Adaptation of structural reliability with measurement information

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Day 2: Structural reliability and measurements

Adaptation of structural reliability with measurement information

- Probabilistic modelling of measurements
  - NDE/NDT measurement performance modelling
  - Damage detection system performance
  - Quantification of measurement uncertainty

- Updating of structural reliability
  - Bayesian updating with indication information
  - Modelling of SHM outcomes

Probabilistic modelling of measurements

Sources of uncertainties for measurements are the measurement technology, the measurement process and human errors.

Measurement technology and process

- E.g. for strain measurements: resistance measurement, amplification, conversion to stresses, data analysis

Model uncertainties

- For calculating the structural performance property

Human errors

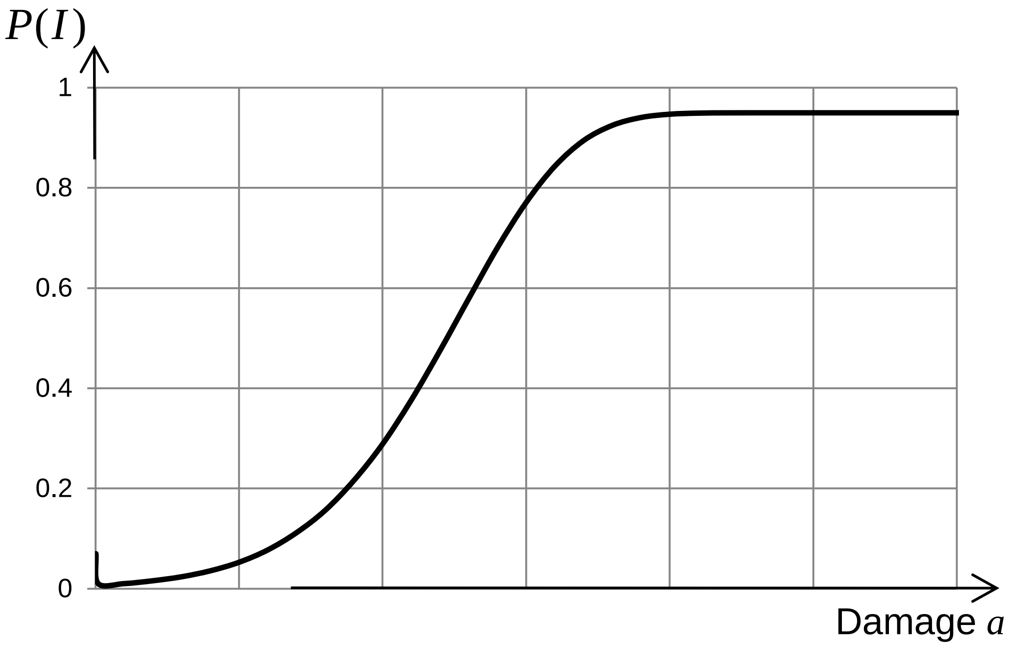
- Position of strain gauges, data analysis

NDE&NDT measurement performance

The NDE&NDT measurement performance is described with the probability of indication (or detection) and the probability of false alarm and the complementary events.

- Probability of indication given a damage size a: $P(I \mid a)$
- Probability of false alarm: $P(I \mid a = 0)$
- Complementary events: $P(\bar{I} \mid a), P(\bar{I} \mid a = 0)$

NDE&NDT measurement performance



The probability of indication/detection (PoD) can shown in a diagram for each damage size.

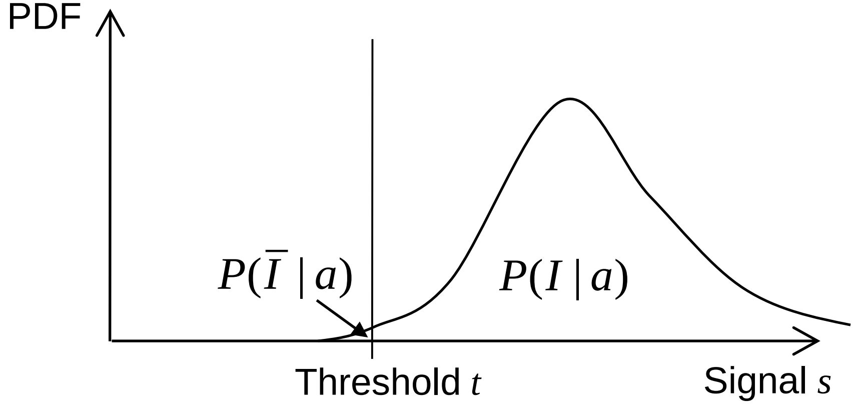
- Contains also the probability of false alarm

The PoD curves may be established based on

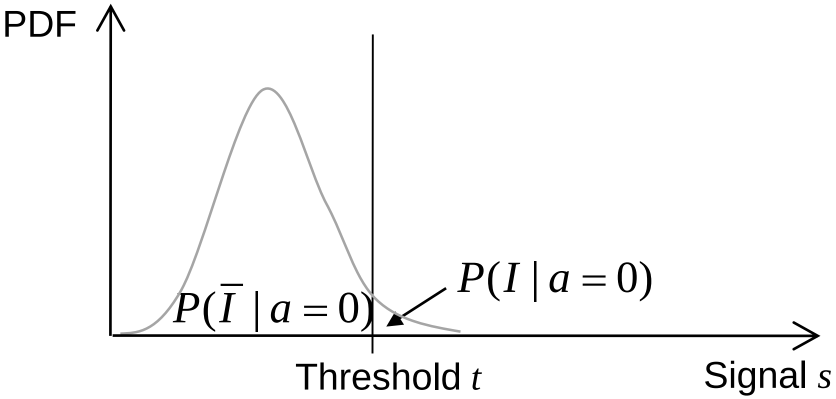
- Simulations
- Round robin tests (interlaboratory test performed independently several times) cover uncertainties associated to measurement technology, the measurement process and human errors.

NDE&NDT measurement performance

Damage state



Reference state



The probabilities of the events can be calculated with the distribution of the signal.

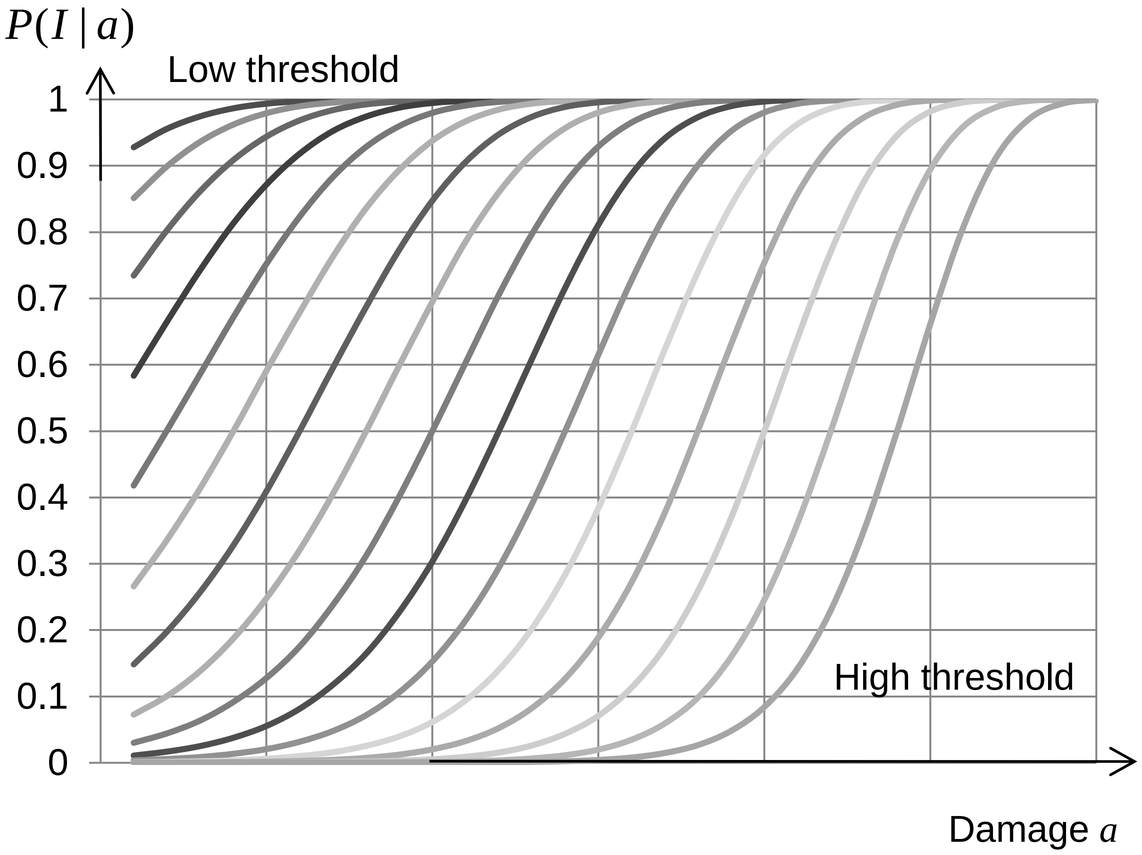
$$P(I \mid a) = \int_t^{\infty} f\left(s \mid a\right) ds$$

$$P(\bar{I} \mid a) = \int_{-\infty}^{t} f\left(s \mid a\right) ds = 1 - P(I \mid a)$$

$$P(I \mid a=0) = \int_t^{\infty} f\left(s \mid a=0\right) ds$$

$$P(\bar{I} \mid a=0) = \int_{-\infty}^{t} f\left(s \mid a=0\right) ds = 1 - P(I \mid a=0)$$

