Structural system reliability analysis

- Load and resistance modelling
- Logical systems, Daniels systems
- Target reliabilities

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Litterature

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Introduction – Uncertainty modelling

Uncertain parameters for buildings, bridges, towers, off-shore structures, wind turbines, …:

- Loads
- Strengths – load bearing capacity
- Models

Modelled by $\mathbf{X} = (X_1, \ldots, X_n)$: stochastic variables

Types of uncertainty:

- **Physical uncertainty**
- **Measurement uncertainty**
- **Statistical uncertainty**: due to limited number of observations
- **Model uncertainty**

Aleatory

Epistemic

Not covered: gross errors / human errors

STOCHASTIC MODELS FOR LOADS AND STRENGTHS

Extreme loads:

- Gumbel distribution: Extreme wind-, snow- and temperature loads
- Weibull distribution: Significant wave heights

Largest load on life time: $Y = \max\{X_1, X_2, \ldots, X_N\}$

X_i : max. load in 1 year : $F_X(x)$

Y : max. load in e.g. 50 years

$$F_Y(y) = F_X(y)^N$$

Fatigue loads:

- LogNormal distribution
- Weibull distribution

Material strengths:

- Normal distribution: if strength
  - can be modelled as a sum of single contributions – e.g. ductile materials
- LogNormal distribution: if strength
  - can be modelled as a product of single contributions
- Weibull distribution: if strength
  - depends of the largest defect in material

JCSS Probabilistic Model Code

1 Basis of Design
2 Loads Models
3 Resistance

2.0	General
2.1	Self weight
2.2	Live load
2.3	Industrial storage
2.4	Cranes
2.5	Traffic
2.6	Car parks
2.7	Silo load
2.8	Liquids/gasses
2.9	Temperature
2.10	Ground
2.11	Water/groundwater
2.12	Snow
2.13	Wind
2.14	Temperature
2.15	Waves
2.16	Avalanches
2.17	Earth quake
2.18	Impact
2.19	Explosion
2.20	Fire
2.21	Chem/Phys agencies

3.0	General
3.1	Concrete
3.2	Reinforcement
3.3	Prestressed steel
3.4	Steel
3.5	Timber
3.6	Aluminium
3.7	Soil
3.8	Masonry
3.9	Model uncertainty
3.10	Dimensions
3.11	Imperfections

JCSS: Joint Committee Structural Safety: http://www.jcss.byg.dtu.dk/

Example - timber

1600 samples from Norway spruce
194 classified as LT20
Bending strength measured

Characteristic value : $x_{0.05}$: 5% quantile

Number	194
Mean [MPa]	39.6
COV	0.26
Min. value [MPa]	15.9
Max. value [MPa]	65.3
$x_{0.05}$ [MPa]	21.6

4 different distribution types are fitted to data:

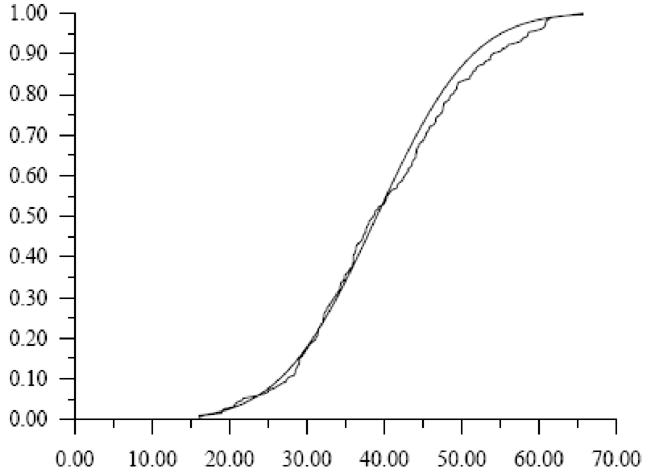
- Normal
- Lognormal
- 2 parameter Weibull
- 3-parameter Weibull with γ chosen as 0.9 times the smallest observed value

2 types of fits:

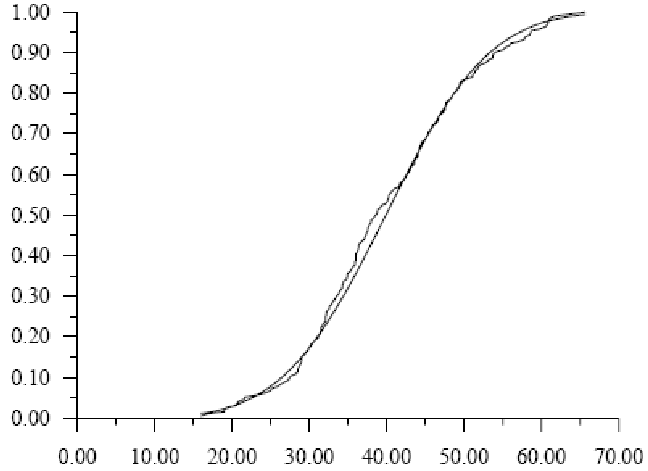
- fit to all data. Maximum Likelihood Method
- tail fit where only the smallest 30% of data are used. Least squares method

	COV	$x_{0.05}$ [MPa]
Non-parametric	0.26	21.6
Normal	0.26	22.4
Normal – tail	0.25	22.7
LogNormal	0.28	24.1
LogNormal – tail	0.38	22.8
Weibull-2p	0.27	21.3
Weibull-2p – tail	0.23	22.8
Weibull-3p	0.26	23.3

Normal:

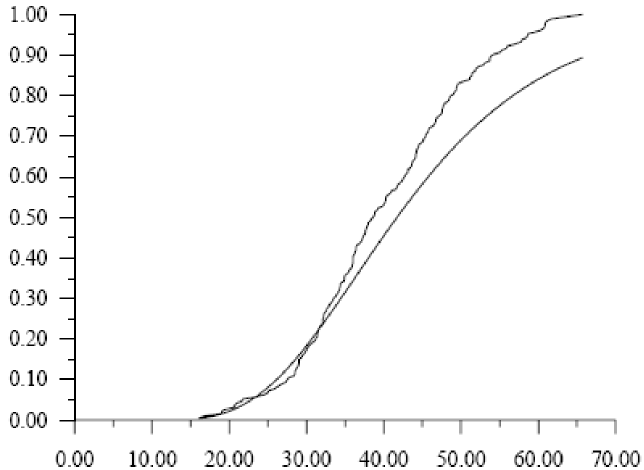


tail-fit

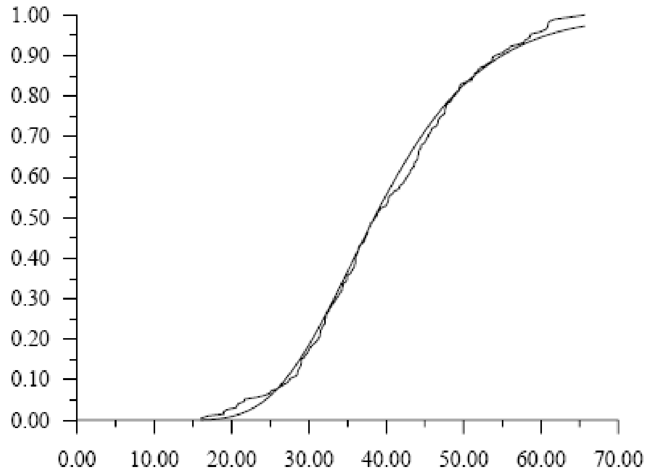


fit with all data

Lognormal:

