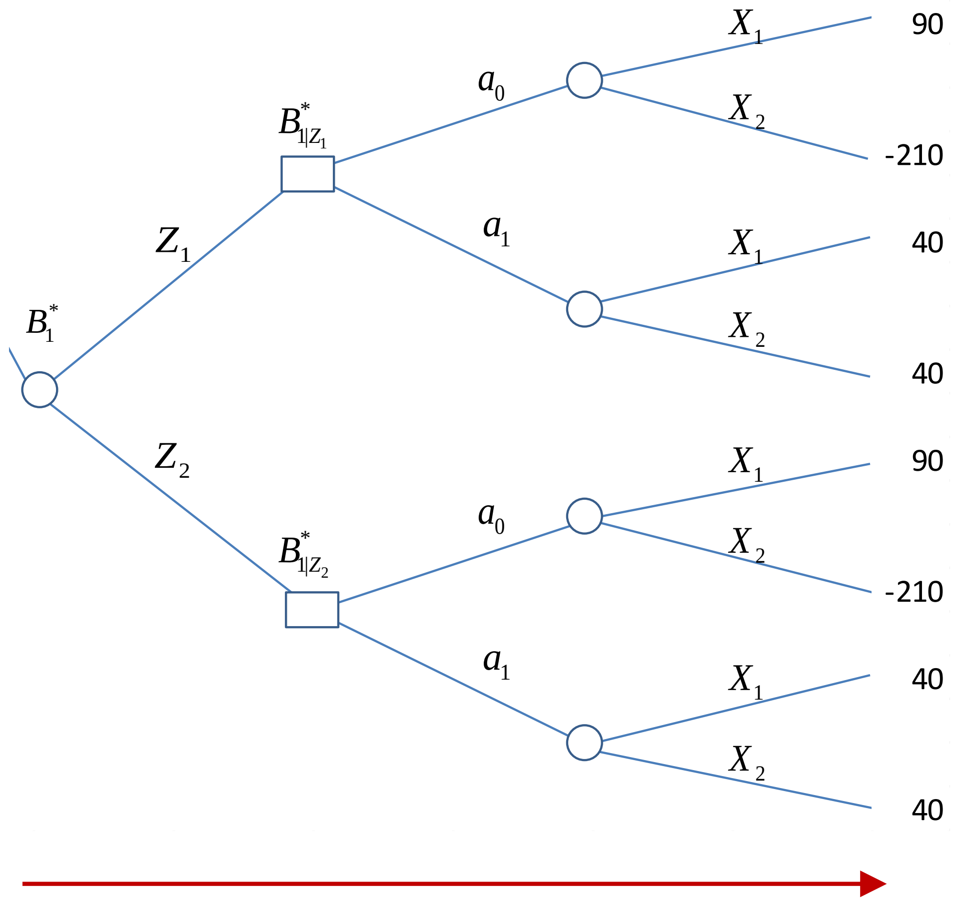
Normal and extensive form decision analyses



Definition of Bayesian updating

$$P''(X_1) = P(X_1 | Z_1) = \frac{P(Z_1 | X_1) P(X_1)}{P(Z_1)}$$

Normal and extensive form decision analyses

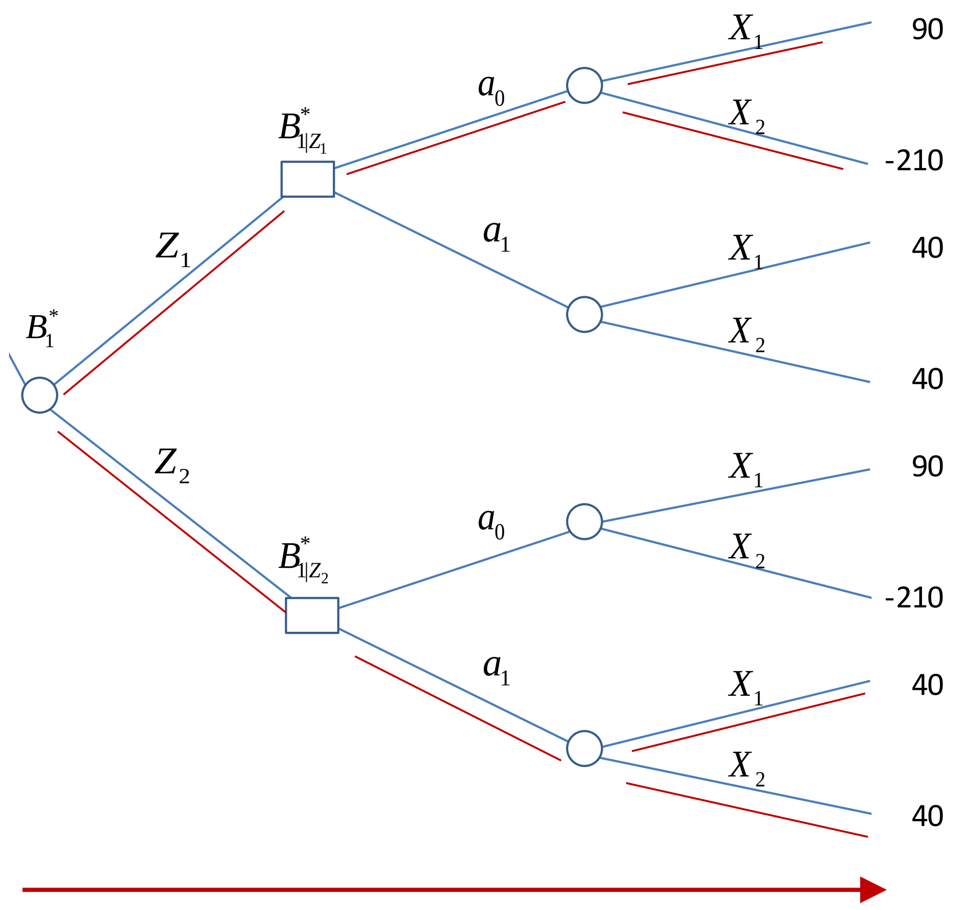


Normal form analysis

$$B_1^* = \max\left[\begin{array}{l}\left(\begin{array}{l}P(Z_1|X_1)\cdot P(X_1)\cdot b(e_1,Z_1,a_0,X_1)\\+P(Z_2|X_1)\cdot P(X_1)\cdot b(e_1,Z_1,a_0,X_2)\end{array}\right)\\ \left(\begin{array}{l}P(Z_1|X_1)\cdot P(X_1)\cdot b(e_1,Z_1,a_1,X_1)\\+P(Z_2|X_1)\cdot P(X_1)\cdot b(e_1,Z_1,a_1,X_2)\end{array}\right)\end{array}\right]$$

$$+\max\left[\begin{array}{l}\left(\begin{array}{l}P(Z_1|X_2)\cdot P(X_2)\cdot b(e_1,Z_2,a_0,X_1)\\+P(Z_2|X_2)\cdot P(X_2)\cdot b(e_1,Z_2,a_0,X_2)\end{array}\right)\\ \left(\begin{array}{l}P(Z_1|X_2)\cdot P(X_2)\cdot b(e_1,Z_2,a_1,X_1)\\+P(Z_2|X_2)\cdot P(X_2)\cdot b(e_1,Z_2,a_1,X_2)\end{array}\right)\end{array}\right]$$

Normal and extensive form decision analyses



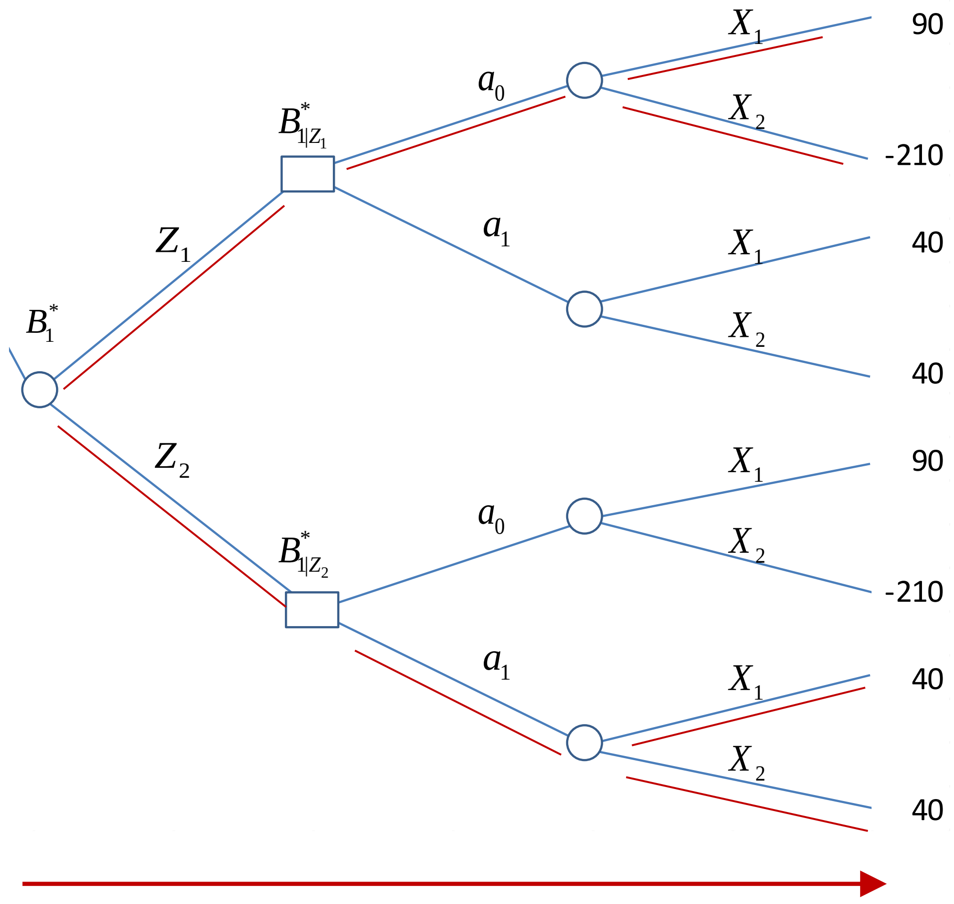
Decision rules can be identified with the branches providing the optimal actions.