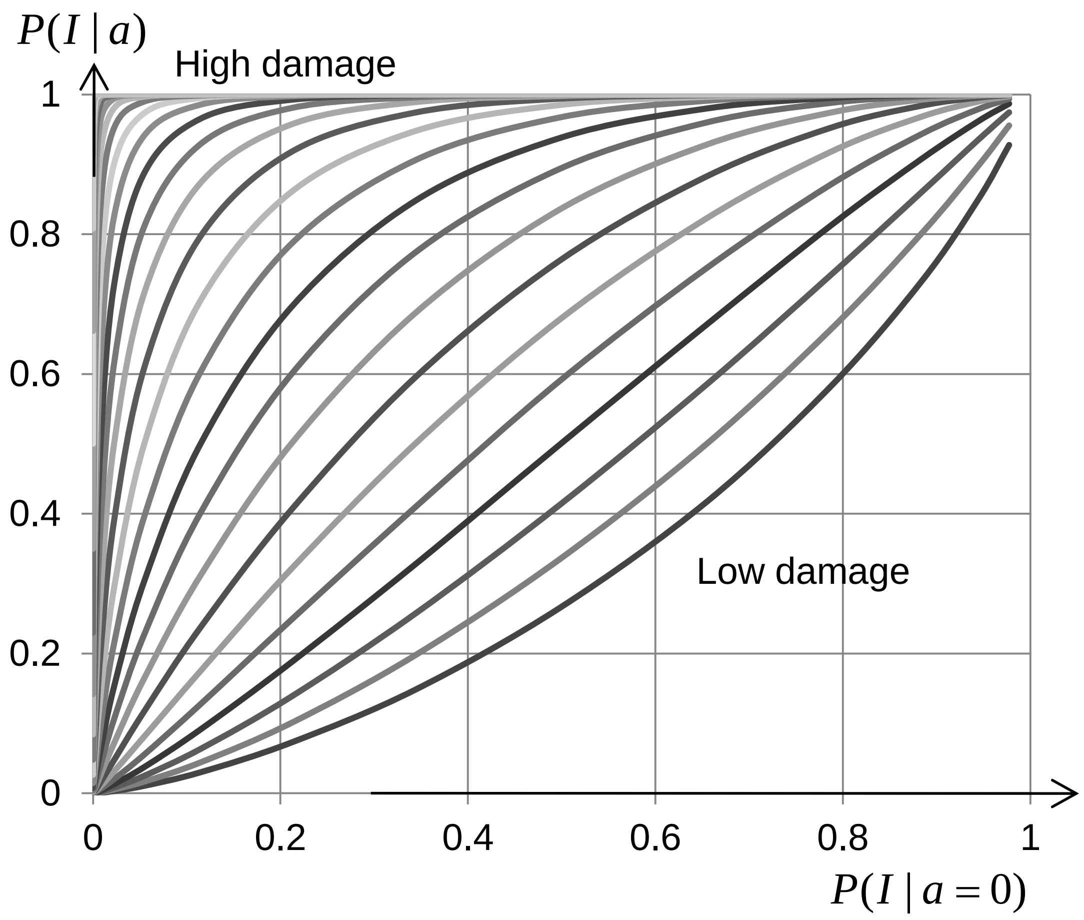
NDE&NDT measurement performance



Probability of indication/detection in dependency of the threshold.

- High threshold causes a shift of the curve towards higher damages
- Higher inclination of the curve towards higher damages

NDE&NDT measurement performance

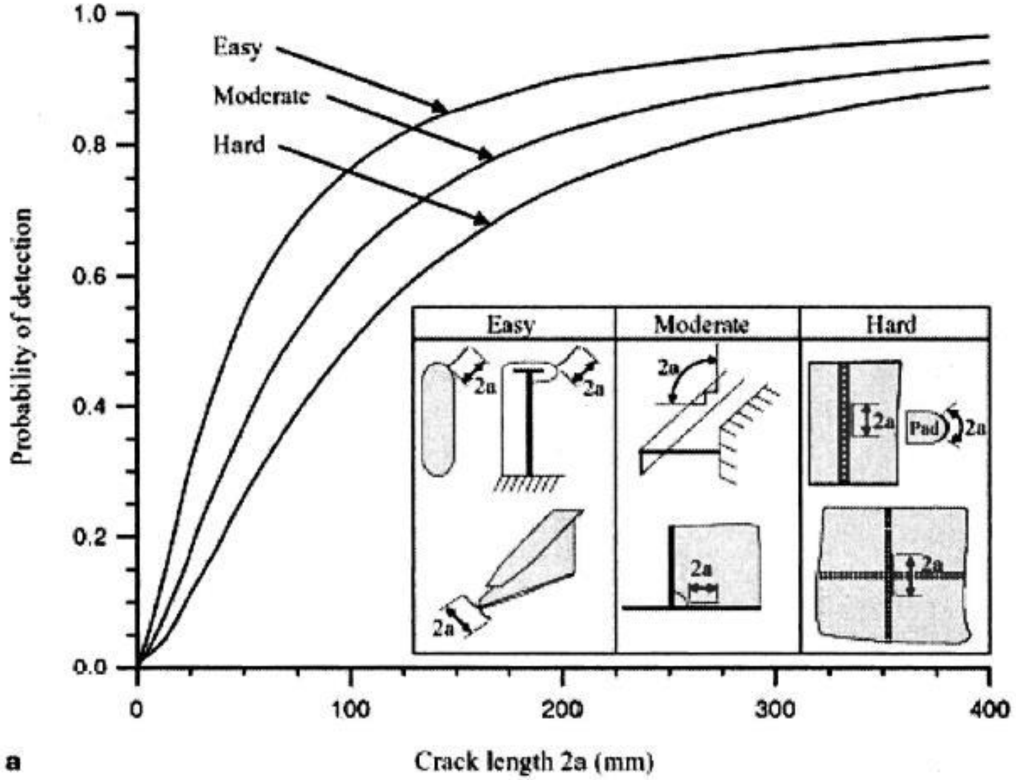


The receiver operating characteristics (ROC) shows the probability of indication vs. the probability of false alarm for a constant damage size.

- The higher the damage sizes, the lower are the associated probabilities of false alarm.

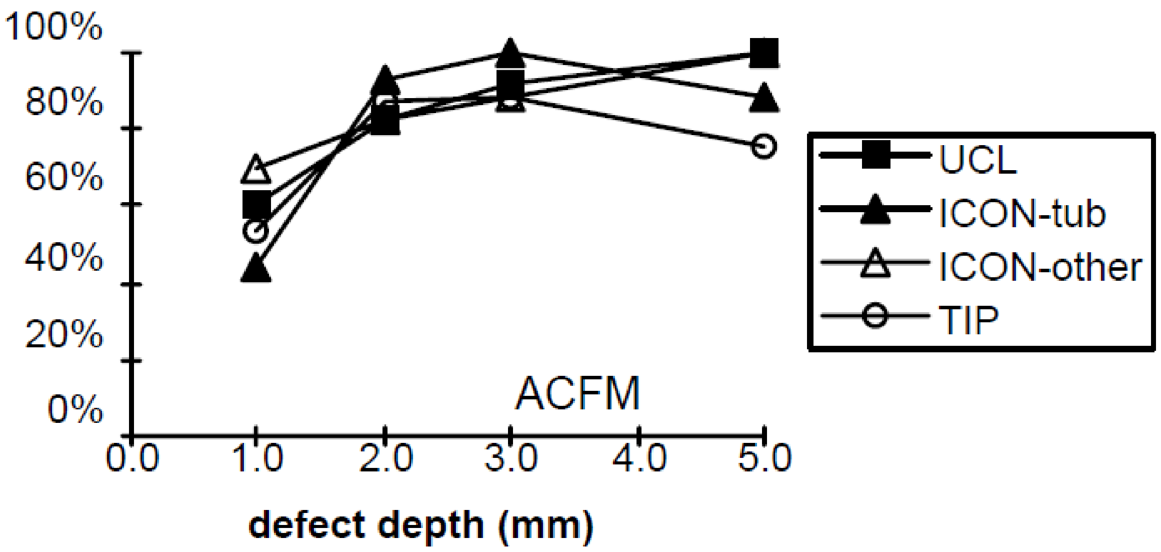
NDE&NDT measurement performance: Examples