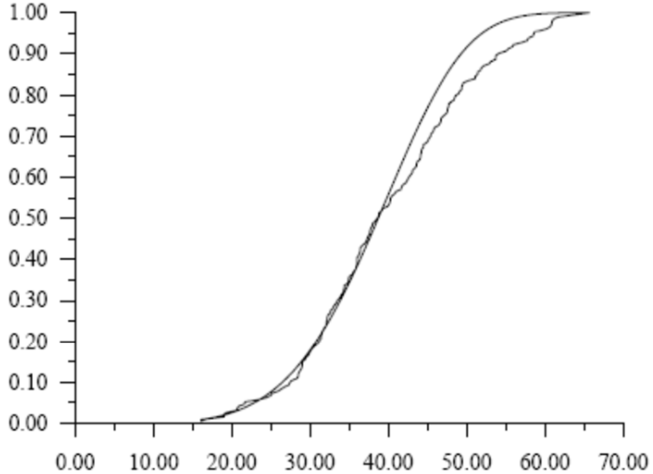


tail-fit

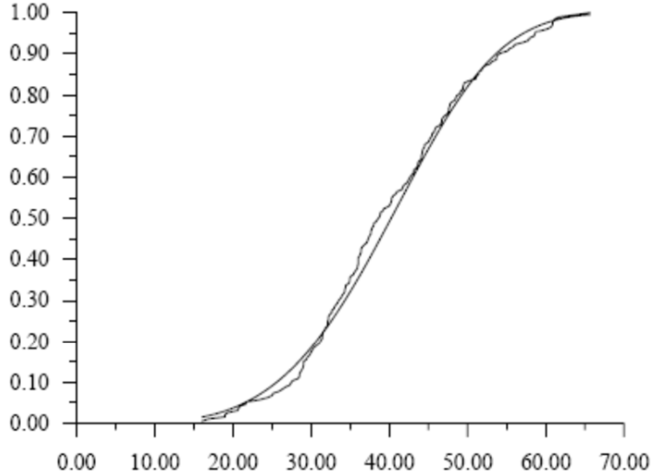


fit with all data

2 parameter Weibull:

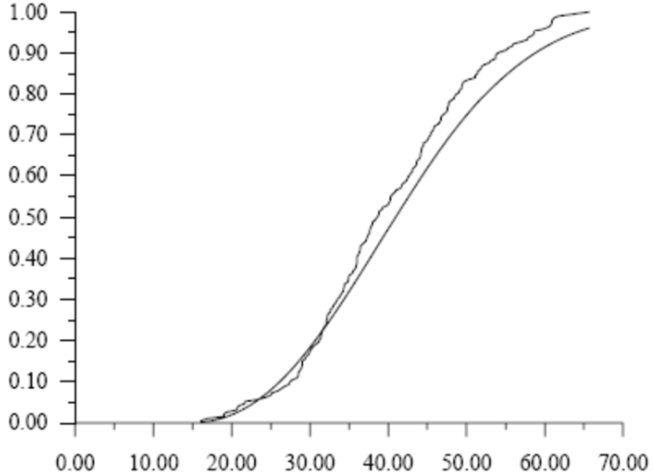


tail-fit

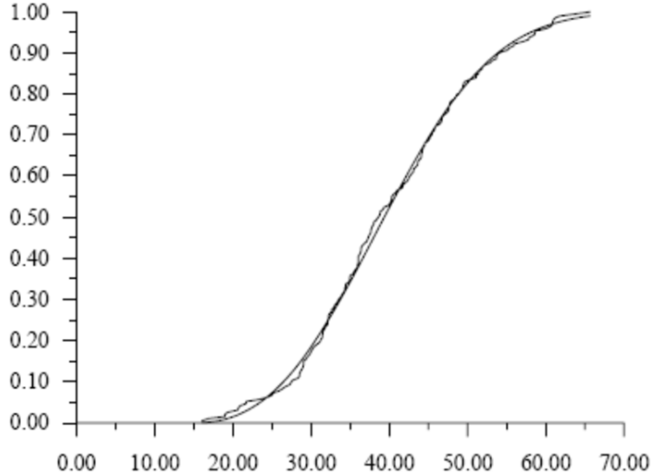


fit with all data

3 parameter Weibull:

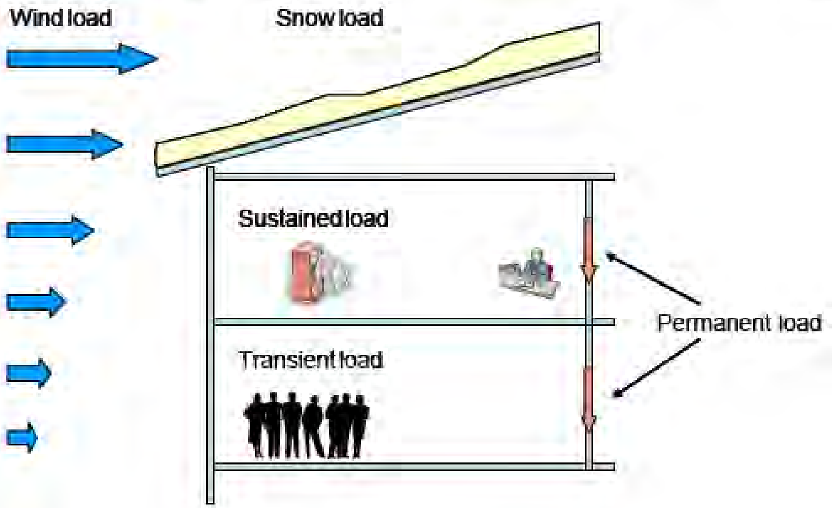


tail-fit



fit with all data

Example – loads on buildings – imposed loads



Permanent loads:	constant in time
Wind load:	mean wind speed constant in 10 minutes periods
Snow load	approximately constant in 14 days periods
Imposed loads:	sustained (5-10 year) and transient (1-3 days)

Mean wind speed

Distribution of all 10-minuttes mean wind speed in a year can be used to

- estimate the expected energy production by a wind turbine
- demonstrate sufficient reliability to fatigue

Weibull distribution

Extreme wind speed

Distribution of yearly maximum wind speed can be used to

- demonstrate sufficient reliability to extreme load

Gumbel distribution

Stochastic model – SAKO - 1999

Parameter	Coefficient of variation, COV			Distribution type
	Conrete	Steel	Gluelam timber	
Actions[1]				
Permanent				
– Self-weight	0,06	0,02	0,06	Normal
– Other	0,10	0,10	0,10	Normal
Variable				
– Environmental	0,40	0,40	0,40	Gumbel
– Imposed	0,20	0,20	0,20	Gumbel
Strength				
Concrete	0,10			Log-Normal
Reinforcement	0,04			Log-Normal
Structural steel		0,05		Log-Normal
Gluelam timber			0,15	Log-Normal
Geometry				
Effective depth	0,02			Normal
Beam depth	0,02	0,01	0,01	Normal
Beam width	0,02	0,01	0,01	Normal
Plate thickness		0,04		Normal
Model uncertainties				
R-model	0,05	0,05	0,05	Normal

[1] Include S-model uncertainties.

Stochastic model – Proqua - 2005

Table 4.1 Conventional models of basic variables for time invariant reliability analyses.