- Optimal decision rule: do experiment and perform actions according to outcomes

$$\mathbf{d} = \begin{bmatrix} e_1 \\ Z_1 : a_0 \\ Z_2 : a_1 \end{bmatrix}$$

If you do not know the optimal branches, the decision rules have to cover all branches of the decision tree.

Normal and extensive form decision analyses

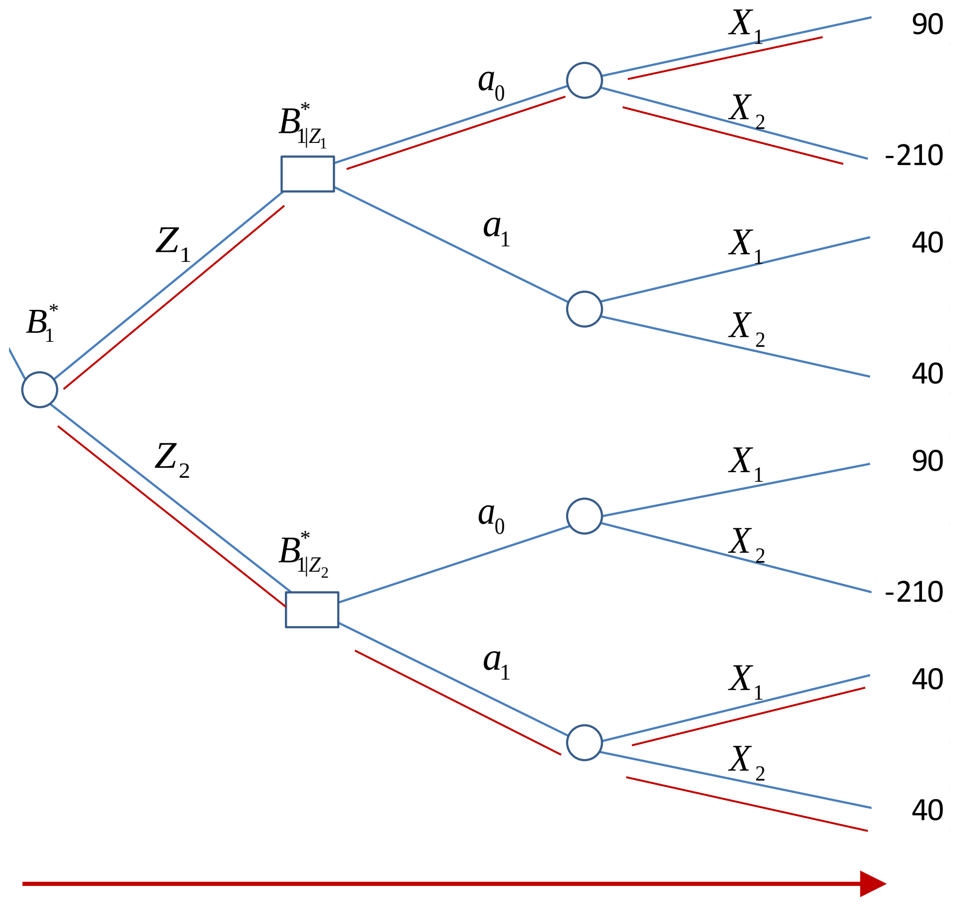


Normal form analysis and identified optimal branches

$$B_1^* = \cancel{\max}\left[\begin{array}{l}\left(\begin{array}{l}P(X_1)\cdot P(Z_1|X_1)\cdot b(e_1,Z_1,a_0,X_1)\\ +P(X_1)\cdot P(Z_2|X_1)\cdot b(e_1,Z_1,a_0,X_2)\end{array}\right)\\ \cancel{\left(\begin{array}{l}P(X_1)\cdot P(Z_1|X_1)\cdot b(e_1,Z_1,a_1,X_1)\\ +P(X_1)\cdot P(Z_2|X_1)\cdot b(e_1,Z_1,a_1,X_2)\end{array}\right)}\end{array}\right]$$

$$+\cancel{\max}\left[\begin{array}{l}\left(\begin{array}{l}P(X_2)\cdot P(Z_1|X_2)\cdot b(e_1,Z_2,a_0,X_1)\\ +P(X_2)\cdot P(Z_2|X_2)\cdot b(e_1,Z_2,a_0,X_2)\end{array}\right)\\ \cancel{\left(\begin{array}{l}P(X_2)\cdot P(Z_1|X_2)\cdot b(e_1,Z_2,a_1,X_1)\\ +P(X_2)\cdot P(Z_2|X_2)\cdot b(e_1,Z_2,a_1,X_2)\end{array}\right)}\end{array}\right]$$

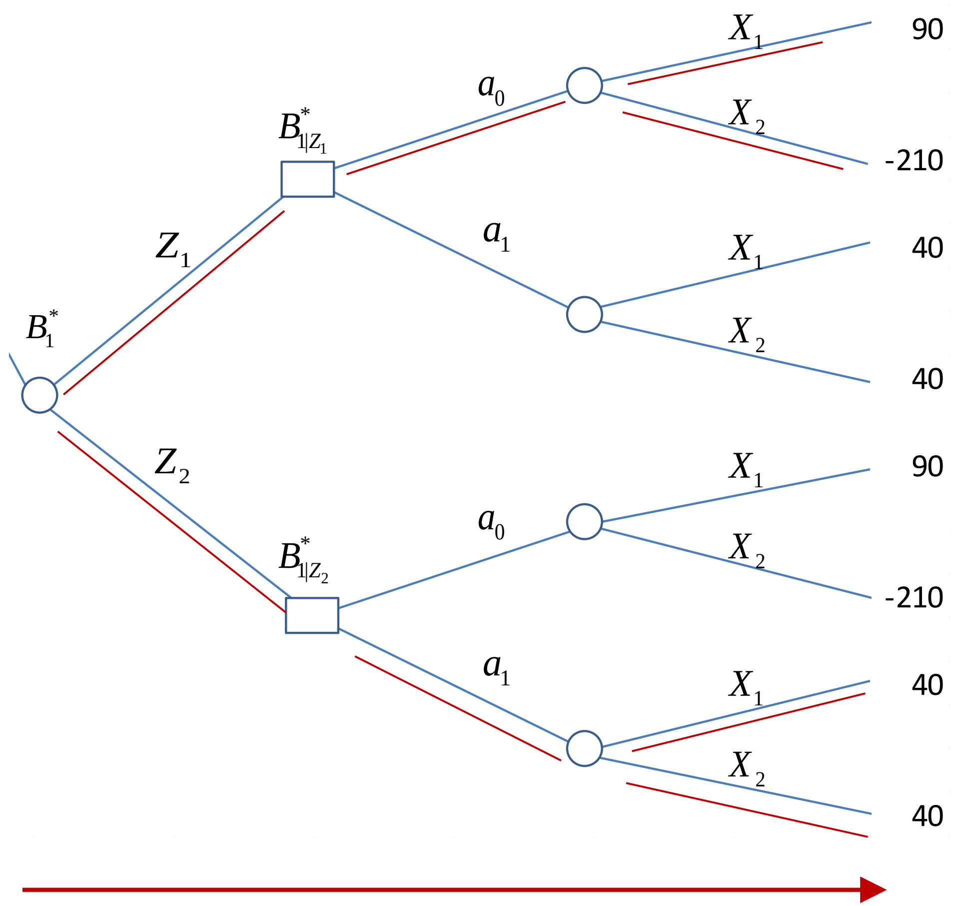
Normal and extensive form decision analyses



Normal form analysis and identified optimal branches

$$
\begin{aligned}
B_1^* = & P(Z_1 \mid X_1) \cdot P(X_1) \cdot b(e_1, Z_1, a_0, X_1) \\
& + P(Z_1 \mid X_2) \cdot P(X_2) \cdot b(e_1, Z_1, a_0, X_2) \\
& + P(Z_2 \mid X_1) \cdot P(X_1) \cdot b(e_1, Z_2, a_1, X_1) \\
& + P(Z_2 \mid X_2) \cdot P(X_2) \cdot b(e_1, Z_2, a_1, X_2)
\end{aligned}
$$

Normal and extensive form decision analyses



We can reproduce the expected benefits with a normal form analysis.