Visual



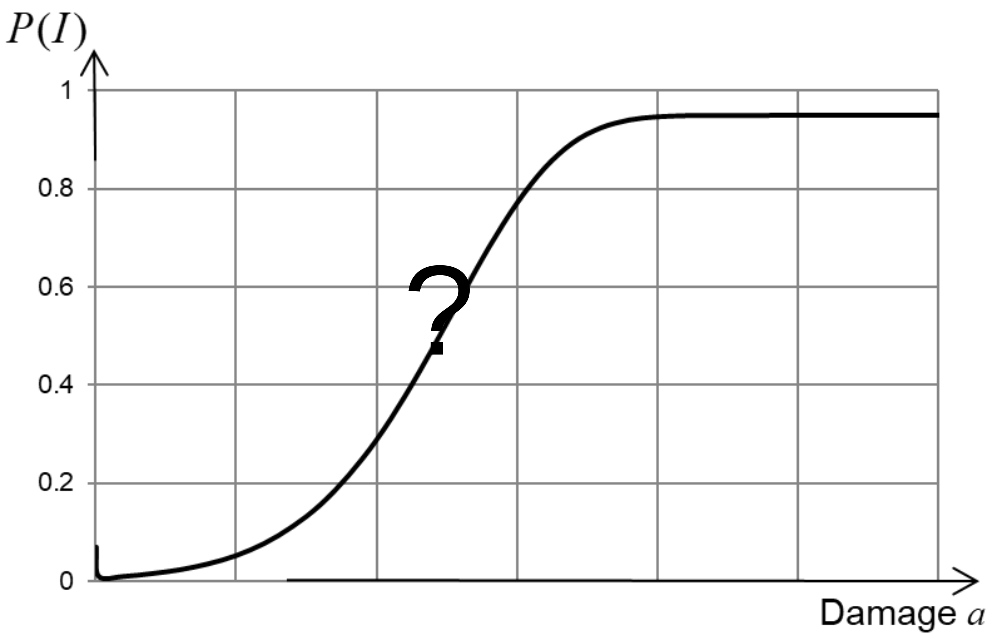
Paik, J. K. and A. K. Thayamballi (2007). Ship-Shaped Offshore Installations - Design, Building, and Operation. New York, USA, Cambridge University Press.

Alternating Current Field Maesurement (ACFM)



Visser Consultancy Limited (2000). POD/POS curves for non-destructive examination. Offshore technology report. Offshore Technology Report 2000/018.

NDE&NDT measurement performance: Task 1



Calculate the probability of indication curve.

The distribution of signal in dependency of the damage size is given:

$$S \sim N(\mu_S, \sigma_S)$$

$$\mu_S = 0.7 + 0.1 \cdot a$$

$$\sigma_S = 0.5 - 0.1 \cdot a$$

$$a = 0.0 \ldots 10.0 \quad mm$$

The noise distribution is given with:

$$S_R \sim N(1.0, 0.5)$$

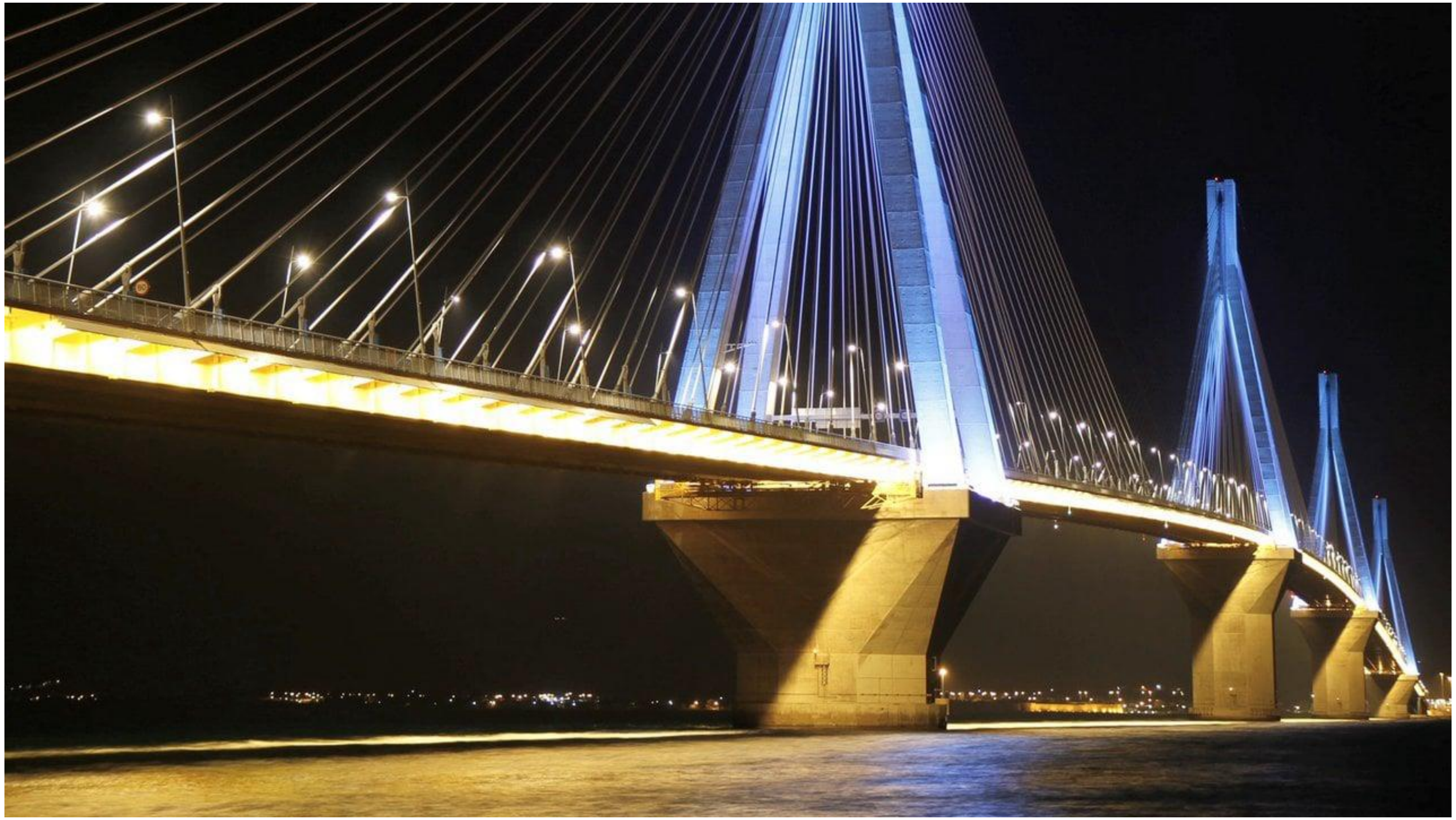
Set the threshold to 1.5.

NDE&NDT measurement performance: Further reading

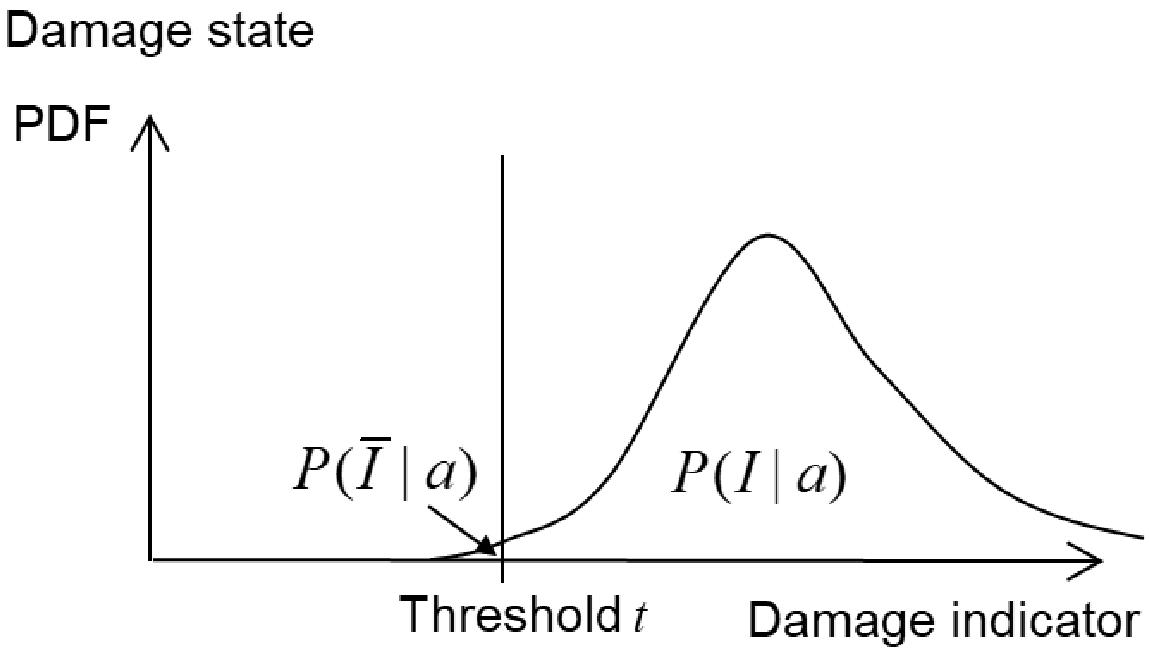
Visser Consultancy Limited (2000). POD/POS curves for non-destructive examination. Offshore technology report. Offshore Technology Report 2000/018.