No.	Category of variables	Name of basic variables	Sym. X	Dim-ension	Dis-trib.	Mean μ_X	St. dev. σ_X	Prob. $\Phi_X(X_k)$	References
1	Actions	Permanent	G	kN/m²	N	G_k	$0{,}03\text{-}0{,}10\mu_X$	0,5	[10],[18]
2		Imposed-5 years	Q		GU	$0{,}2Q_k$	$1{,}1\mu_X$	0,995	[10],[28]
3		Imposed-50 years	Q		GU	$0{,}6Q_k$	$0{,}35\mu_X$	0,953	
4		Wind-1 year *	W		GU	$0{,}3W_k$	$0{,}5\mu_X$	0,999	[10],[29]
5		Wind-50 years *	W		GU	$0{,}7W_k$	$0{,}35\mu_X$	0,890	
6		Snow - 1 year **	S		GU	$0{,}35\ S_k$	$0{,}70\ \mu_X$	0,998	[10],[30]
7		Snow -50 year**	S		GU	$1{,}1\ S_k$	$0{,}30\ \mu_X$	0,437	
8	Material strengths	Steel yield point	f_y	N/mm²	LN	$f_{yk}+k_s\sigma^{1)}$	$0{,}08\mu_X$	-	[10], [23] to [25]
9		Steel strength	f_u		LN	$\kappa\,\mu_{fy}{}^{2)}$	$0{,}05\mu_X$	-	
10		Concrete	f_c		LN	$f_{ck}+k_c\sigma^{3)}$	$0{,}10\text{-}0{,}18\mu_X$	-	
11		Reinforcement	f_y		LN	$f_{yk}+2\sigma$	30 N/mm²	0,02	

Stochastic model – Proqua - 2005

12	Geometry steel sect.	IPE profiles	A,W,I	$m^{2,3,4}$	N	$0,99X_{nom}$	$0,01\text{-}0,04\ \mu_X$	$\cong 0,73$	[10],[25]
13		L-section, rods	A,W,I		N	$1,02X_{nom}$	$0,01\text{-}0,02\ \mu_X$	$\cong 0,16$	
14	Geometry concrete	Cross-section	b, h	m	N	b_k,h_k	0,005-0,01	0,5	[10]
15		Cover of reinf.	a		BET	a_k	0,005-0,015	0,5	
16	cross-sect.	Additional ecc.	e		N	0	0,003-0,01	-	
17	Model uncertainties	Load effect factor	θ_E	-	N	1	0,05-0.10	-	[10],[26]
18		Resistance factor$^+$	θ_R		N	1-1,25	0,05-0,20	-	

Notes: * See also Table 4.2.

** See also Table 4.3.

1) The coefficient k_s can be expected within the interval from 2 to 2,5 depending on execution control (2,0 for mills which do not regularly control the quality and 2,5 for efficient quality control. [51].

2) The coefficient κ can be expected within the interval from 1,1 to 1,5 depending on the type of steel [10].

3) The coefficient k_c can be expected within the interval from 1,5 to 2 depending on execution control [10] (1,5 for in situ concrete and 2,0 for prefabricated concrete with efficient quality control).

Stochastic model – Baravalle et al. 2017

*Table B.2. Stochastic models based on [13] unless otherwise specified (*yearly maxima).*

Random variable		Distr. type	Mean (μ)	COV	Ch. Fract. (value)	Ref. and notes
Resistance model unc. (steel)	$\Theta_{R,1}$	Logn.	1.00	0.05	(μ)	
Resistance model unc. (concrete)	$\Theta_{R,2}$	Logn.	1.00	0.10	(μ)	
Resistance model unc. (rebar)	$\Theta_{R,3}$	Logn.	1.00	0.10	(μ)	
Resistance model unc. (glulam)	$\Theta_{R,4}$	Logn.	1.00	0.10	(μ)	
Resistance model unc. (solid timber)	$\Theta_{R,5}$	Logn.	1.00	0.10	(μ)	
Resistance model unc. (masonry)	$\Theta_{R,6}$	Logn.	1.16	0.175	(μ)	
Mat. property (steel yielding strength)	R_1	Logn.	1.00	0.07	$\mu - 2\sigma$	
Mat. property (concrete compr. capacity)	R_2	Logn.	1.00	0.15	0.05	
Mat. Property (rebar yielding strength)	R_3	Logn.	1.00	0.07	0.05	
Mat. property (glulam bending strength)	R_4	Logn.	1.00	0.15	0.05	
Mat. property (solid timber bending strength)	R_5	Logn.	1.00	0.20	0.05	
Mat. property (masonry compr. strength)	R_6	Logn.	1.00	0.16	0.05	

Stochastic model – Baravalle 2017

Self-weight (steel)	$G_{S,1}$	Norm.	1.00	0.04	0.50	See 0
Self-weight (concrete)	$G_{S,2}$	Norm.	1.00	0.05	0.50	
Self-weight (rebar)	$G_{S,3}$	Norm.	1.00	0.05	0.50	
Self-weight (glulam)	$G_{S,4}$	Norm.	1.00	0.10	0.50	
Self-weight (solid timber)	$G_{S,5}$	Norm.	1.00	0.10	0.50	
Self-weight (masonry)	$G_{S,6}$	Norm.	1.00	0.065	0.50	
Permanent load	G_P	Norm.	1.00	0.10	0.50	
Permanent load (large COV)	G_P^*	Norm.	1.00	0.20	0.95	
Wind time-invariant part (gust C_g ,pressure C_{pe} and roughness C_r coefficients)	Θ_{Q_1}	Logn.*	0.79*	0.24*	(1.095)*	* Parameters of the Logn. distribution approximating the upper tail (> 0.90 fractile) of the distribution representing $\Theta_Q = C_g C_r C_{pe}$ with: C_{pe} Gumbel [134, 139], $\mu_{C_{pe}} = 1$; $COV_{C_{pe}} = 0.15$ and ch. fractile 0.78 [99, 140] (0.80 is suggested in [107]); C_r Logn., $\mu_{C_r} = 0.80$; $COV_{C_r} = 0.15$ and ch. value = 1.00; C_g Logn., $\mu_{C_g} = 1$; $COV_{C_g} = 0.10$ and ch. value = 1.00.
Snow time-invariant part (model uncertainty and shape coefficient)	Θ_{Q_2}	Logn.	1.00	0.30	($\mu + \sigma$)	Ch. value equal to $\mu + \sigma$ given in [13, 141]; ch. Value equal to the mean given in [58].
Wind mean reference velocity pressure *	Q_1	Gumb.	1.00	0.25	0.98	When the COV varies over the country and only one PSFs is sought the mean COV over the country can be used, see [101]. Alternatively, PSFs can vary over the territory; this is a national choice.
Snow load on roof *	Q_2	Gumb.	1.00	0.40	0.98	
Imposed load model uncertainty	Θ_{Q_3}	Logn.	1.00	0.10	(1.00)	The COV is assumed since no data are found in the literature. To be further assessed. [Not yet discussed in CEN/TC250-SC10/WG1].
Imposed load *	Q_3	Gumb.	1.00	0.53	0.98	See B.3.2.2. [Not yet discussed in CEN/TC250-SC10/WG1].

Stochastic model – wind turbines (IEC 61400-1)
Extreme load cases

Variable	Distribution	Mean	COV	Comment
R	Lognormal	-	V_R	Strength
δ	Lognormal	-	V_δ	Model uncertainty
L – DLC 1.1	Weibull	-	0.15	Annual maximum load effect obtained by load extrapolation
L – DLC 6.1	Gumbel	-	0.2	Annual maximum wind pressure – European wind conditions
$Xdyn$	Lognormal	1.00	0.05	
$Xexp$	Lognormal	1.00	0.15	
$Xaero$	Gumbel	1.00	0.10	
$Xstr$	Lognormal	1.00	0.03	

Stochastic model – Fatigue – JCSS PMC

Units: mm and N

Variable		**Distribution**	**Mean**	**V**	
C	Material parameter S-N curve	lognormal	$1.0 \cdot 10^{13}$	0.58	
m	Slope value		3	-	
$\log C_1$	Material parameter 2 par S-N curve	normal	Depends		C_1 and C_2 fully correlated
$\log C_2$	Material parameter 2 par S-N curve	normal	Depends		
m_1 (air)	Slope value 1st branch	deterministic	5	-	
m_2 (air)	Slope value 2nd branch	deterministic	3	-	
D_{cr}	Miner's sum at failure	lognormal	1.0	0.3	
A_1 (air)*	Paris Law Parameter 1	lognormal	$4.80 \cdot 10^{-18}$	1.70	
A_2 (air)*	Paris Law Parameter 2	lognormal	$5.86 \cdot 10^{-13}$	0.60	
m_1 (air)	Slope value 1st branch	deterministic	5.10	-	
m_2 (air)	Slope value 2nd branch	deterministic	2.88	-	
ΔK_0 (air)	Threshold value for ΔK	lognormal	140	0.40	
A_1 (marine)*	Paris Law Parameter 1	lognormal	$5.37 \cdot 10^{-14}$	1.10	
A_2 (marine)*	Paris Law Parameter 2	lognormal	$5.67 \cdot 10^{-7}$	0.16	
m_1 (marine)	Slope value 1st branch	deterministic	3.42	-	
m_2 (marine)	Slope value 2nd branch	deterministic	1.11	-	
ΔK_0 (marine)	Threshold value for ΔK	lognormal	0.0	-	
a_0	Initial crack depth	lognormal	0.15	0.66	
a_0/c_0	Initial aspect ratio	lognormal	0.62	0.40	