$$B_0^* = 40.0 + 10.0 = 50.0$$

$$\begin{aligned} B_1^* &= P(Z_1 \mid X_1)\cdot P(X_1)\cdot b(e_1, Z_1, a_0, X_1) \\ &+ P(Z_1 \mid X_2)\cdot P(X_2)\cdot b(e_1, Z_1, a_0, X_2) \\ &+ P(Z_2 \mid X_1)\cdot P(X_1)\cdot b(e_1, Z_2, a_1, X_1) \\ &+ P(Z_2 \mid X_2)\cdot P(X_2)\cdot b(e_1, Z_2, a_1, X_2) \\ &= 64.8 - 6.3 + 3.2 + 6.8 = 68.5 \end{aligned}$$

$$V = B_1^* - B_0^* = 68.5 - 50.0 = 18.5$$

Normal and extensive form decision analyses

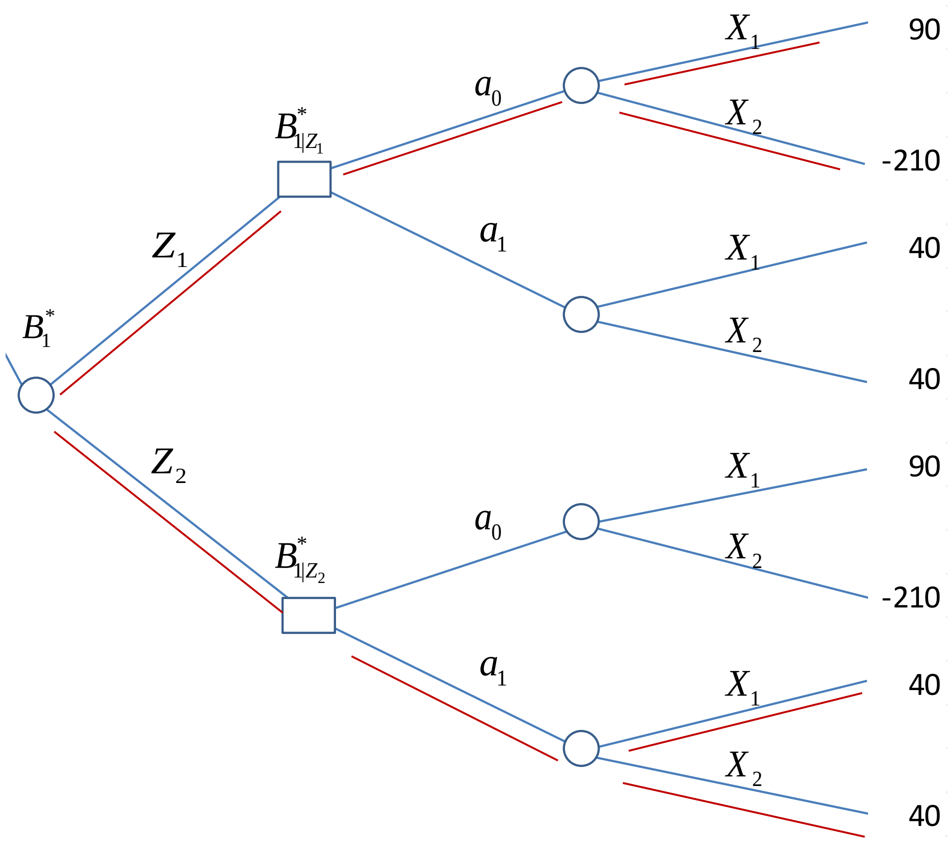
The extensive form and normal form analysis are equivalent.

Extensive form
- Analysis form the right hand side to the left hand side
- Bayesian updating required

Normal form
- Analysis form the left hand side to the right hand side
- Definition of decision rules is required.
- Computationally more efficient
  - No unnecessary operations
  - Only the optimal branches need to be considered (which may be known before)

Decision rules are required for the implementation of a decision process.

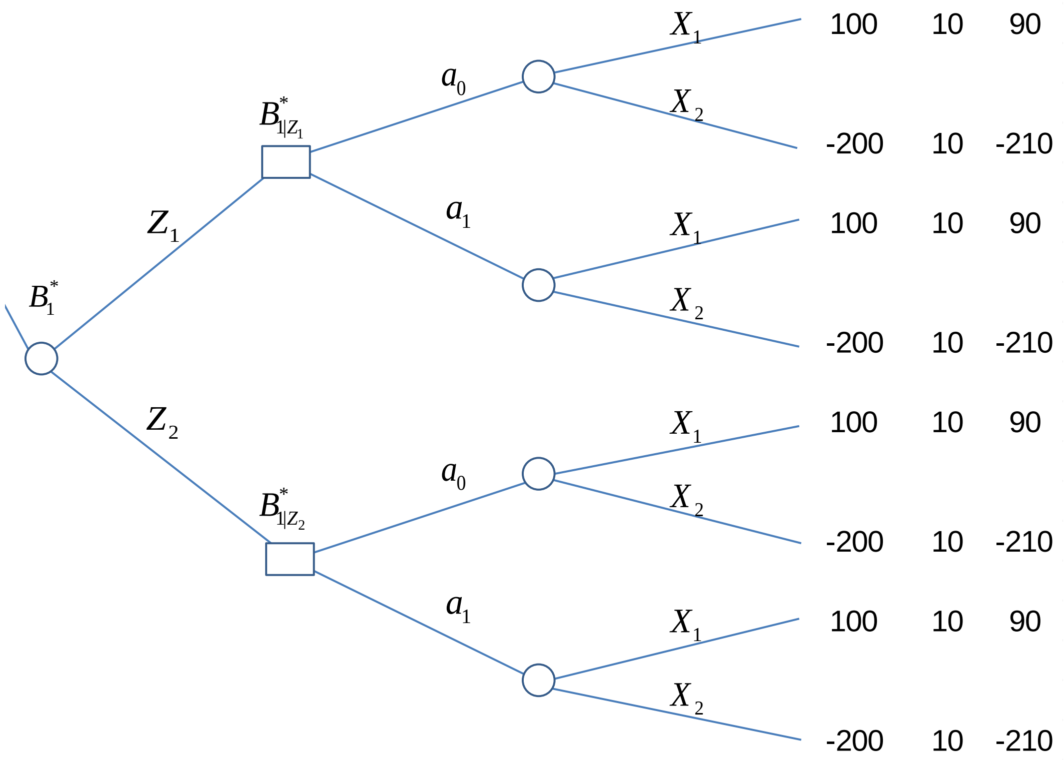
Branch eliminating?



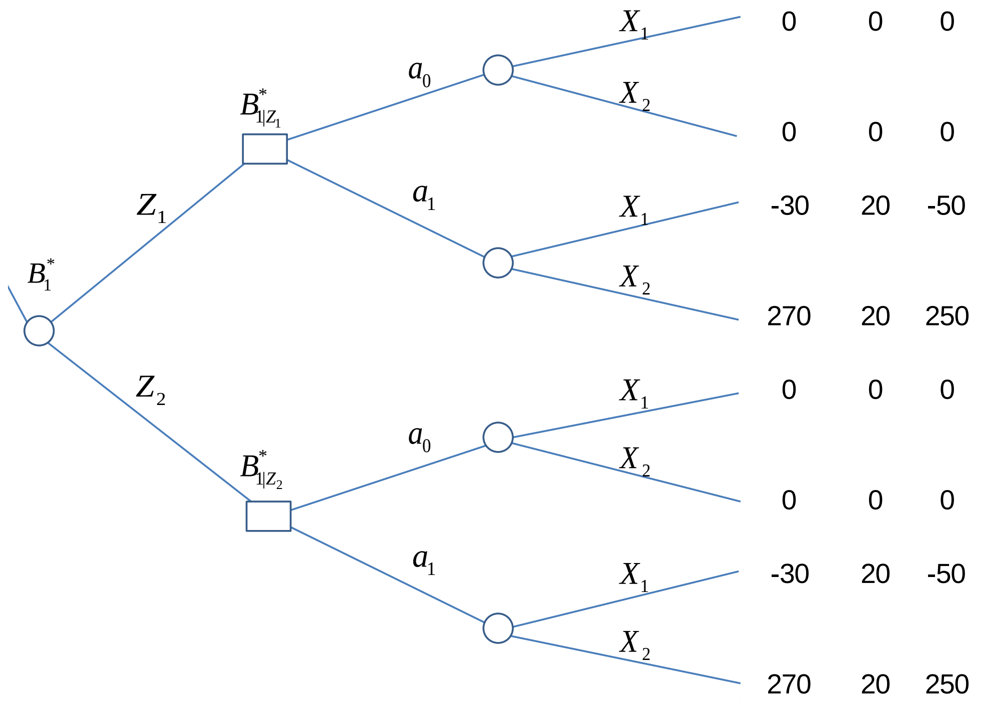
Branch eliminating by separation

The decision analysis is separated in two decision trees. The summation leads to the original decision tree.

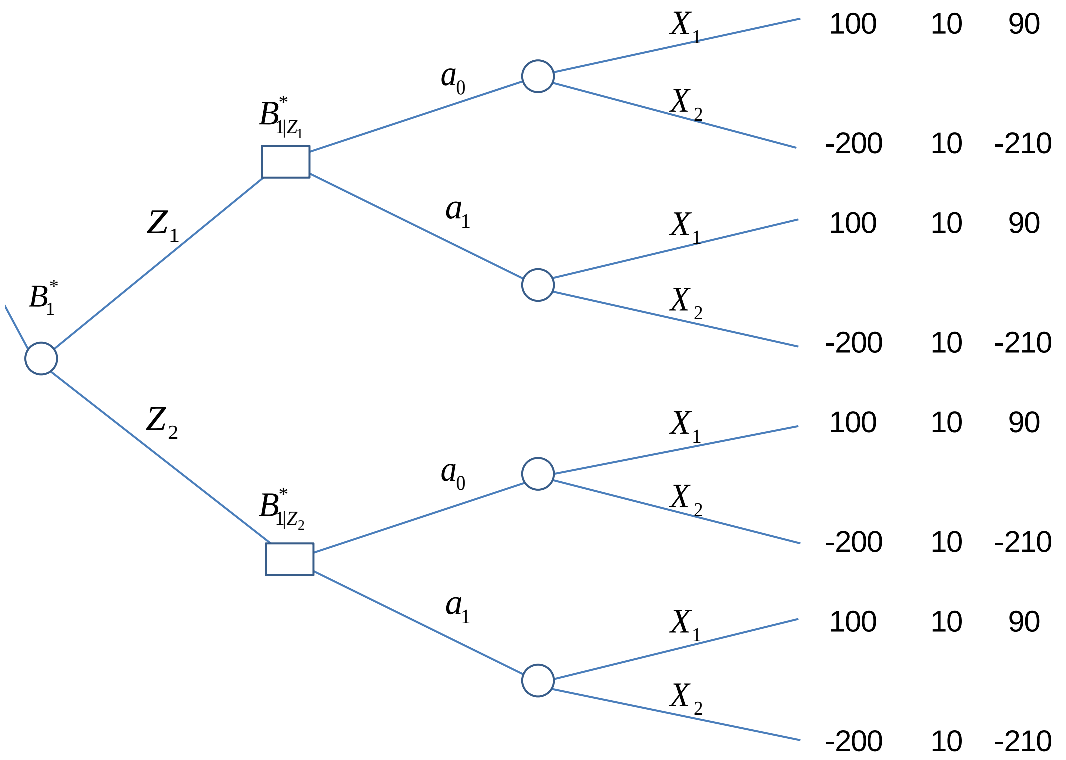
Decision tree 1 is only associated with benefits and costs associated with the system states and SHM.