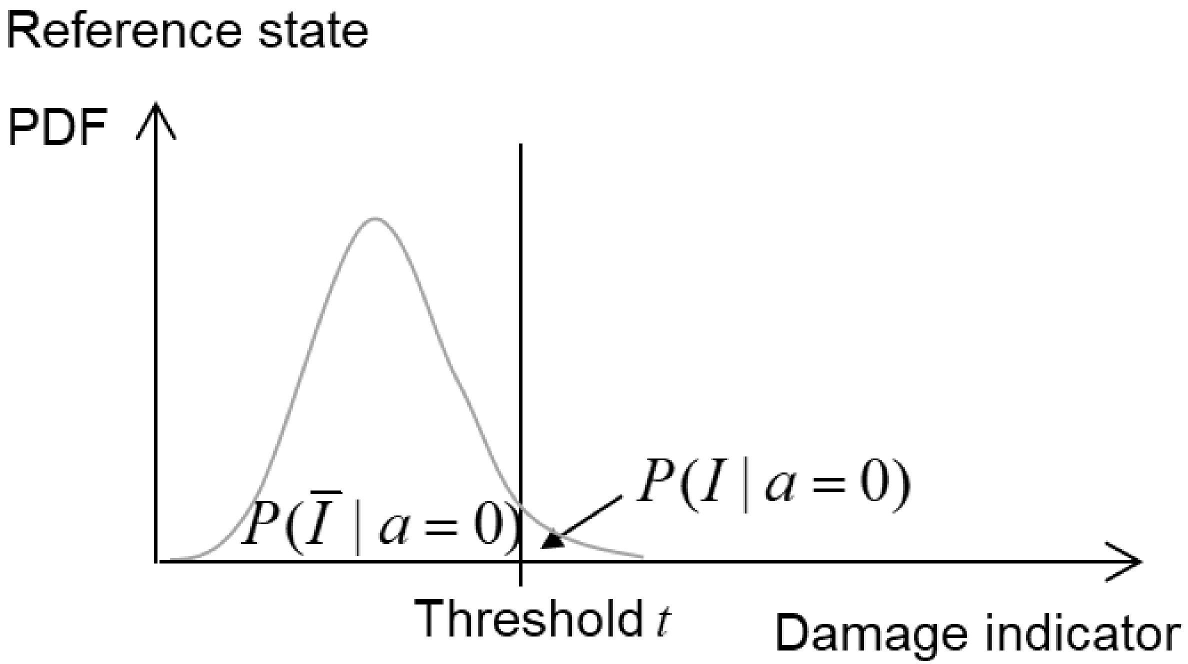
Gandossi, L. and C. Annis (2010). Probability of Detection Curves: Statistical Best-Practices, European Commission, Joint Research Centre, Institute for Energy. ISBN: 978-92-79-16105-6.

Damage detection system performance

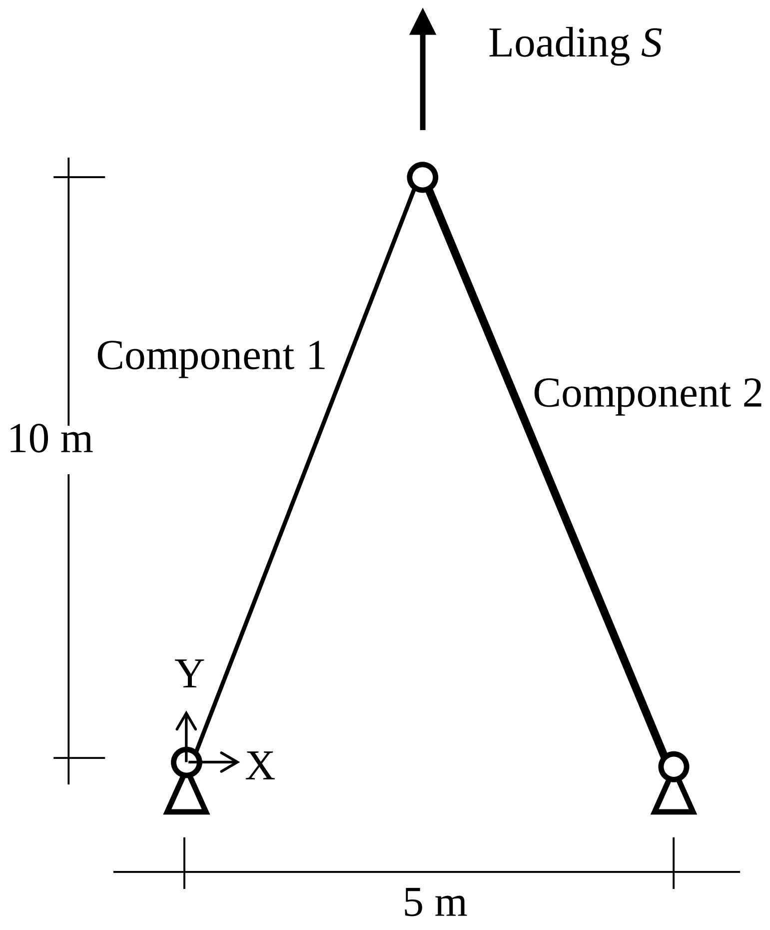


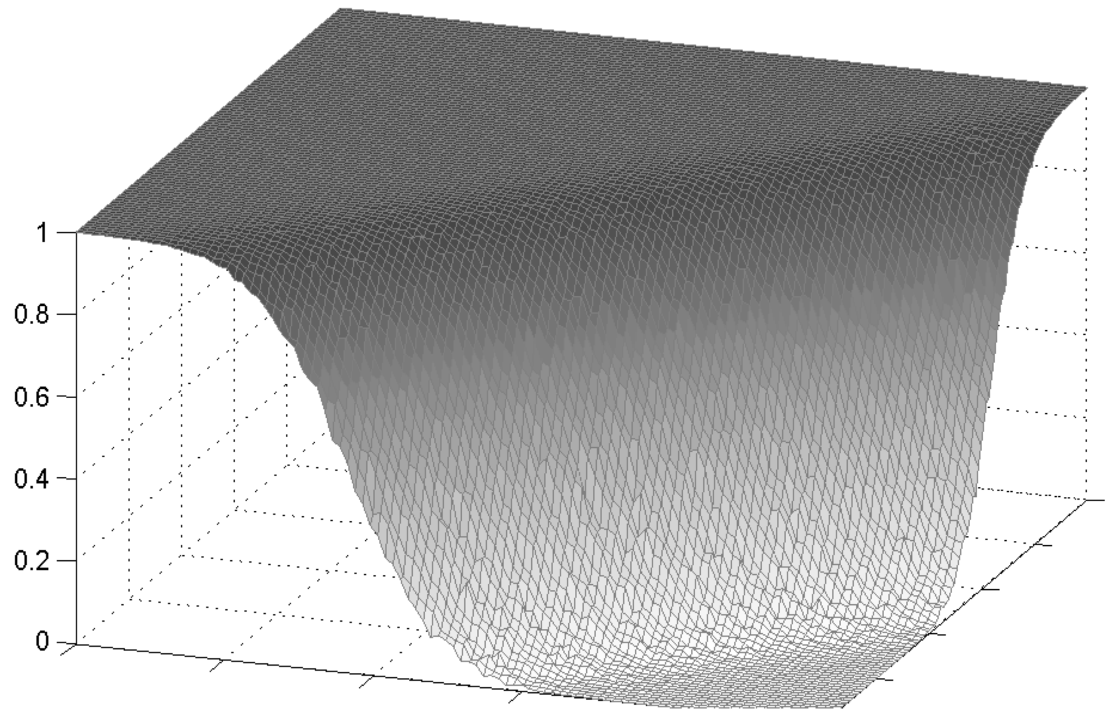
Damage detection system performance



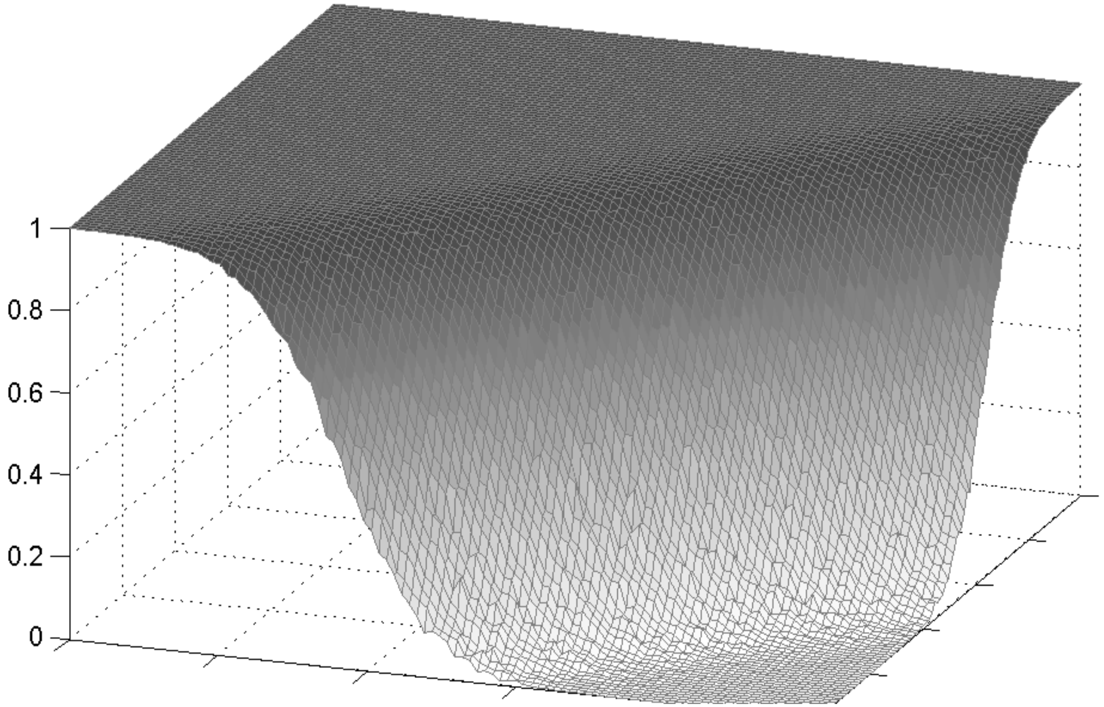


Damage detection system performance





Damage detection system performance



$$P(I \mid a_1, a_2) = \int_t^{\infty} f\left(s \mid a_1, a_2\right) ds$$

$$P(I \mid a_1 = a_2 = 0) = \int_t^{\infty} f\left(s \mid a_1 = a_2 = 0\right) ds$$

$$P(I \mid a_1, a_2 = 0) = \int_t^{\infty} f\left(s \mid a_1, a_2 = 0\right) ds$$

$$P(I \mid a_1 = 0, a_2) = \int_t^{\infty} f\left(s \mid a_1 = 0, a_2\right) ds$$