Stochastic model – Fatigue – JCSS PMC

B_{glob}	MU global stress model*	lognormal	1.0	0.10	
B_{scf}	MU stress concentration	lognormal	1.0	0.20	
B_{sif}	MU stress intensity factor (hand)	lognormal	1.0	0.20	
B_{sif}	MU stress intensity factor (FEM)	lognormal	1.0	0.07	
σ_{res}	Residual stresses	lognormal	300	0.20	
R	Resistance fracture toughness	lognormal	1.7	0.18	
K_{mat}	Fracture toughness	Weibull	See (4.1)		

Structural system reliability analysis

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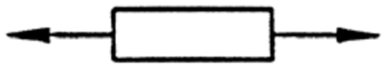
MODELLING OF SYSTEMS

A system model consists of:

- A number of failure elements each modelled by a failure function:

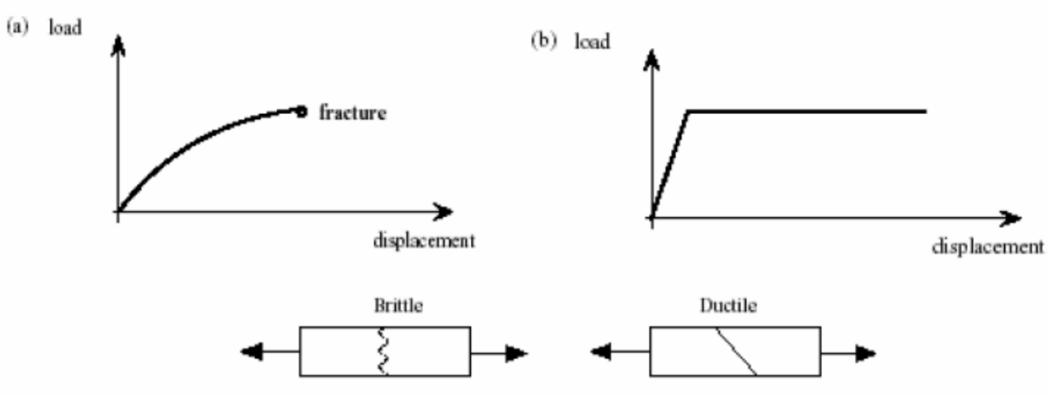
$$g_i(\mathbf{x}) = 0\ , \quad i = 1, 2, \ldots, m$$



- Common basic variables in failure functions:

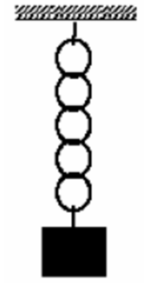
$$X_i\ , \quad i = 1,\ 2, \ldots, n$$

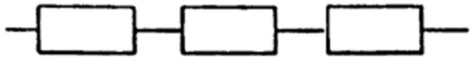
- Behaviour of elements: ductile / brittle



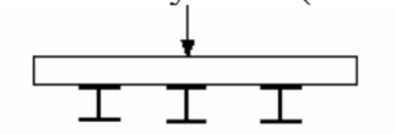
- Types of systems models:

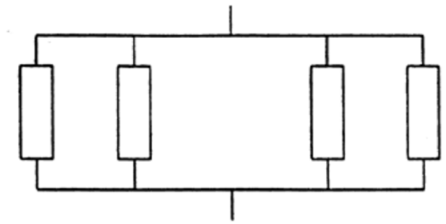
  - Series system:



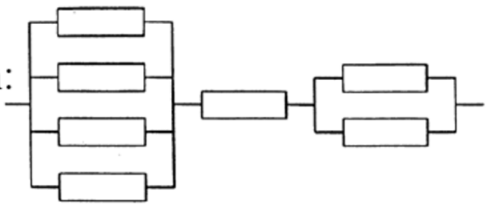


  - Parallel system: (next lecture)



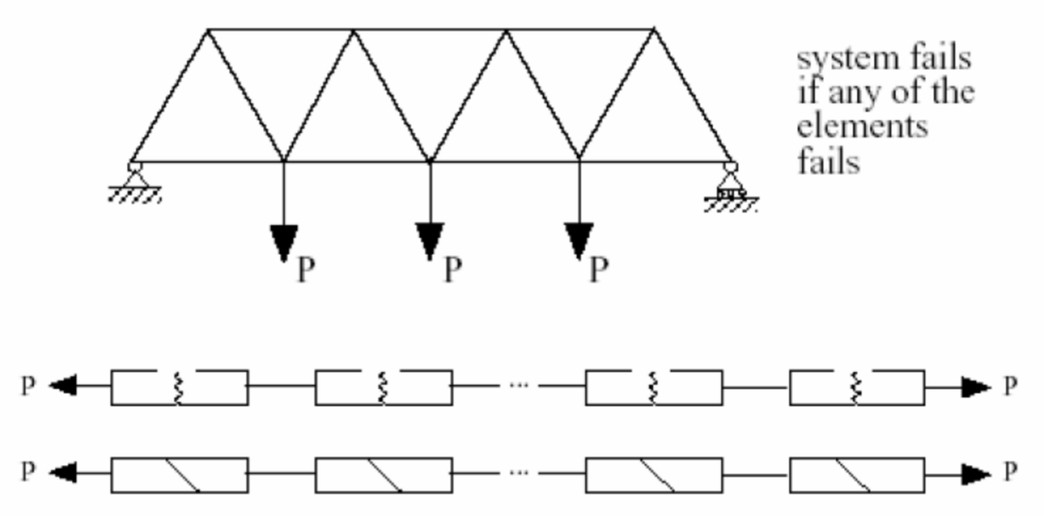


  - Series/parallel system: (next gang)



Example of Series system

Statically determinate truss system:



Series system fails if one element fail – thus:

$$
\begin{aligned}
P_f^S &= P\left(M_1 \le 0 \cup \ldots \cup M_m \le 0\right) \\
&= P\left(\bigcup_{i=1}^{m} \{M_i \le 0\}\right) \\
&= P\left(\bigcup_{i=1}^{m} \{\, g_i(\mathbf{X}) \le 0\}\right) \\
&= P\left(\bigcup_{i=1}^{m} \{\, g_i(\mathbf{T}(\mathbf{U})) \le 0\}\right) \\
&\approx P\left(\bigcup_{i=1}^{m} \left\{\, \beta_i - \boldsymbol{\alpha}_i^T \mathbf{U} \le 0\right\}\right) \\
&= 1 - P\left(\bigcap_{i=1}^{m} \left\{\, \beta_i - \boldsymbol{\alpha}_i^T \mathbf{U} \ge 0\right\}\right) \\
&= 1 - P\left(\bigcap_{i=1}^{m} \left\{\, \boldsymbol{\alpha}_i^T \mathbf{U} \le \beta_i \right\}\right) \qquad (*) \\
&= 1 - \Phi_m(\boldsymbol{\beta}; \boldsymbol{\rho})
\end{aligned}
$$