Decision tree 2 is only associated to action dependent costs.



Branch eliminating by separation

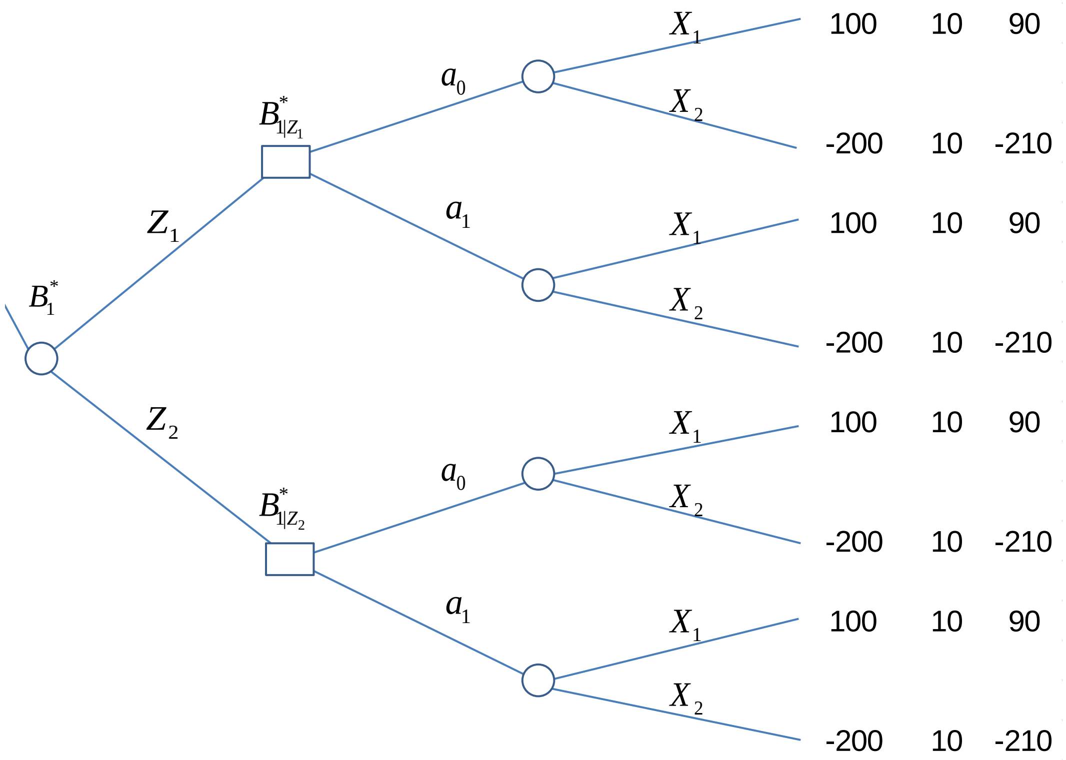


Decision tree 1, Normal form analysis

$$B_{1,1}^* = C_{e_1} + \max\begin{bmatrix} \begin{pmatrix} P(Z_1|X_1)\cdot P(X_1)\cdot b(X_1) \\ +P(Z_2|X_1)\cdot P(X_1)\cdot b(X_2) \end{pmatrix} \\ \begin{pmatrix} P(Z_1|X_1)\cdot P(X_1)\cdot b(X_1) \\ +P(Z_2|X_1)\cdot P(X_1)\cdot b(X_2) \end{pmatrix} \end{bmatrix}$$

$$+\max\begin{bmatrix} \begin{pmatrix} P(Z_1|X_2)\cdot P(X_2)\cdot b(X_1) \\ +P(Z_2|X_2)\cdot P(X_2)\cdot b(X_2) \end{pmatrix} \\ \begin{pmatrix} P(Z_1|X_2)\cdot P(X_2)\cdot b(X_1) \\ +P(Z_2|X_2)\cdot P(X_2)\cdot b(X_2) \end{pmatrix} \end{bmatrix}$$

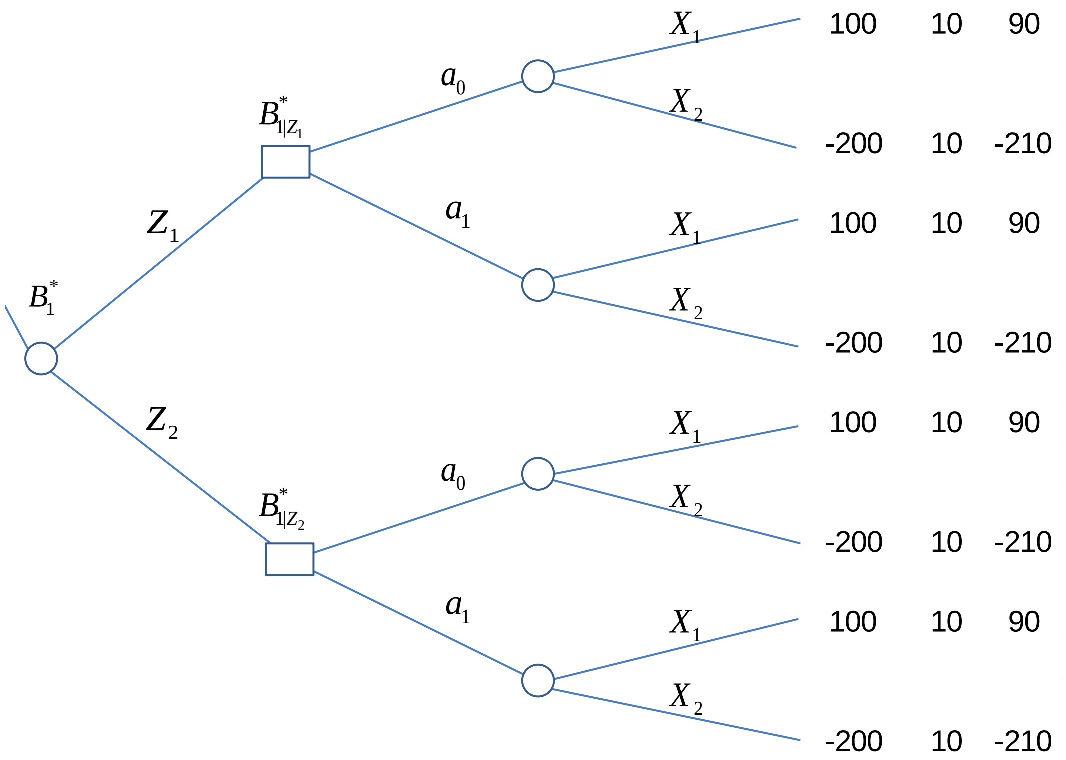
Branch eliminating by separation



Decision tree 1, Normal form analysis

$$B_{1,1}^* = C_{e_1}$$
$$+\left(P\left(Z_1 \mid X_1\right)\cdot P\left(X_1\right)+P\left(Z_1 \mid X_2\right)\cdot P\left(X_2\right)\right)\cdot b\left(X_1\right)$$
$$+\left(P\left(Z_2 \mid X_1\right)\cdot P\left(X_1\right)+P\left(Z_2 \mid X_2\right)\cdot P\left(X_2\right)\right)\cdot b\left(X_2\right)$$

Branch eliminating by separation

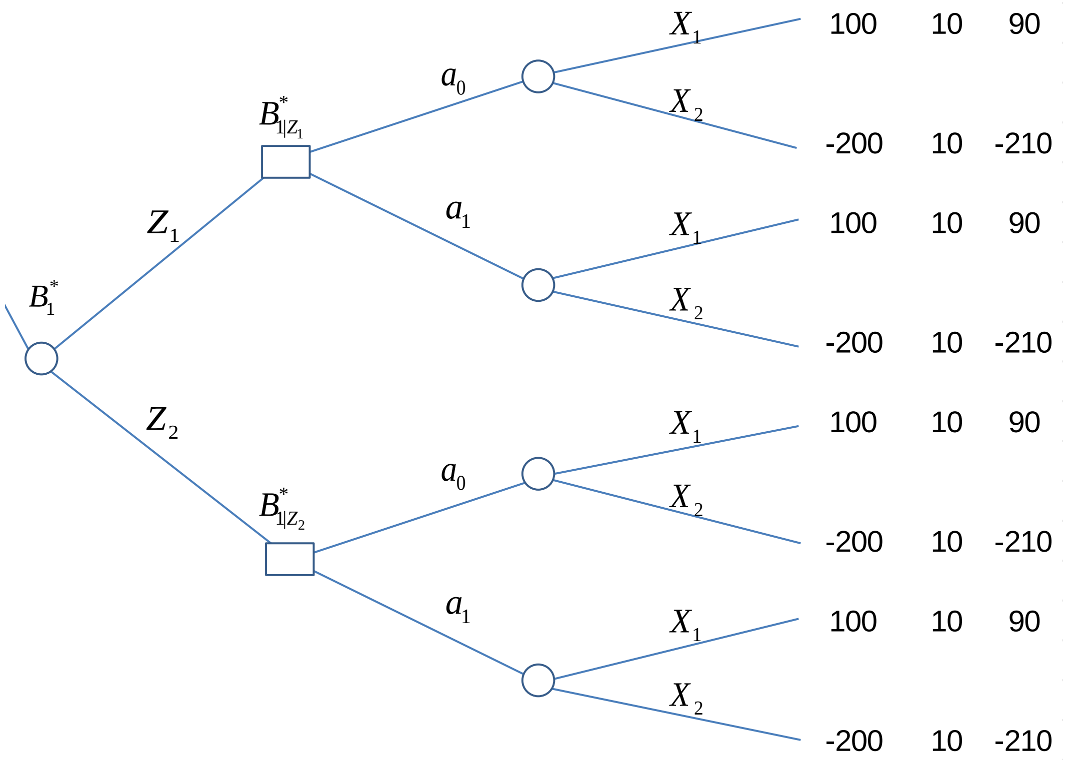


Decision tree 1, Normal form analysis

Definition of the conditional probability and commutative law of intersection operator, e.g.:

$$P(Z_1 \mid X_1) \cdot P(X_1) = P(Z_1 \cap X_1) =$$
$$P(X_1 \cap Z_1) = P(X_1 \mid Z_1) \cdot P(Z_1)$$