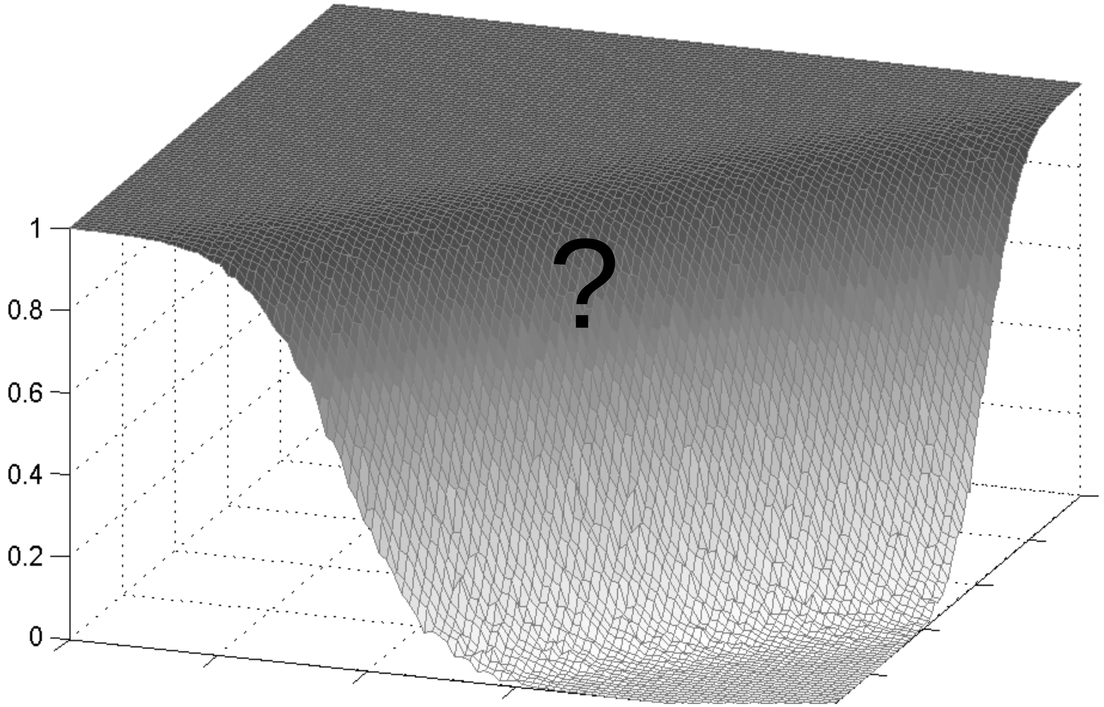
Damage detection system performance

1. The NDE and NDT performance modelling can be extended to damage indicators.

2. For a measurement system where the signal depend on the response of the structural system: the measurement information is structural system level.

3. Determination of the joint probability of indication may only be possible by simulation and direct calculation.

Damage detection system performance: Task 2



Calculate the joint probability of indication.

- Take basis in the NDE/NDT signal modelling
- Define Multivariate Normal distributions for the noise and signal distribution.
- Noise model

$$s_R \sim N_2\left(\left[1.0, 1.0\right], \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}\right)$$

Probabilistic modelling of measurements

Sources of uncertainties for measurements are the measurement technology, the measurement process and human errors.

Measurement technology and process

- E.g. for strain measurements: resistance measurement, amplification, conversion to stresses, data analysis

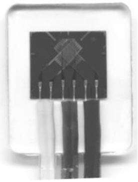
Model uncertainties

- For calculating the structural performance property

Human errors

- Position of strain gauges, data analysis

Measurement uncertainties



The measurement uncertainty can be determined with a process equation and with observations.

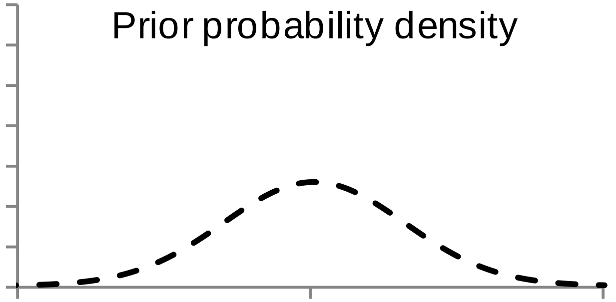
The process equation describes the measurement process to come to a measurand Y with a probabilistic physical model including the measurement equation E and model uncertainties M.

The measurand Y can also be determined with observations and the measurement equation E subjected to a model uncertainty M.

Measurement uncertainties

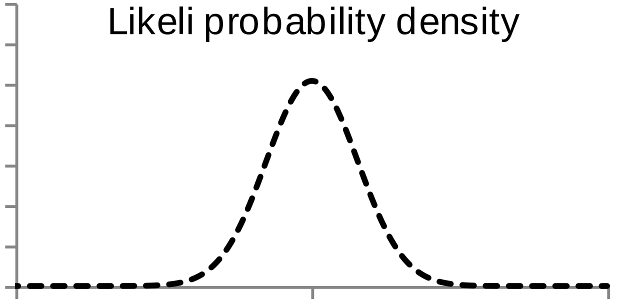
Process equation

$$Y \sim F\left(M_E, E\left(\mathbf{X}_E\right), M_P, p\left(\mathbf{X}_p\right)\right)$$



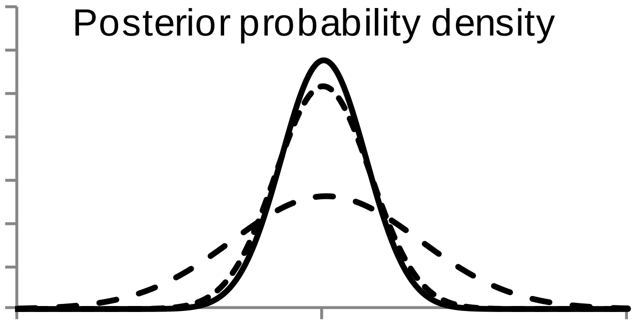
Observations

$$L\left(Y\right) = F\left(M_E, E\left(\hat{\mathbf{x}}_E\right)\right)$$

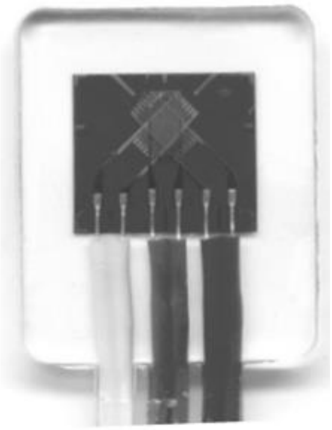


Posterior measurement uncertainty

$$f''\left(Y\right) = f'\left(Y\right) \cdot L\left(Y\right) \cdot const$$



Measurement uncertainties: strain gauge measurements



$$E_{mech} = M_E + E_{amp} + E_{app}$$

Measurement equation

- E_{amp}: strain which is measured with the amplifier
- E_{app}: apparent strain: caused by temperature effects in the strain gauge

Measurement uncertainties: strain gauge measurements

$$E_{P,amp} = f_{a,a} \frac{4}{k\left(1 + f_{s,v} + M_s + f_{s,q} + \alpha_{s,k} \Delta T_{20°C}\right)} \frac{U_A}{U_B}$$

$$+ f_{a,z}$$

$$f_{s,q} = \frac{q}{1 - q\nu_0}\left(\frac{\varepsilon_q}{\varepsilon_l} + \nu_0\right)$$

$$E_{P,app} = \varepsilon'_{app}\left(\Delta T_{20°C}\right) + M_{\varepsilon_T} \Delta T_{20°C}$$