De Morgans rule:

$$A_1 \cup \ldots \cup A_m = \overline{\overline{A}_1 \cap \ldots \cap \overline{A}_m}$$

$$\boxed{\rho_{ij} = \boldsymbol{\alpha}_i^T \boldsymbol{\alpha}_k}$$

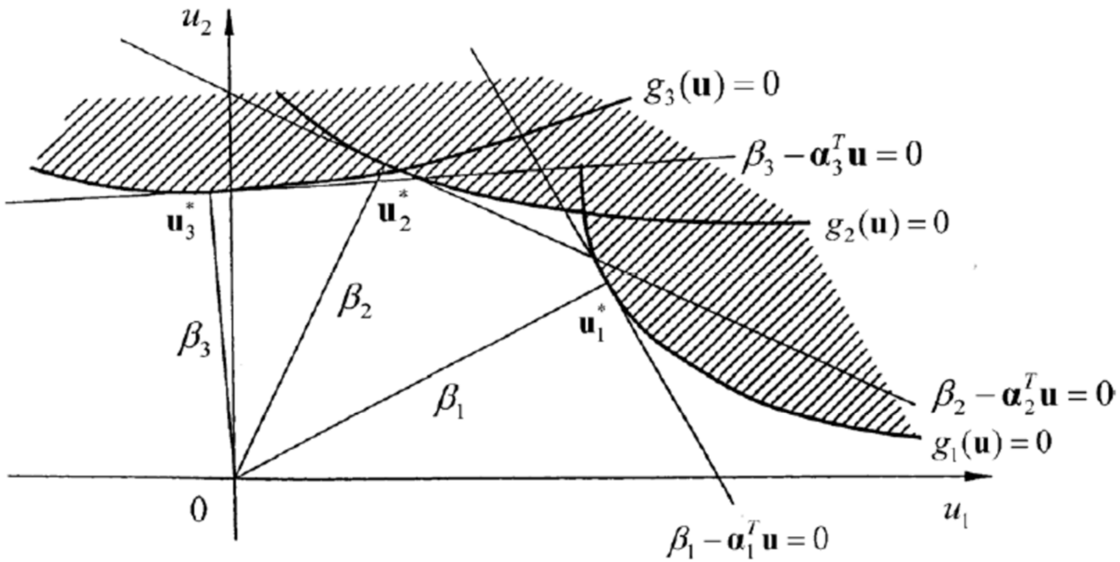
Φ_m m-dimensional standard normal distribution

Can be estimated e.g. by the Hohenbichler approximation

Illustration of FORM - Approximation

2 stochastic variables
3 failure elements in a series system:

$$g_i(\mathbf{T}(\mathbf{u})) = 0\ , \quad i = 1,2,3$$



- Hatched area: exact failure domain / failure probability
- In FORM – approximation: linearization of failure surfaces in β-points

Bounds for Probability of Failure for a Series system

Simple Bounds:

$$\max_{i=1}^{m} P(M_i \leq 0) \leq P_f^S \leq \sum_{i=1}^{m} \left(P(M_i \leq 0) \right)$$

Lower bound is exact if all safety margins are fully correlated

Ditlevsen Bounds

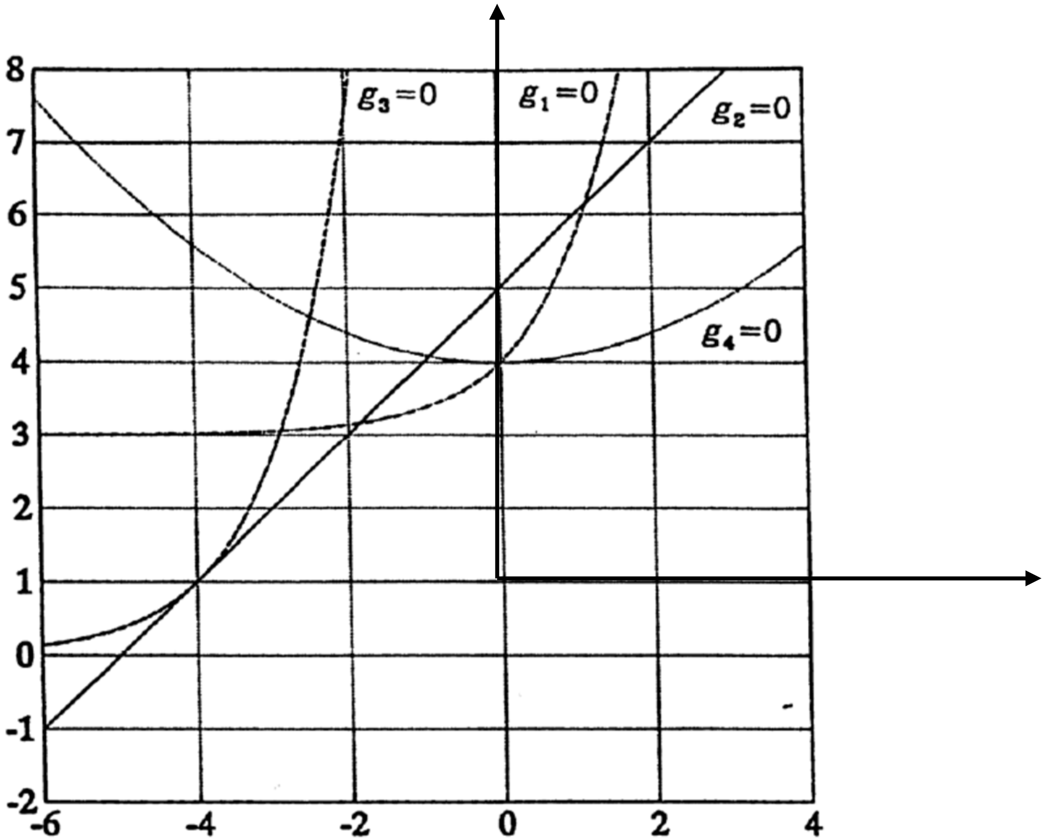
$$P_f^S \geq P(M_1 \leq 0) + \sum_{i=2}^{m} \max \left\{ P(M_i \leq 0) - \sum_{j=1}^{i-1} P(M_i \leq 0 \bigcap M_j \leq 0), 0 \right\}$$

$$P_f^S \leq \sum_{i=1}^{m} P(M_i \leq 0) - \sum_{i=2}^{m} \max_{j<i} \left\{ P(M_i \leq 0 \bigcap M_j \leq 0) \right\}$$

Calculation of β^S for series system

Series system with 4 elements.

$$g_1(\mathbf{u}) = \exp(u_1) - u_2 + 3$$
$$g_2(\mathbf{u}) = u_1 - u_2 + 5$$
$$g_3(\mathbf{u}) = \exp(u_1 + 4) - u_2$$
$$g_4(\mathbf{u}) = 0.1u_1^2 - u_2 + 4$$



Reliability index calculation for each element:

i	β_i	$\Phi(-\beta_i)$	α_{i1}	α_{i2}	u_{i1}^*	u_{i2}^*
1	3.51	$2.276 \cdot 10^{-4}$	–0.283	0.959	–0.99	3.36
2	3.54	$2.035 \cdot 10^{-4}$	–0.707	0.707	–2.50	2.50
3	3.86	$5.738 \cdot 10^{-5}$	–0.875	0.483	–3.38	1.86
4	4.00	$3.174 \cdot 10^{-5}$	0.000	1.000	0.00	4.00

Correlation between safety margins:

$$\rho_{ij} = \boldsymbol{\alpha}_i^T \boldsymbol{\alpha}_j$$

$$\boldsymbol{\rho} = \begin{bmatrix} 1.000 & & & \text{sym.} \\ 0.878 & 1.000 & & \\ 0.712 & 0.961 & 1.000 & \\ 0.962 & 0.714 & 0.492 & 1.000 \end{bmatrix}$$