Branch eliminating by separation



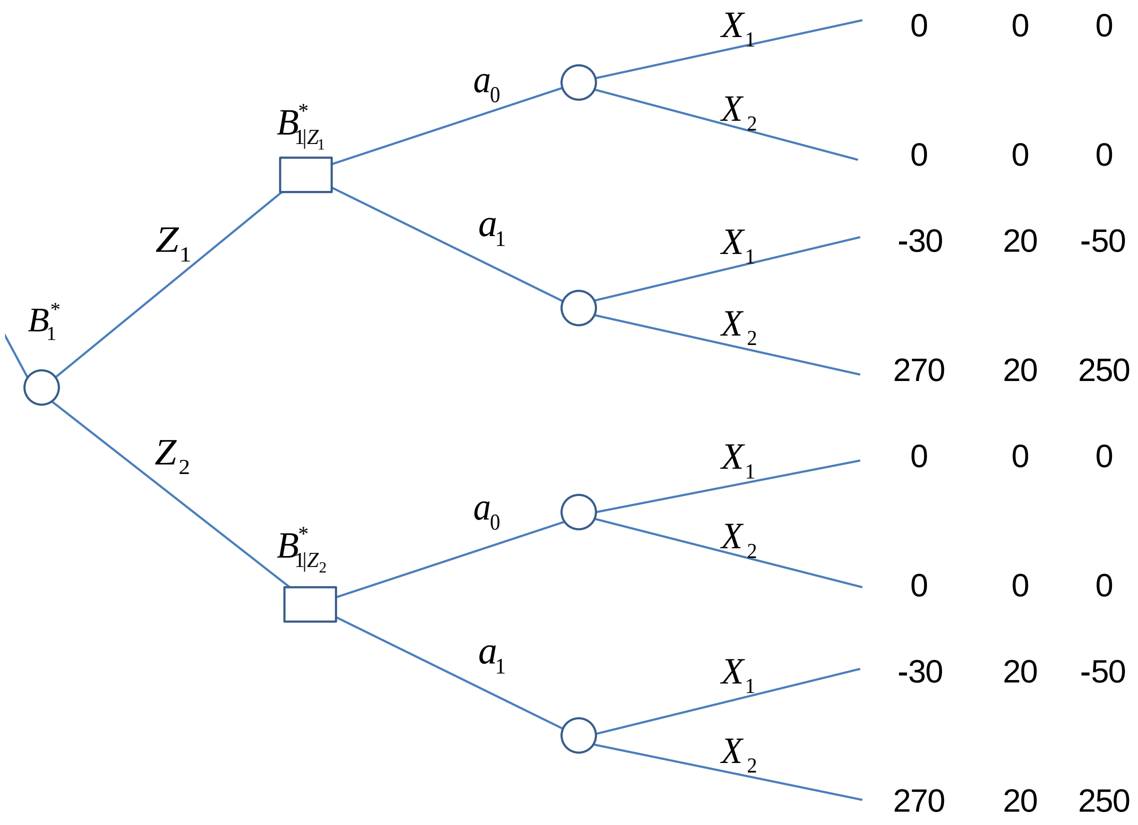
Only system state consequences and experimental costs are considered.

$$B_{1,1}^* = C_{e_1}$$
$$+\left(P(Z_1)\cdot P(X_1 | Z_1) + P(Z_2)\cdot P(X_1 | Z_2)\right)\cdot b(X_1)$$
$$+\left(P(Z_1)\cdot P(X_2 | Z_1) + P(Z_2)\cdot P(X_2 | Z_2)\right)\cdot b(X_2)$$

$$B_{1,1}^* = C_{e_1} + P(X_1)\cdot b(X_1) + P(X_2)\cdot b(X_2)$$

$$B_{1,1}^* = -10 + 0.8\cdot 100 + 0.2\cdot(-200) = 30.0$$

Branch eliminating by separation



Decision tree 2 is only associated to action dependent costs or benefits.

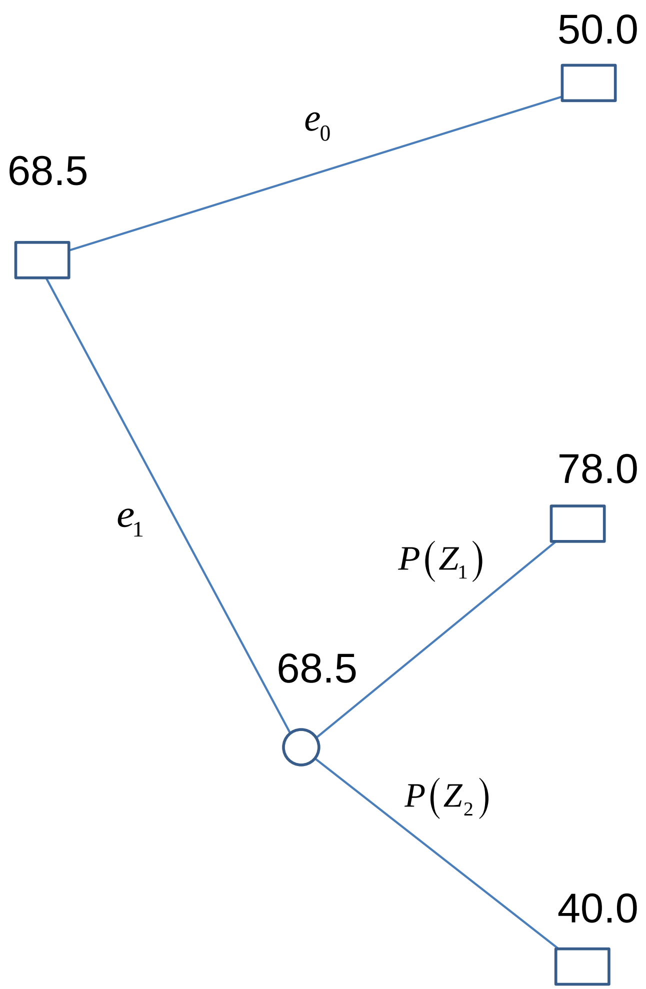
Normal form with decision rule:

$$\mathbf{d}\begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$B_{1,2}^* = P(Z_2 \mid X_1) \cdot P(X_1) \cdot b(a_1, X_1(a_1)) + P(Z_2 \mid X_2) \cdot P(X_2) \cdot b(a_1, X_2(a_1))$$

$$B_{1,2}^* = 0.1 \cdot 0.8 \cdot (-50) + 0.85 \cdot 0.2 \cdot 250 = 38.5$$

Branch eliminating by separation



$$B_1^* = B_{1,1}^* + B_{1,2}^* = 30.0 + 38.5 = 68.5$$

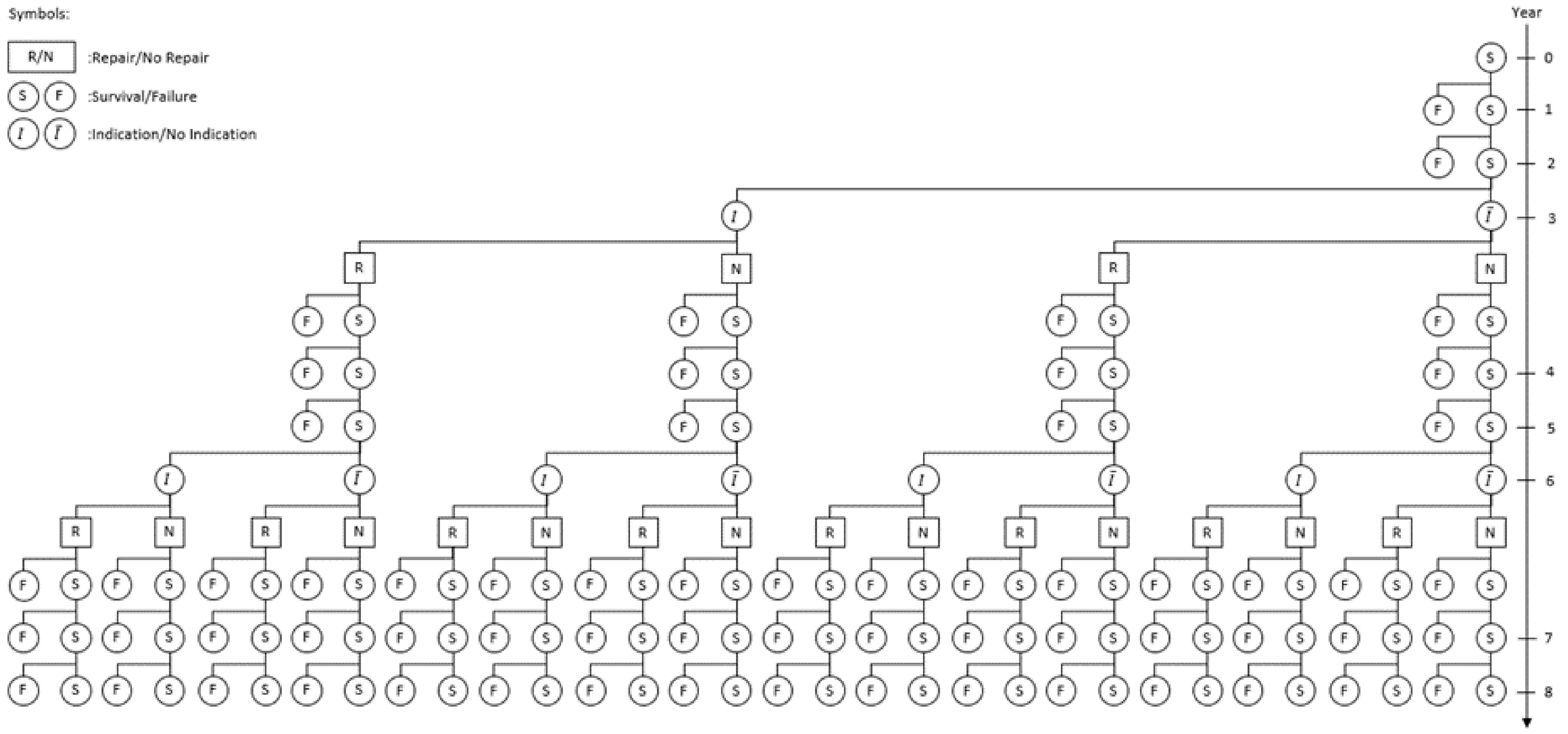
Reliability and risk based inspection planning