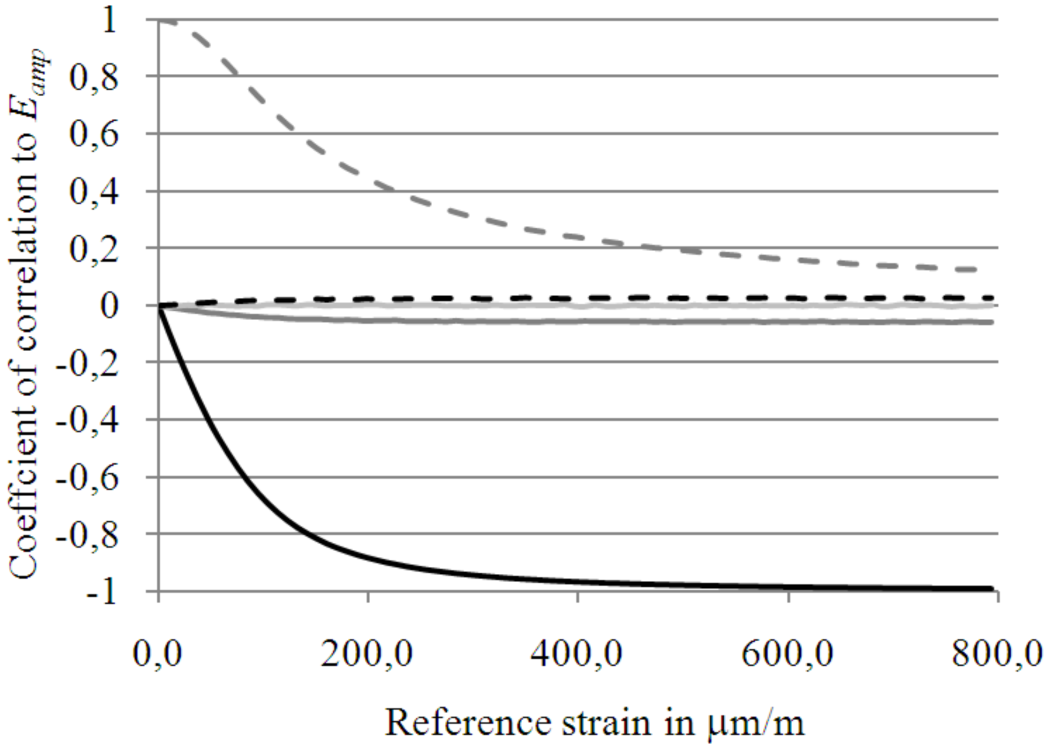
The amplifier strain is dependent on

- Gauge factor and gauge factor variation
- Model uncertainty of gauge factor variation
- Transverse sensitivity
- Specimen Possion Ratio
- Poisson’s Ratio of gauge calibration beam
- Amplifier zero deviation
- Amplifying deviation factor

The apparent strain is dependent on

- Temperature coefficient of gauge factor
- Model uncertainty of the temperature-variation curve

Measurement uncertainties: strain gauge measurements



The amplifier strain is mainly dependent on

- Gauge factor and gauge factor variation
- Amplifier zero deviation

Measurement uncertainties: further reading

ISO Guide 98: Uncertainty of measurement

Books and standards on measurement technologies, such as e.g. for strain gauges:

- Keil, S. (1995). Beanspruchungsermittlung mit Dehnungsmessstreifen, Cuneus.
- VDI/VDE/GESA (2007). Experimental structure analysis, Metallic bonded resistance strain gages, Characteristics and test conditions. VDI/VDE/GESA 2635, Part 1.

Day 2: Structural reliability and measurements

Adaptation of structural reliability with measurement information

- Probabilistic modelling of measurements
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  - Quantification of measurement uncertainty

- Updating of structural reliability
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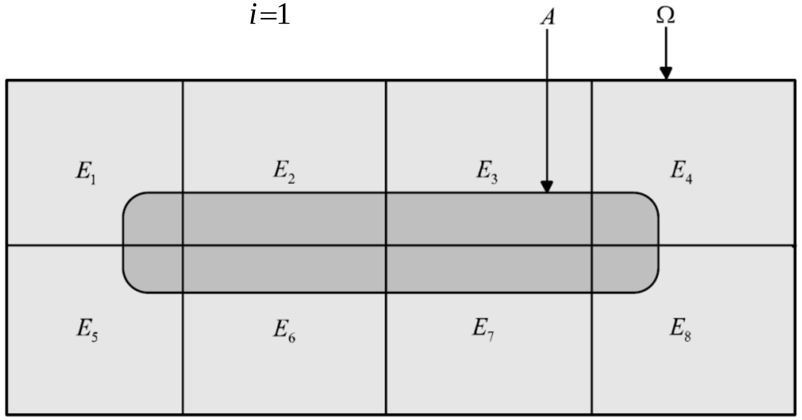
  - Modelling of SHM outcomes

Updating of structural reliability

Posterior Likelihood Prior

$$P(E_i|A) = \frac{P(A|E_i)P(E_i)}{P(A)}$$

$$P(E_i|A) = \frac{P(A|E_i)P(E_i)}{\sum_{i=1}^{n} P(A|E_i)P(E_i)}$$



Bayes theorem

- Prior probability
- Likelihood
- Posterior
- Total probability theorem

Updating of structural reliability

Posterior Likelihood Prior

$$\underset{\text{Posterior}}{f_X''(x \mid E)} = \frac{\overset{\text{Likelihood}}{L(E \mid x)}\,\overset{\text{Prior}}{f_X'(x)}}{\int_{-\infty}^{\infty} L(E \mid x)\, f_X'(x)\, dx}$$

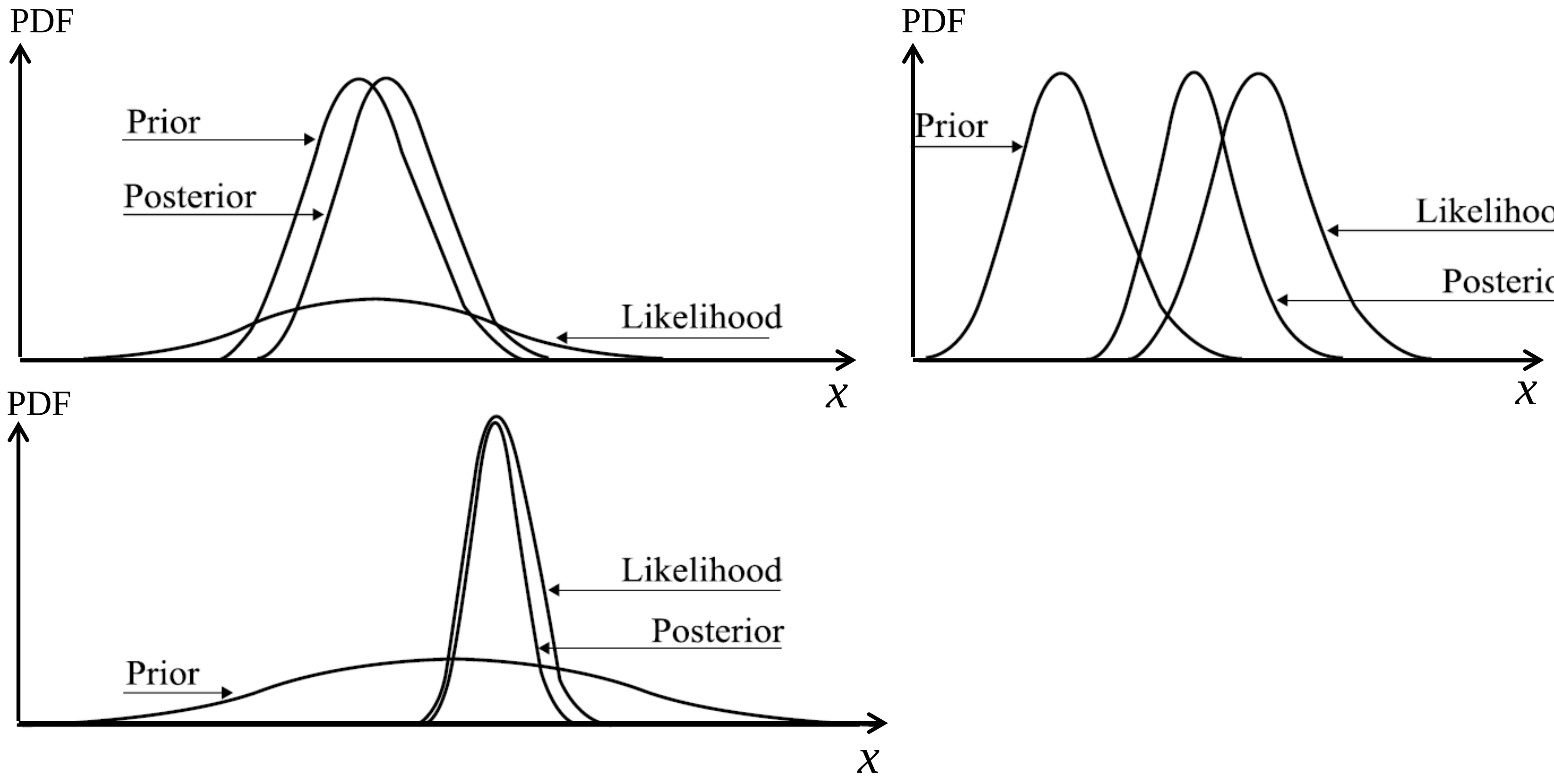
Posterior Likelihood Prior

$$f_{\boldsymbol{\Theta}}''(\boldsymbol{\theta} \mid \hat{\mathbf{x}}) = \frac{L(\hat{\mathbf{x}} \mid \boldsymbol{\theta})\, f_{\boldsymbol{\Theta}}'(\boldsymbol{\theta})}{\int_{-\infty}^{\infty} L(\hat{\mathbf{x}} \mid \boldsymbol{\theta})\, f_{\boldsymbol{\Theta}}'(\boldsymbol{\theta})\, d\boldsymbol{\theta}}$$