Simple Bounds

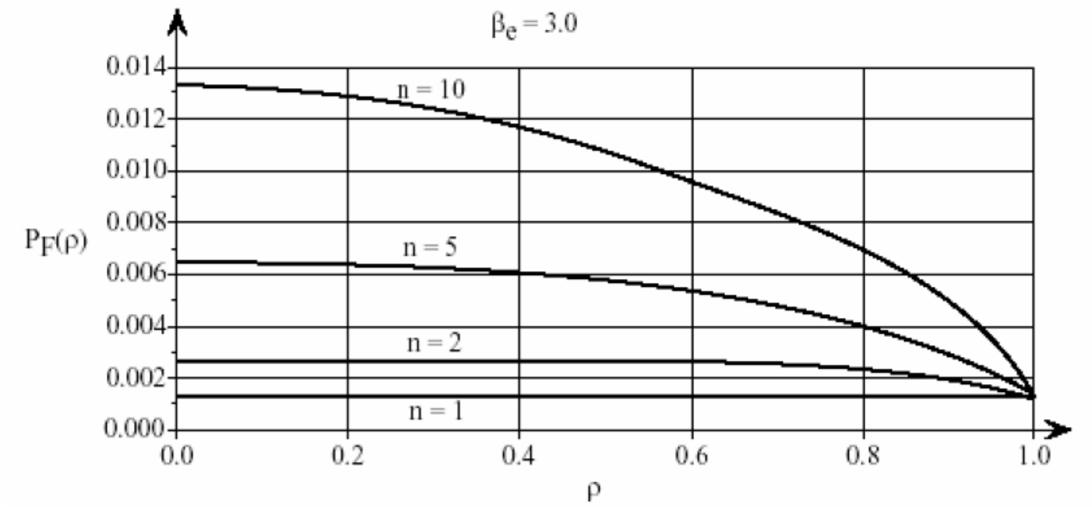
$$\beta^S \geq -\Phi^{-1}(2.276 \cdot 10^{-4} + 2.035 \cdot 10^{-4} + 5.738 \cdot 10^{-5} + 3.174 \cdot 10^{-5})$$
$$= 3.28$$
$$\beta^S \leq \min\{3.51; 3.54; 3.86; 4.00\} = 3.51$$

Ditlevsen Bounds

$$3.381 \leq \beta^S \leq 3.383$$

Probability of failure for series system with n equal-correlated elements with same element reliability index β_e:

$$P_f(\rho) = P_f^S(\rho) = 1 - \int_{-\infty}^{\infty} \varphi(t) \left\{ \Phi\left(\frac{\beta_e - \sqrt{\rho}t}{\sqrt{1-\rho}} \right) \right\}^n dt = \Phi(-\beta)$$



SENSITIVITY ANALYSIS

Differentiation of:

$$\Phi(-\beta^S) = 1 - \Phi_m(\boldsymbol{\beta};\boldsymbol{\rho})$$

gives:

$$\frac{d\beta^S}{dp} = \frac{1}{\varphi(\beta^S)} \sum_{i=1}^{m} \left\{ \frac{\partial \Phi_m(\boldsymbol{\beta};\boldsymbol{\rho})}{\partial \beta_i} \frac{d\beta_i}{dp} + 2 \sum_{j=1}^{i-1} \frac{\partial \Phi_m(\boldsymbol{\beta};\boldsymbol{\rho}) d\rho_{ij}}{\partial \rho_{ij}} \frac{d\rho_{ij}}{dp} \right\}$$

Often it is enough to use:

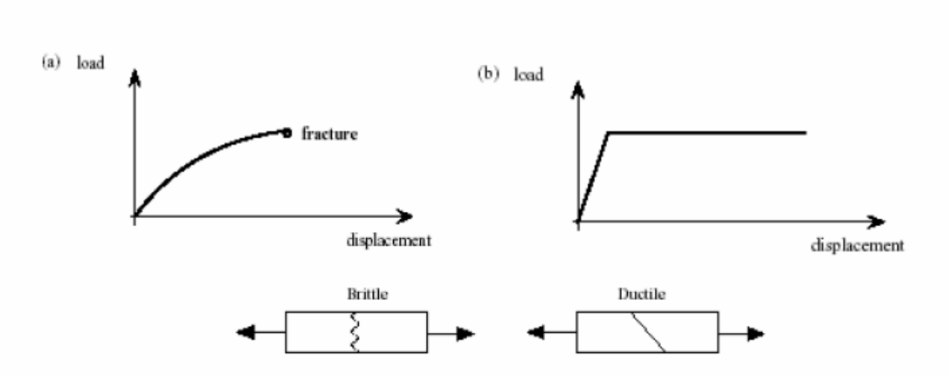
$$\frac{d\beta^S}{dp} \approx \frac{1}{\varphi(\beta^S)} \sum_{i=1}^{m} \frac{\partial \Phi_m(\boldsymbol{\beta};\boldsymbol{\rho})}{\partial \beta_i} \frac{d\beta_i}{dp}$$

$d\beta_i / dp$ is determined as described in note 4

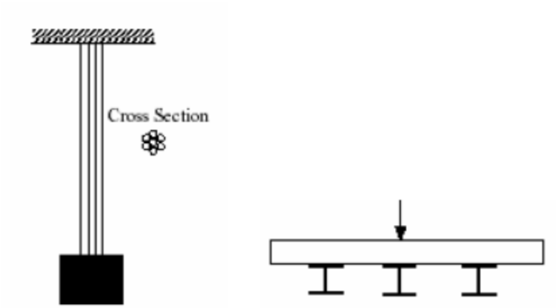
$\partial \Phi_m(\boldsymbol{\beta},\boldsymbol{\rho}) / \partial \beta_i$ is determined numerically

MODELLING OF PARALLEL SYSTEM

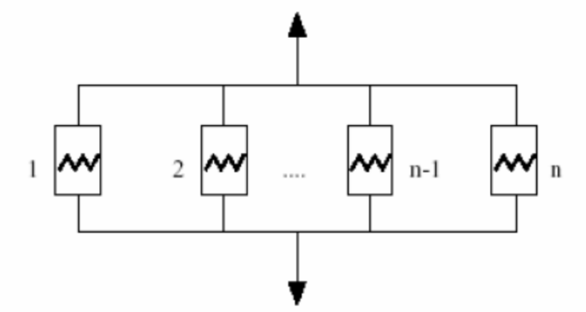
Elements:



Examples:

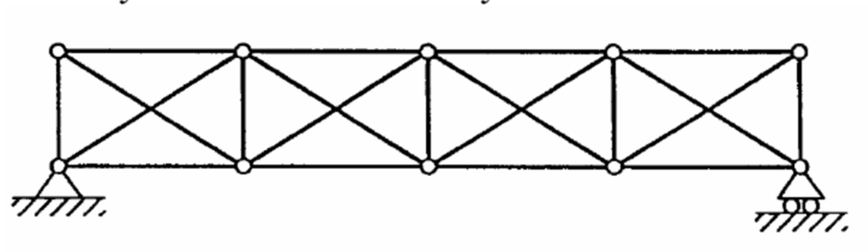


Parallel system:

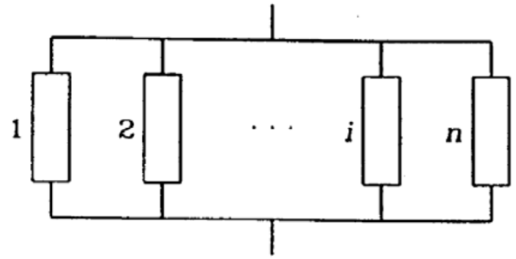


MODELLING AF PARALLEL SYSTEM

Statically indeterminate truss system:



A given failure sequence can be modelled by a parallel system with n-elements:



Failure functions $g_i(\mathbf{u})$, $i = 1,\ldots, n$ models:

1: Failure of weakest element - other intact

2: Failure of next weakest element, after failure of weakest element

…

i: Failure of ith weakest element, after failure of all other weaker elements

…

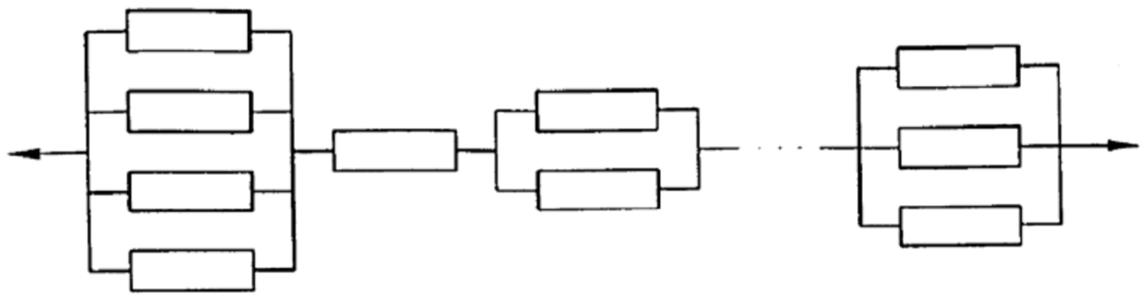
n: Failure of nth weakest element, resulting in global failure (truss system has become statically determinate)

GENERAL SYSTEM MODEL

- Each sequence of failures of the truss system is modelled as a parallel system

- Each failure sequence (parallel system) can be modelled as an element in a series system

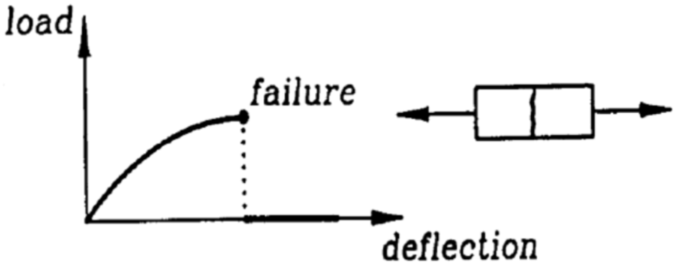
Generalised series/parallel system:



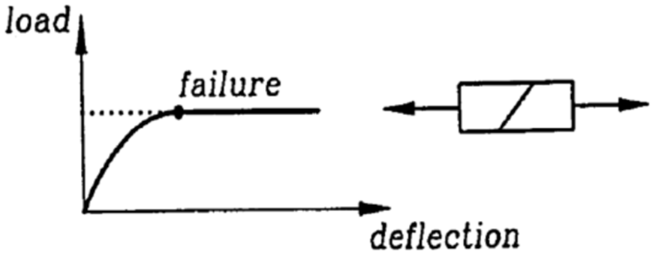
Modelling of failure elements in parallel system

For a parallel system it is important to model the structural behaviour after failure

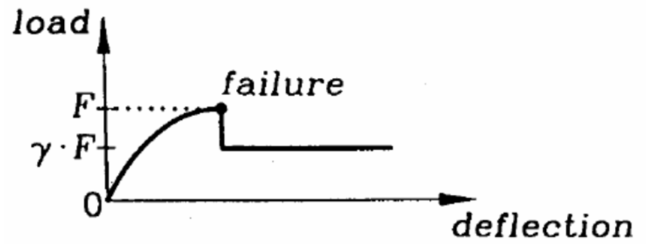
Perfect brittle element:



Perfect ductile element:

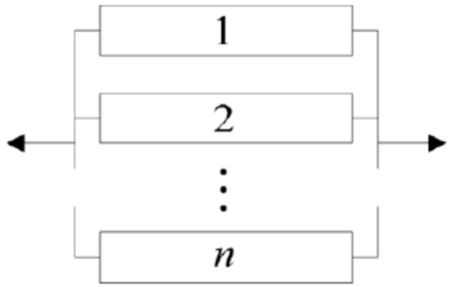


Other:



Parallel system with equal-correlated ductile elements

Consider a fibre bundle of n perfect ductile fibre.



- Strengths $R_i, i = 1,2,\ldots,n$, identically normal distributed $\mathrm{N}(\mu,\sigma)$ and all correlated with ρ.

- Deterministic load on system $S = nS_e$, where S_e is load on each fibre.

- Reliability index for each fibre:
  $$\beta = \frac{\mu - S_e}{\sigma}$$

- Strength R of fibre bundle is the sum of the single strengths.

- Expected value and standard deviation of R :

$$\mu_R = \sum_{i=1}^{n} \mu = n\mu$$

$$\sigma_R^2 = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} \sigma^2 & \cdots & \rho\sigma^2 \\ \vdots & \ddots & \vdots \\ \rho\sigma^2 & \cdots & \sigma^2 \end{bmatrix} \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

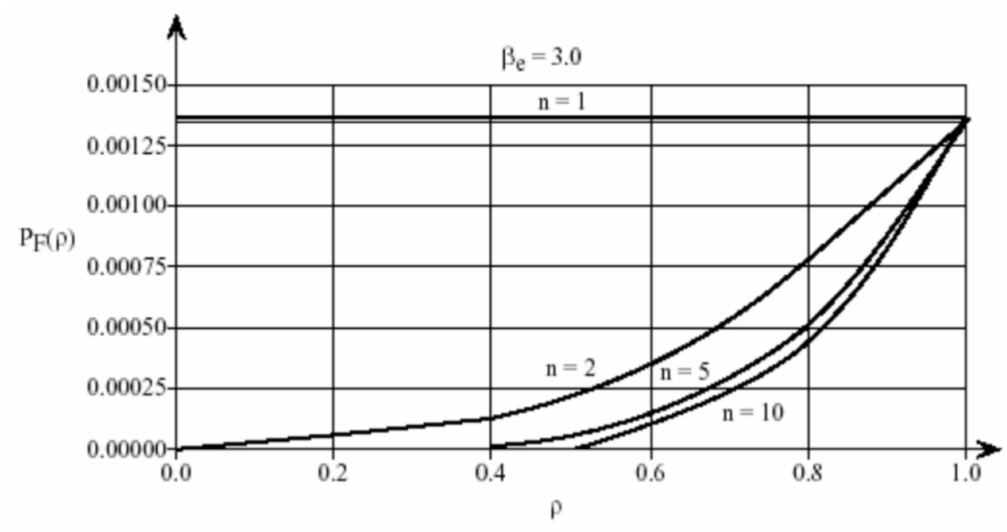
$$= n\sigma^2 + n(n-1)\sigma^2\rho$$

- Reliability index for fibre bundle:

$$\beta^P = \frac{\mu_R - S}{\sigma_R} = \frac{n\mu - n(\mu - \beta\sigma)}{\sqrt{n\sigma^2 + n(n-1)\sigma^2\rho}} = \beta\sqrt{\frac{n}{1+\rho(n-1)}}$$

Note: $S = nS_e = n(\mu - \beta\sigma)$

Example: n ductile elements in parallel system with $\beta_e = 3$



Parallel system of brittle fibres (Daniels system)

- If fibre system consists of perfect brittle fibres with strengths $r_1 \le r_2 \le \cdots \le r_n$:

  $r = \max\{ nr_1, (n-1)r_2, \ldots, 2r_{n-1}, r_n \}$

  It is assumed that load effects in each fibre are equal

  Example:

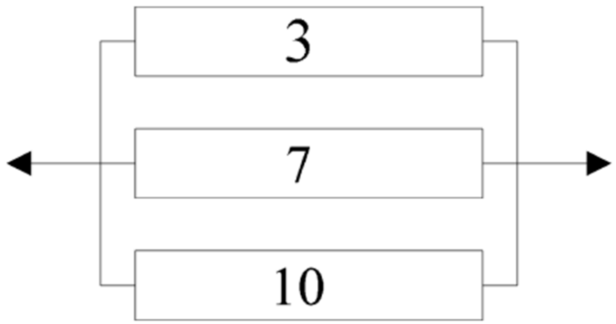
  $r_1 = 3,\ r_2 = 7,\ r_3 = 10$

  $\Downarrow$

  $r = \max\{3 \cdot 3,\ \underline{2 \cdot 7},\ 1 \cdot 10\}$

  $\Downarrow$

  $r = 14$



- If $r_i,\ i = 1, 2, \ldots, n$: outcomes of independent identical distributed variables - then for $n \to \infty$