The planning of inspections should account for structural condition an should be optimised over the service life: Value of Information challenge.

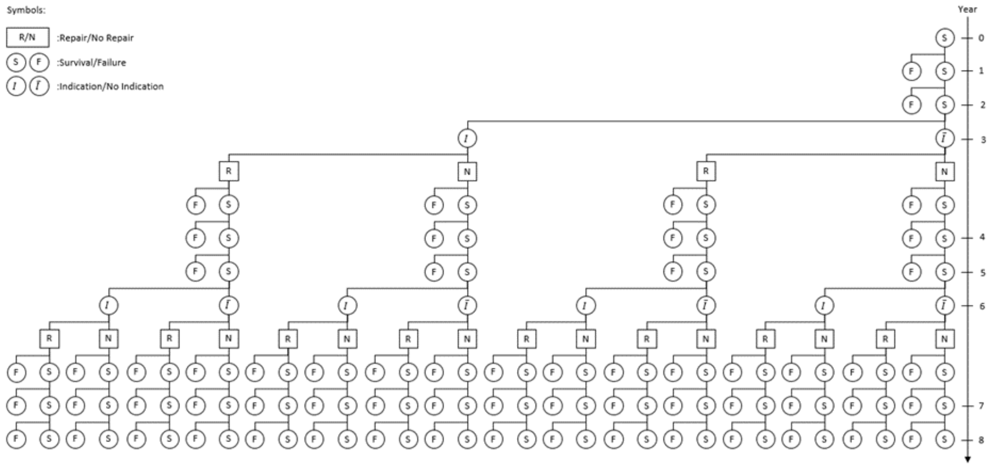
Let us have a look to an example:

- A component with 8 years of service life with 2 inspections
- The probabilities of safe and failure states are described with a fracture mechanics (FM) model
- An inspection provides information about the presence of a crack (indication or no indication) – which is predicted by the FM model

Reliability and risk based inspection planning



Reliability and risk based inspection planning

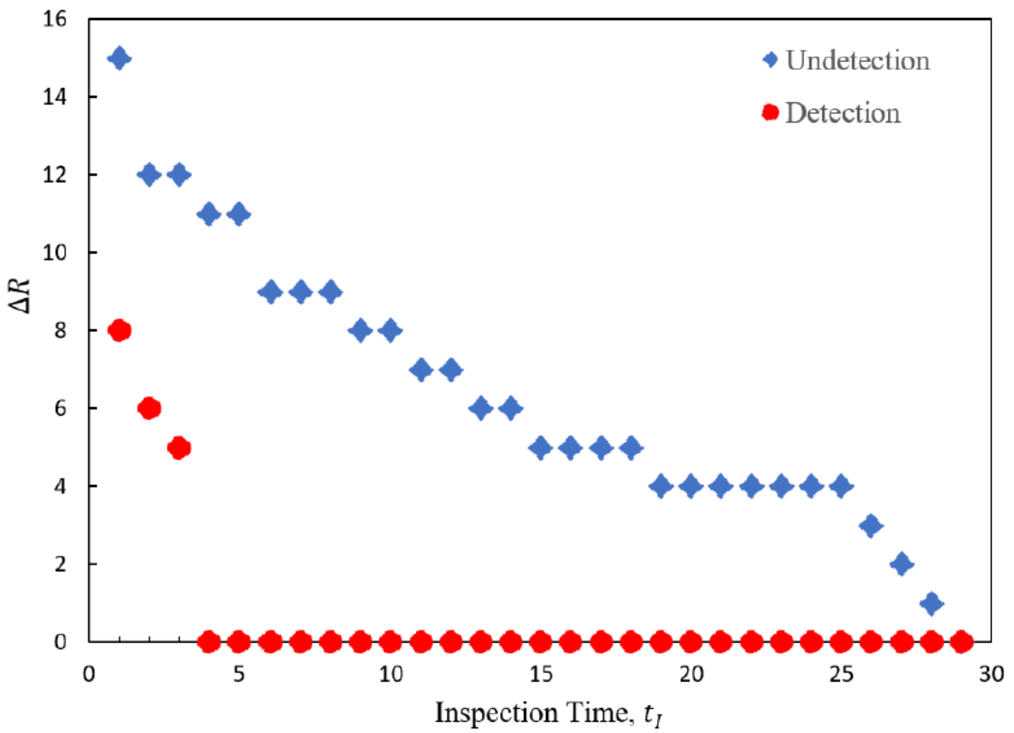


The number of branches increases exponentially with the inspection times and proportionally with the number of inspection outcomes and actions.

$$n_{branch} = \left(n_I \cdot n_a\right)^{n_{ins}}$$

We have a computational challenge. What can we do?

Reliability and risk based inspection planning



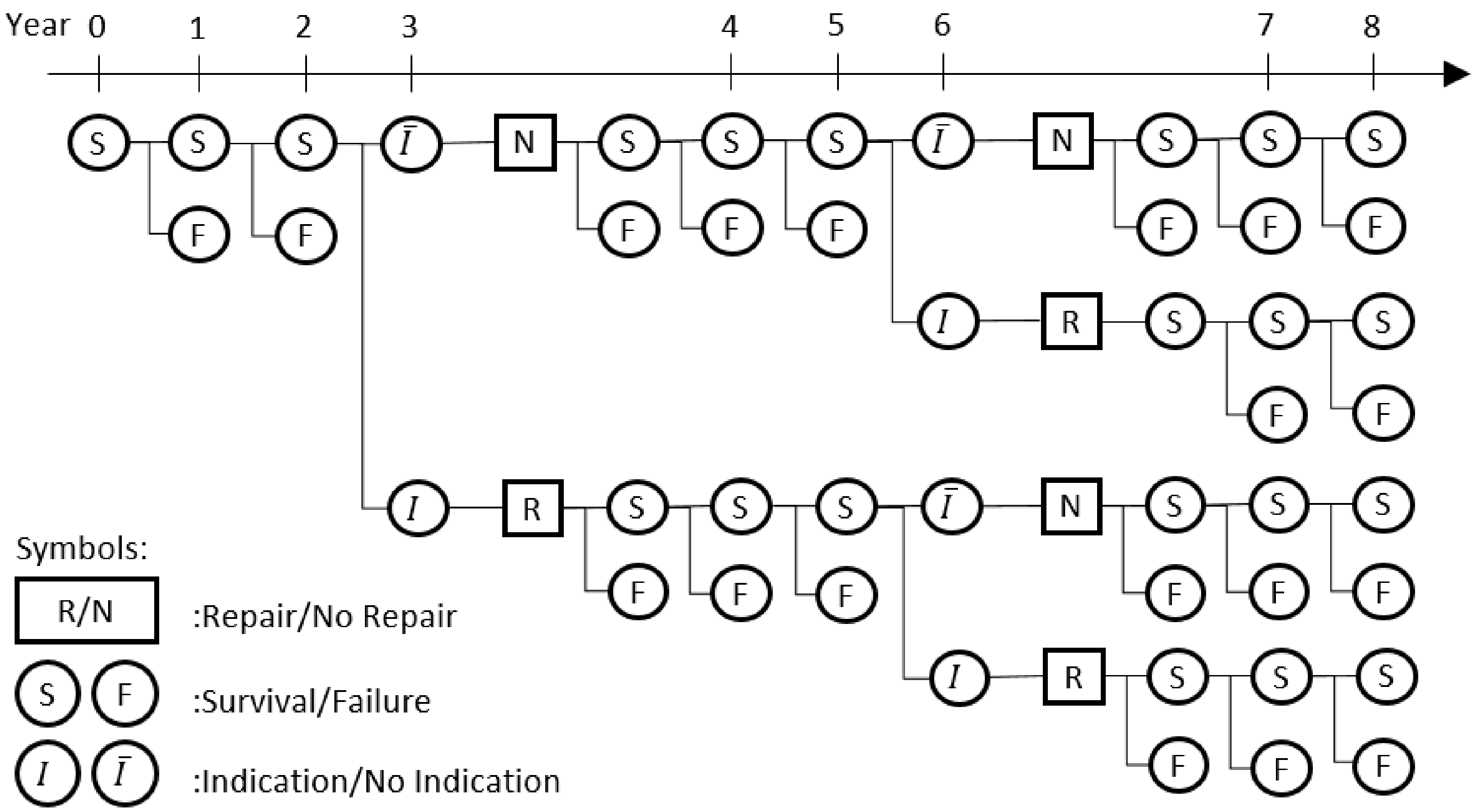
Irman, A. A., S. Thöns and B. J. Leira (2017). Value of information-based inspection planning for offshore structures. 36th International Conference on Ocean, Offshore and Artic Engineering (OMAE), Trondheim, Norway, 25-30 June, 2017.