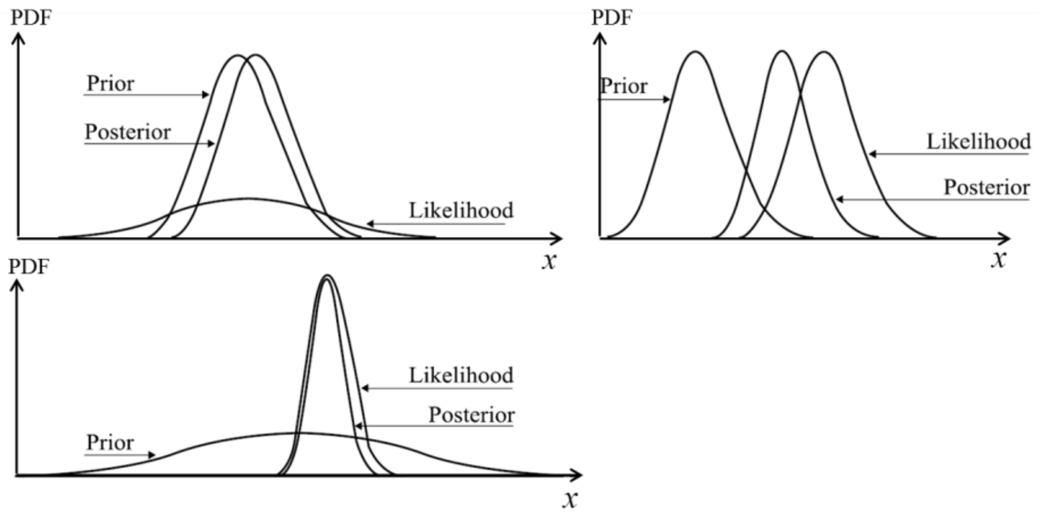
Bayesian updating can be formulated for random variables.

- Bayesian updating for continuous random variables
- Bayesian updating for the parameters of a random variable

Updating of structural reliability



Updating of structural reliability



Conjugate Prior: If the posterior distributions are in the same family as the prior probability distribution, then the prior and posterior are defined as conjugate distributions. The prior is called a conjugate prior for the likelihood function.

- The Gaussian distributions are conjugate to their selves.
- Concept introduced by Raiffa and Schlaifer (1961): Applied statistical decision theory

Updating of structural reliability

The prior distribution is often chosen such that the posterior distribution will be of the same type as the prior distribution. Analytical solutions can be found e.g. for:

- Normal distribution with unknown mean
- Normal distribution with unknown standard deviation
- Normal distribution with unknown mean and standard deviation
- Gumbel distribution
- Weibull distribution
- Exponential distribution
- Bernoulli distribution
- Poisson distribution
- Multidimensional Normal distribution with unknown means
- Multidimensional Normal distribution with unknown standard deviations
- Multidimensional Normal distribution with unknown means and standard deviations

Updating of structural reliability: inspection and damage detection information

$$P(D|\bar{I}) = \frac{P(\bar{I}|D)P(D)}{P(\bar{I})} = \frac{P\left(\bar{I} \cap D\right)}{P(\bar{I})}$$

The calculation of the probability of damage given no indication requires the probability of indication.

$$g_{\bar{I}} = \Phi^{-1}\left\{P\left(I \mid d\right)\right\} - u$$

The probability of no indication can be calculated with a limit state function.

Day 2: Structural reliability and measurements

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Modelling of SHM outcomes

$$X = f_{num}\left(\mathbf{V}, M_{num}\right)$$

$$X = M_{num} \cdot f_{num}\left(\mathbf{V}\right)$$

$$x_i = m_{num,i} \cdot f_{num}\left(\mathbf{V}\right)$$

The true system performance X can be described with a numerical model f_{num} subjected to the uncertainties $\mathbf{V}$ and the model uncertainties M_{num}.

- Model uncertainties account for assumptions and limitation of models.

Model uncertainties are determined with experiments, i.e. observations on a number of real structures or structural components. The experiments are setup so that all other uncertainties can be neglected.

Modelling of SHM outcomes

$$X_i = U \cdot m_{num,i} \cdot f_{num}(\mathbf{V})$$

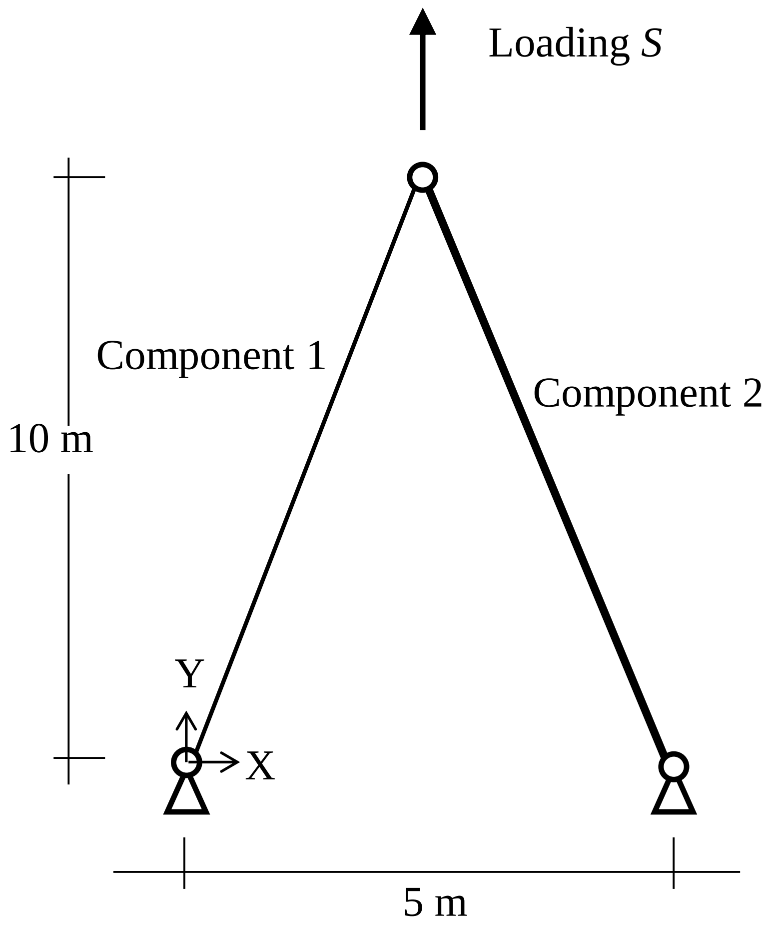
$$X = \int U \cdot f_{num}(\mathbf{V}) \cdot m_{num} \cdot f_{num}(M_{num})\, dm_{num}$$
$$= U \cdot f_{num}(\mathbf{V}) \cdot \int m_{num} \cdot f_{num}(M_{num})\, dm_{num}$$
$$= U \cdot f_{num}(\mathbf{V}) \cdot E[M_{num}]$$

The system performance X subjected to monitoring can be described with a numerical model f_{num} subjected to the uncertainties $\mathbf{V}$ and the realisations of the model uncertainties M_{num}.

- Monitoring reveals the realisation of the model uncertainty.
- Integrating over all possible outcomes leads to the expected value of the model uncertainties.

Be aware: Model uncertainty outcomes may require actions!

Updating of structural reliability: Task 3



Update the component and system damage probabilities with inspection, damage detection and monitoring information.

Damage development (uncorrelated; in mm) with model uncertainty (fully correlated):

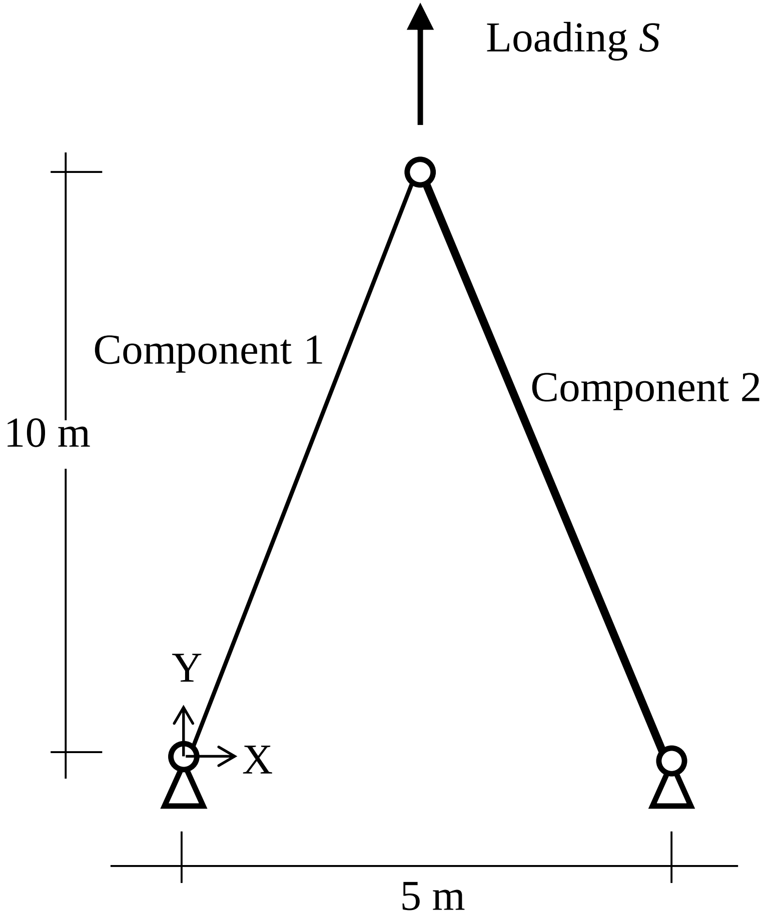
$$D_1 \sim N(5.0, 1.0)$$

$$D_2 \sim N(7.0, 1.5)$$

$$M_D \sim LN(1.0, 0.2)$$

Damage resistance (fully correlated): Lognormal distribution with a standard deviation of 1.0 mm also including model uncertainties.

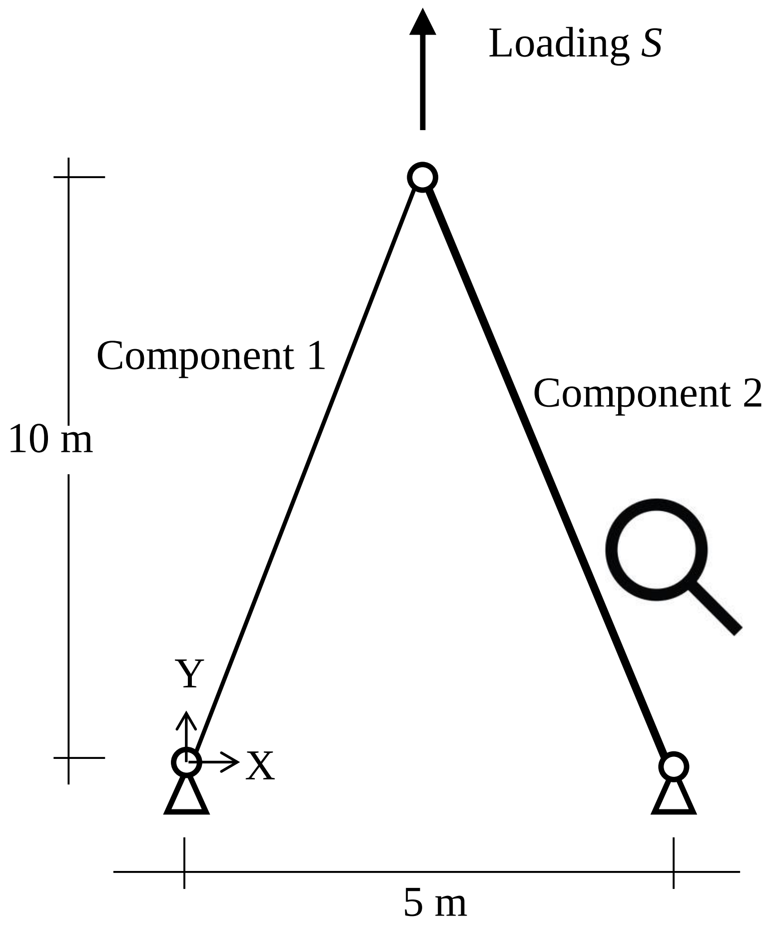
Updating of structural reliability: Task 3



1. Design the structure

- Derive the limit state equation.
- Calibrate the component probability of damage so that it equals 1×10^{-2}.
- How are component probabilities of damage correlated?

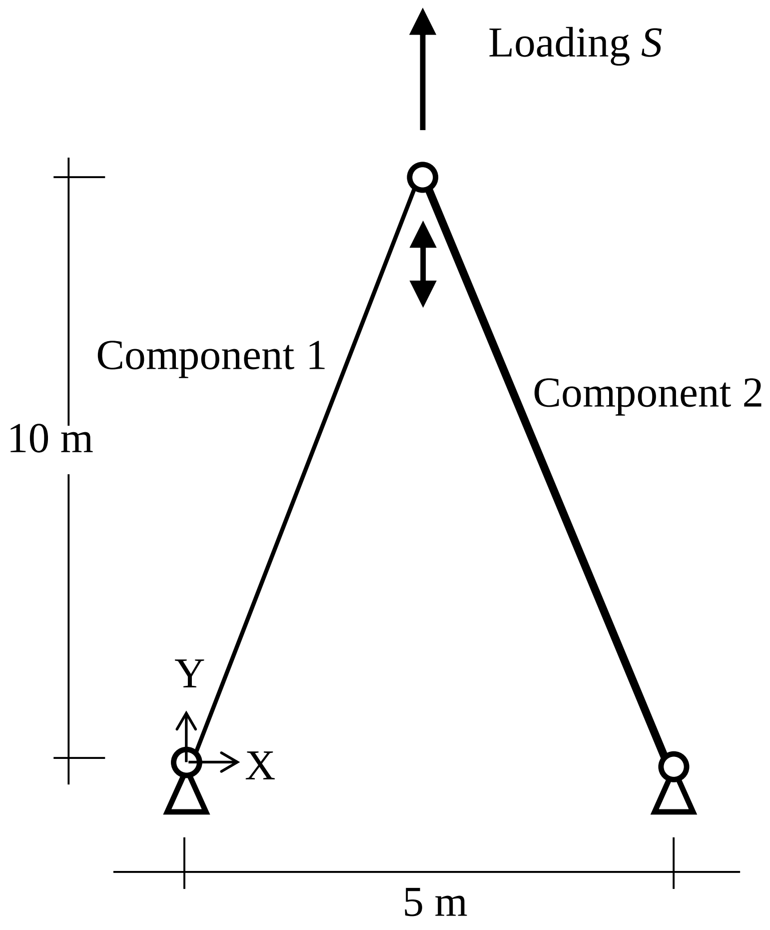
Updating of structural reliability: Task 3



2. Update the component probabilities of damage with inspection information of a finding and no finding.

- Take the inspection information model from task 1.
- Show the effect of relevant influencing factors.

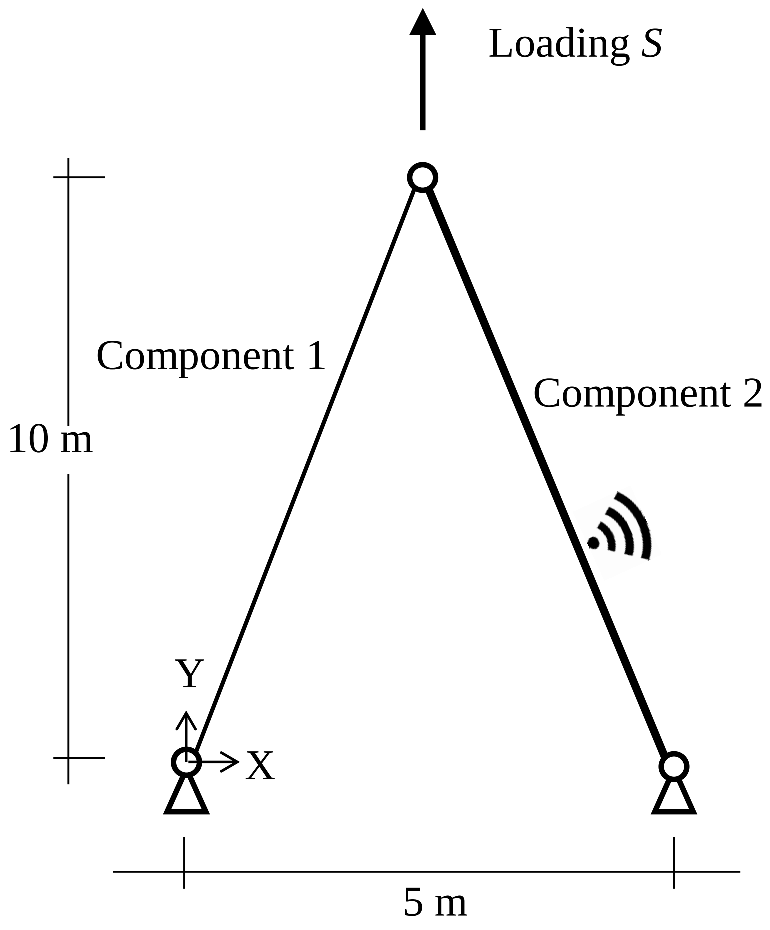
Updating of structural reliability: Task 3



3. Update the component probabilities of damage with damage detection information of a finding and no finding.

- Calculate the system probability of damage.
  - What structural system reliability model applies and why?
- Take the damage detection information model from task 2.

Updating of structural reliability: Task 3



4. Update the component probabilities of damage with monitoring information.

- What is the probability that the probability of system failure is lower than designed?

Updating of structural reliability: Further reading

Papadimitriou, C., J. L. Beck and L. S. Katafygiotis (2001). Updating robust reliability using structural test data. Probabilistic Engineering Mechanics 16(2): 103-113. DOI: https://doi.org/10.1016/S0266-8920(00)00012-6.

Straub, D. (2011). Reliability updating with equality information. Probabilistic Engineering Mechanics 26(2): 254–258.

Thöns, S. and Miraglia, S.: TU1402 Fact Sheet No. WG1-5: Classification for a Value of SHM quantification.

Thöns, S. and M. Döhler (Under review). On Damage Detection System Information for Structural Systems.

Döhler, M. and S. Thöns (2016). Efficient Structural System Reliability Updating with Subspace-Based Damage Detection Information. European Workshop on Structural Health Monitoring (EWSHM), Bilbao, Spain, 5-8 July 2016.

Thank you for your attention.