Introduction of a decision rule: A repair is performed immediately after an indication of a crack.

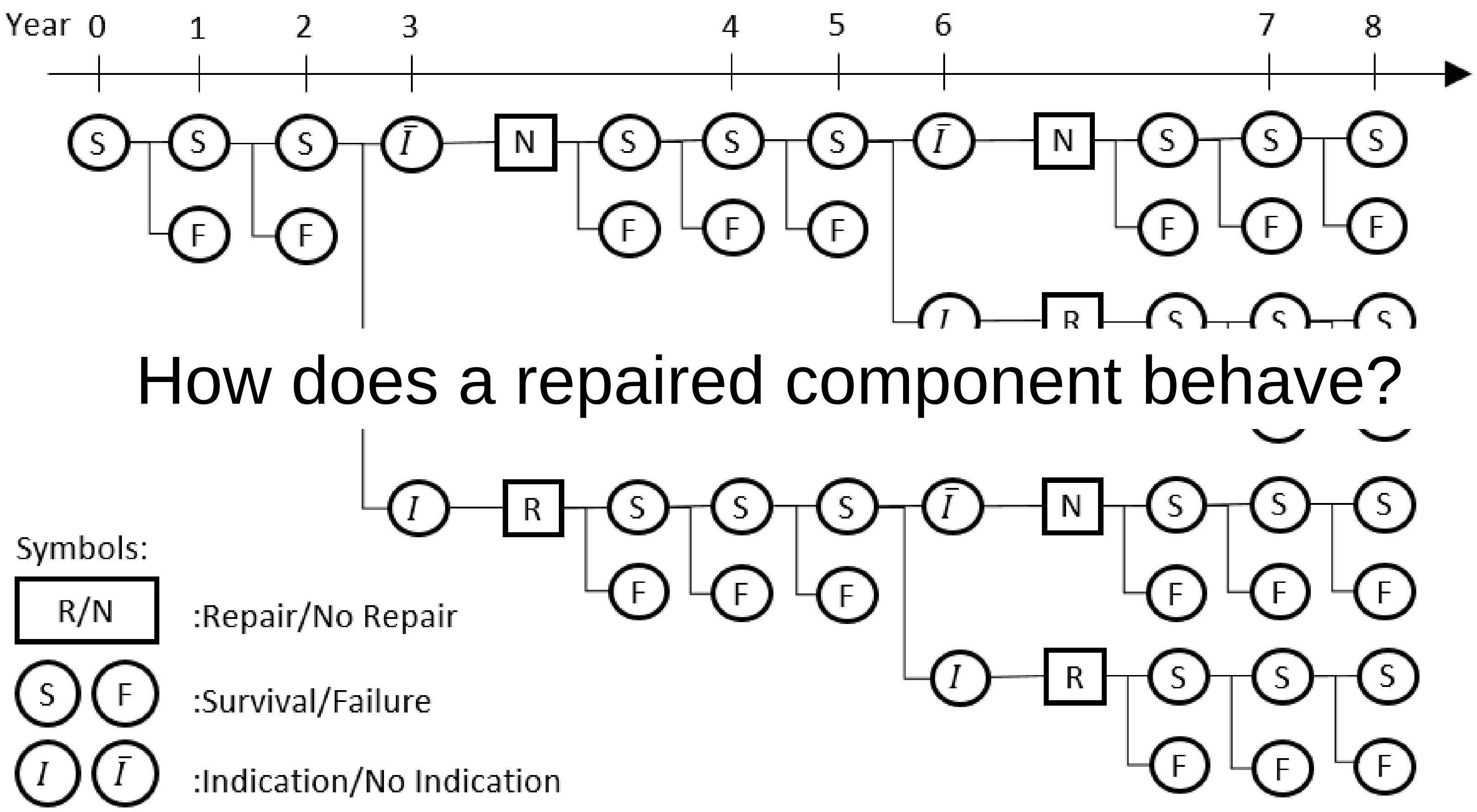
Can this heuristic be substantiated?

1. It is Value of Information optimal.
2. It is common practice in industry.

Reliability and risk based inspection planning

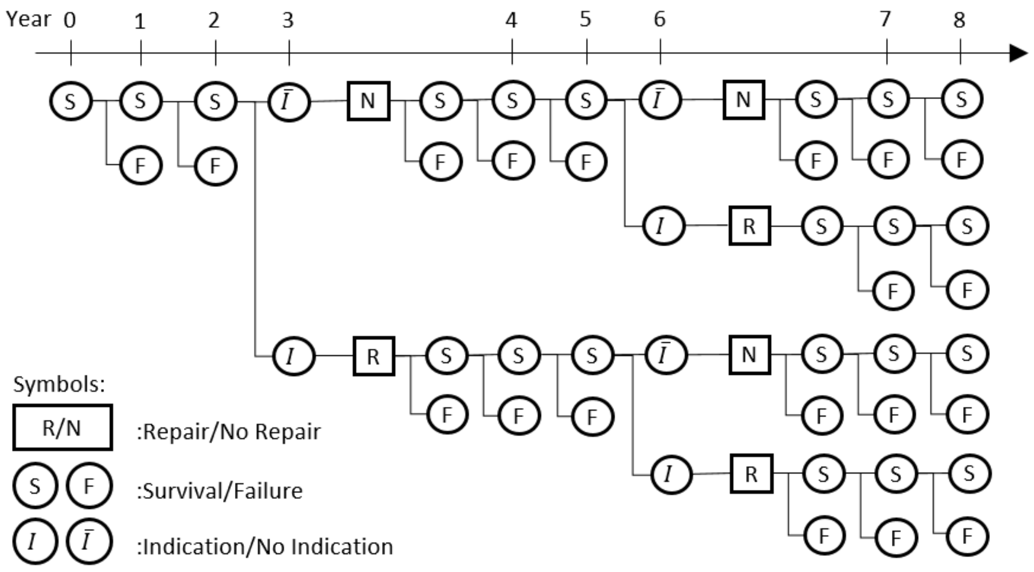


Reliability and risk based inspection planning



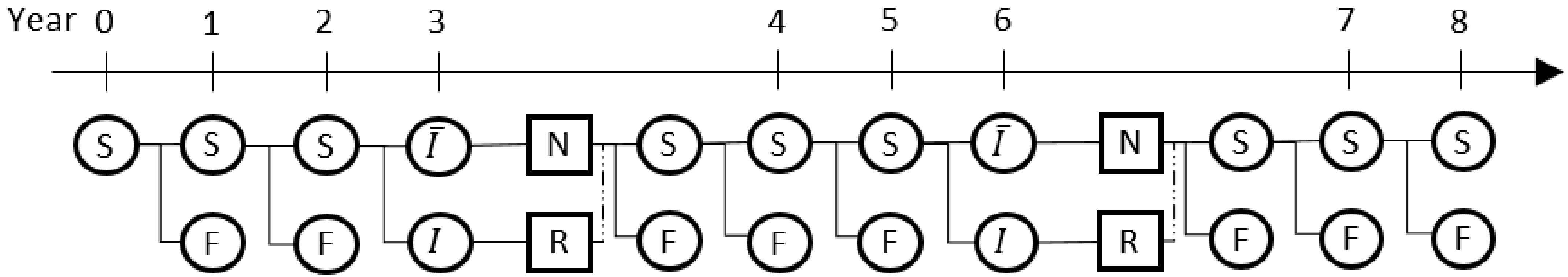
How does a repaired component behave?

Reliability and risk based inspection planning



A repaired component may behave like a component with no finding.

Reliability and risk based inspection planning



Reliability and risk based inspection planning: further reading

Faber, M. H., S. Engelund, J. D. Sørensen and A. Bloch (2000). Simplified and Generic Risk Based Inspection Planning. Proceedings OMAE2000, 19th Conference on Offshore Mechanics and Arctic Engineering, New Orleans, Louisiana, USA.

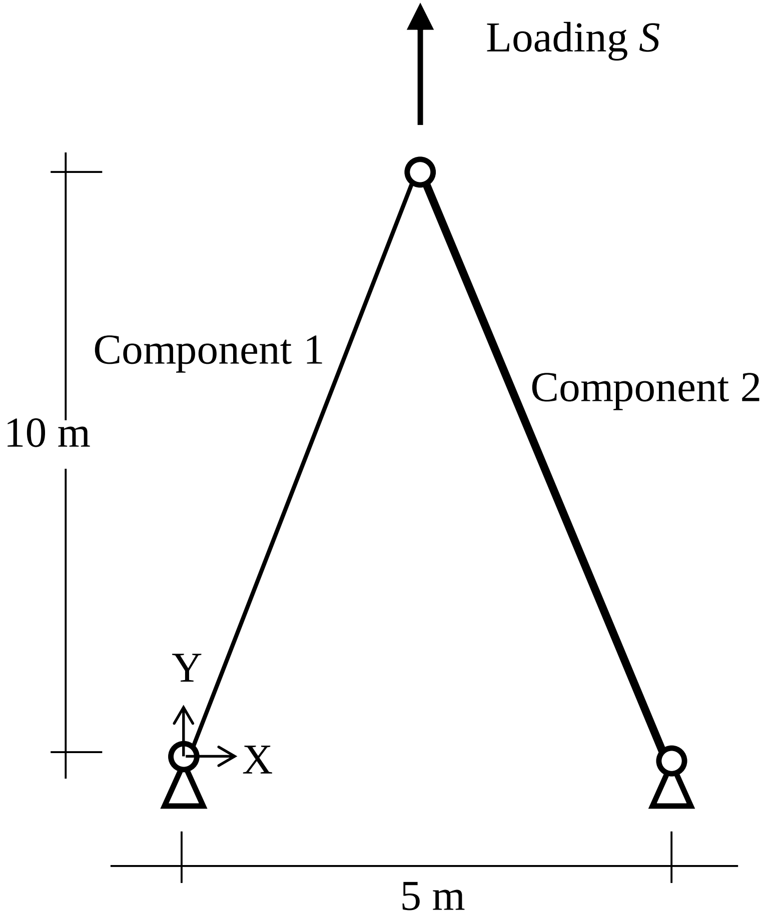
Straub, D. (2004). Generic Approaches to Risk Based Inspection Planning for Steel Structures. PhD. thesis. Chair of Risk and Safety, Institute of Structural Engineering. ETH Zürich.

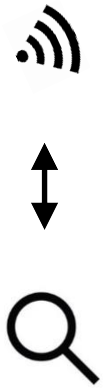
Further reading

Raiffa, H. and R. Schlaifer (1961). Applied statistical decision theory. New York, Wiley (2000). ISBN: 047138349X.

Benjamin, J. R. and C. A. Cornell (1970). Probability, Statistics and Decision for Civil Engineers, McGraw-Hill, New York. ISBN: 070045496.

Value of Information analysis: Task

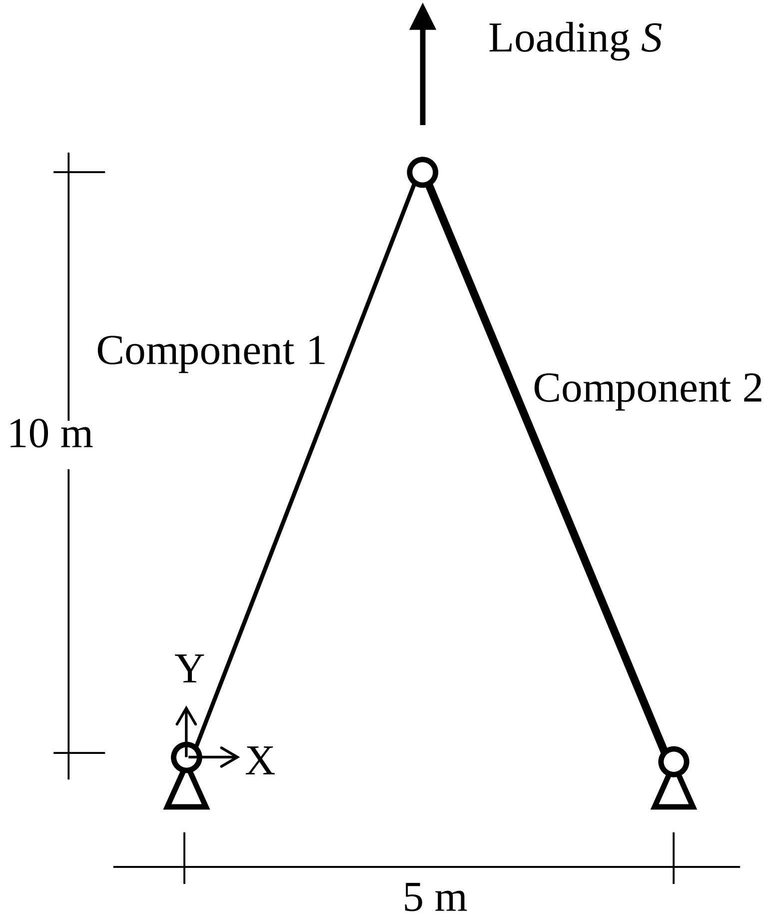




The system on the left experiences high consequences in case of system damage. A mitigation can be implemented with a cost of 2.5 leading to a reduction of 5 of the system damage consequences.

Which information acquirement strategy leads to the highest Value of Information: an inspection, damage detection or monitoring?

Value of Information analysis: Task



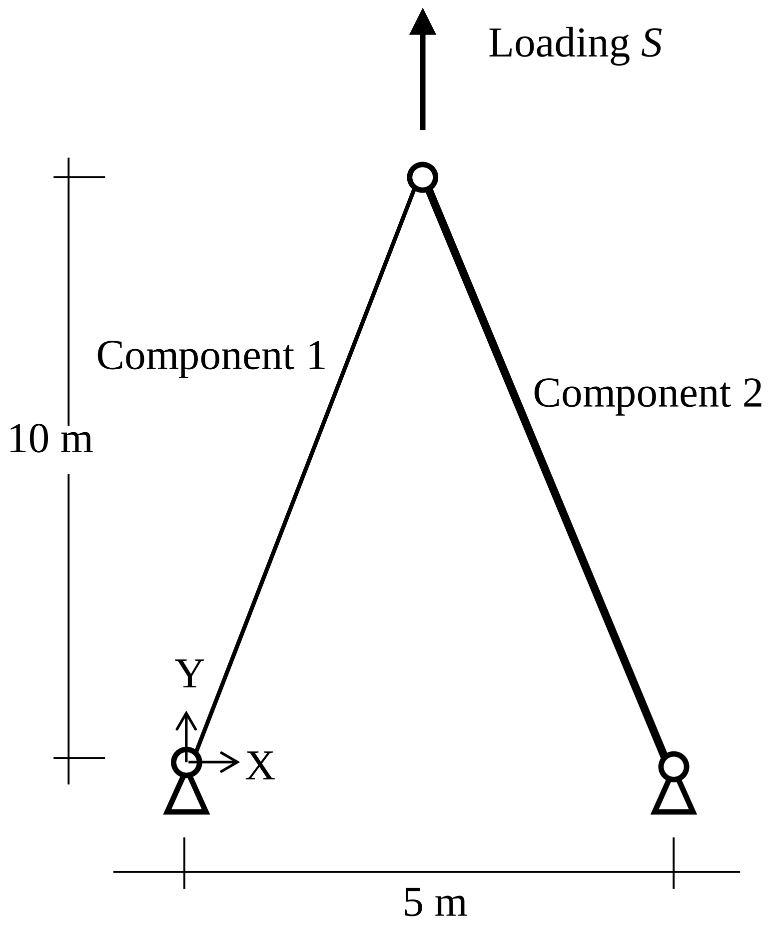
Damage development (uncorrelated; in mm) with model uncertainty (fully correlated):

$$D_2 \sim N(7.0, 1.5)$$

$$M_D \sim LN(1.0, 0.2)$$

Damage resistance (fully correlated): Lognormal distribution with a standard deviation of 1.0 mm also including model uncertainties.

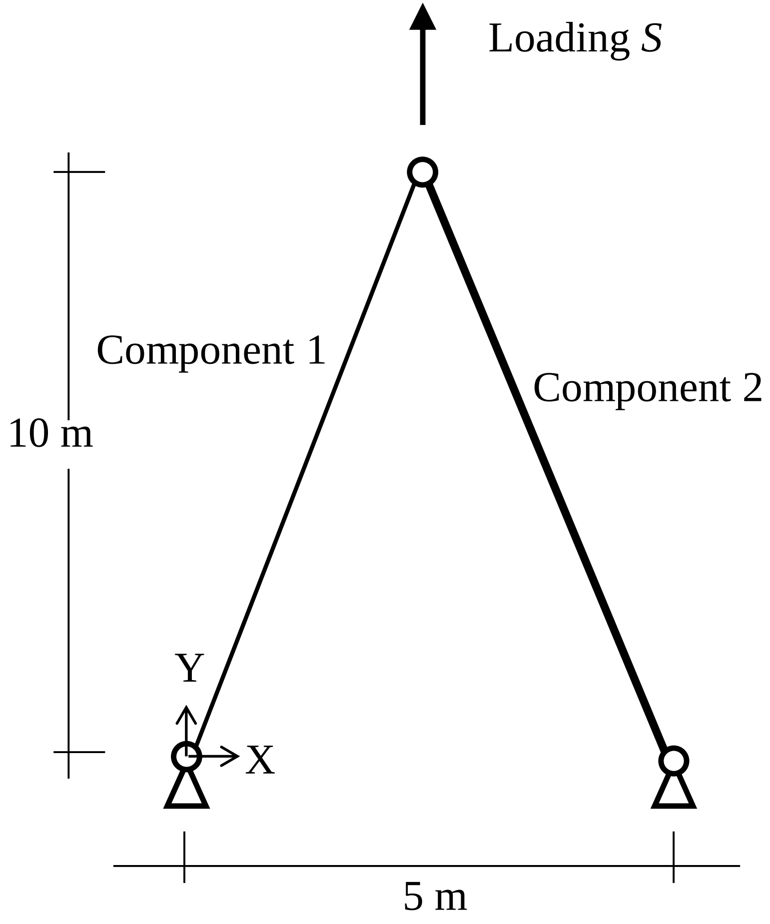
Value of Information analysis: Task

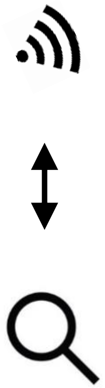


A probabilistic model for the damage of component 1 is to developed by using the observations below and the Maximum Likelihood method.

5.19	3.83
6.80	3.20
4.63	4.17
5.07	5.43
4.84	3.95
6.14	4.64
4.14	1.84
6.40	3.68
5.02	5.82
4.48	5.78

Value of Information analysis: Task





Assume the following consequences and costs.

- System damage: 20.0
- Damage of one component: 1.0

- Inspection of one component: 0.001
- Damage detection system: 0.005
- Monitoring of one component: 0.0005

Thank you for your attention.

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Engineering Risk and Decision Analysis