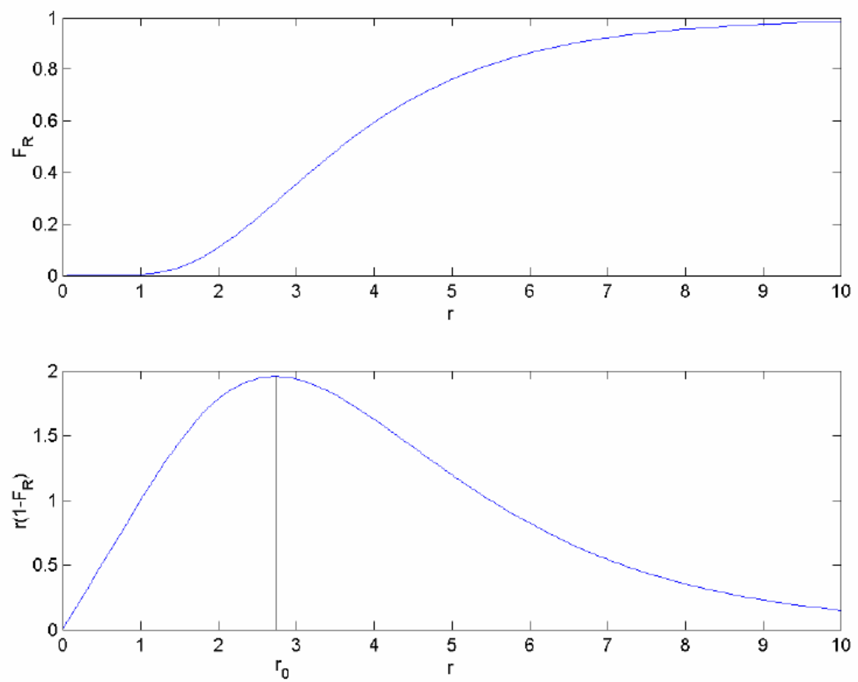
  $R \sim N(\mu_R, \sigma_R)$

  $$\mu_R = n r_0 [1 - F_{R_i}(r_0)]$$

  $$\sigma_R^2 = n r_0^2 F_{R_i}(r_0)[1 - F_{R_i}(r_0)]$$

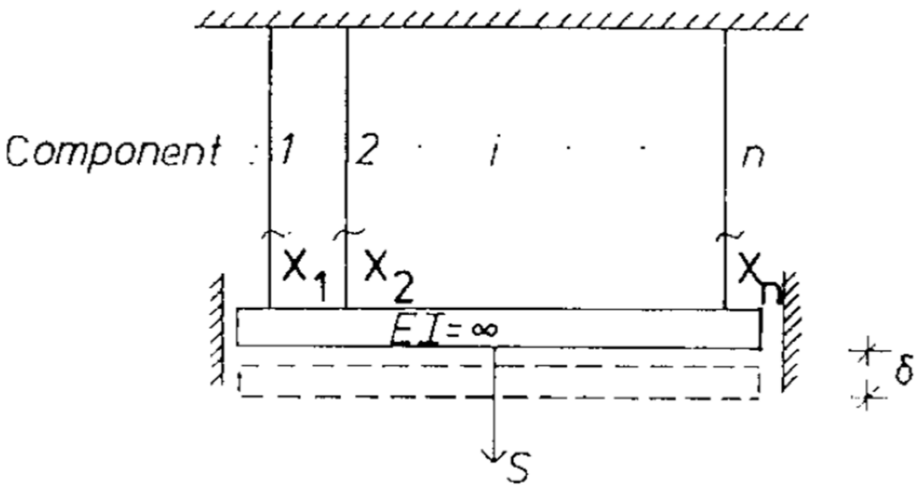
  where r_0 is the value with maximum of:

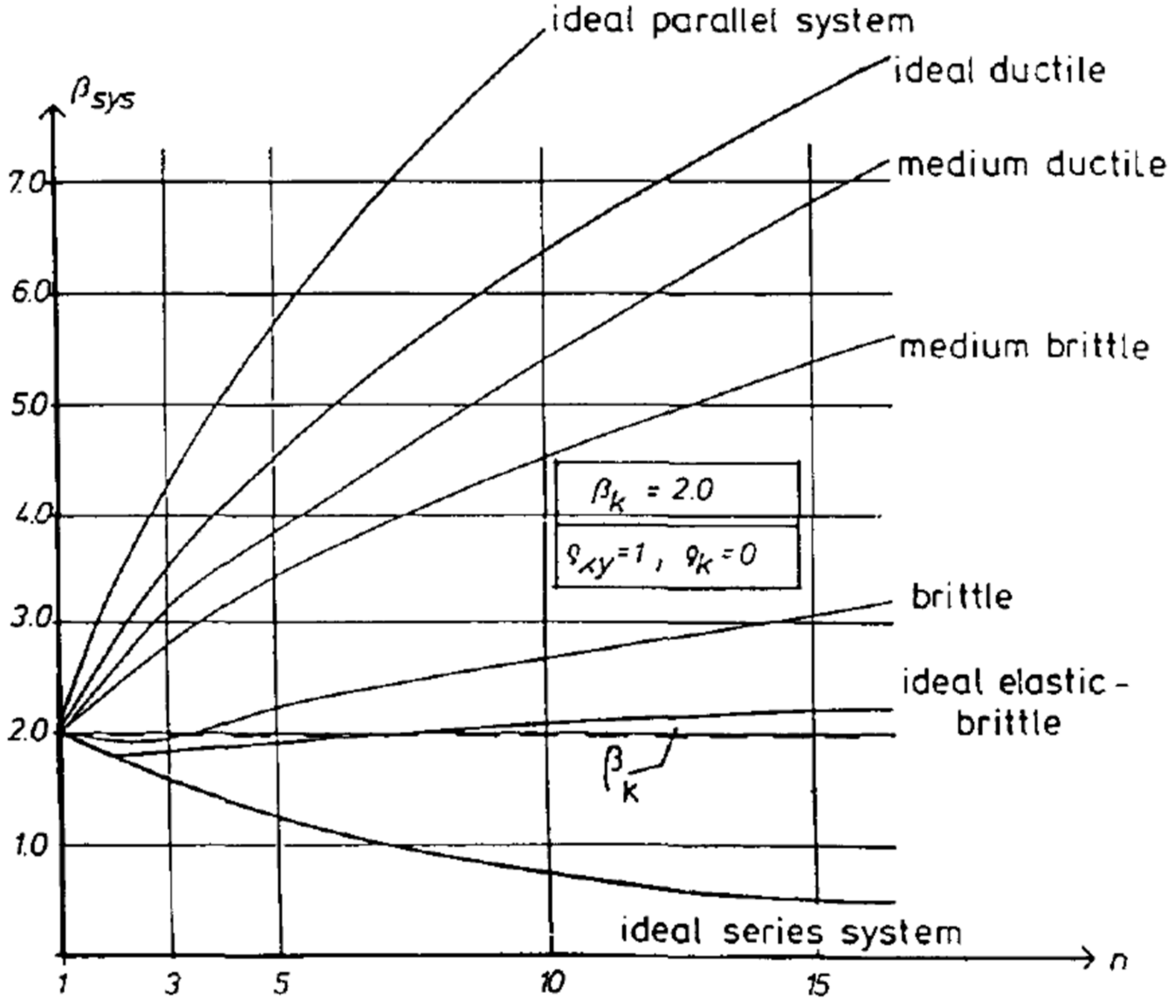
  $r[1 - F_{R_i}(r)]$

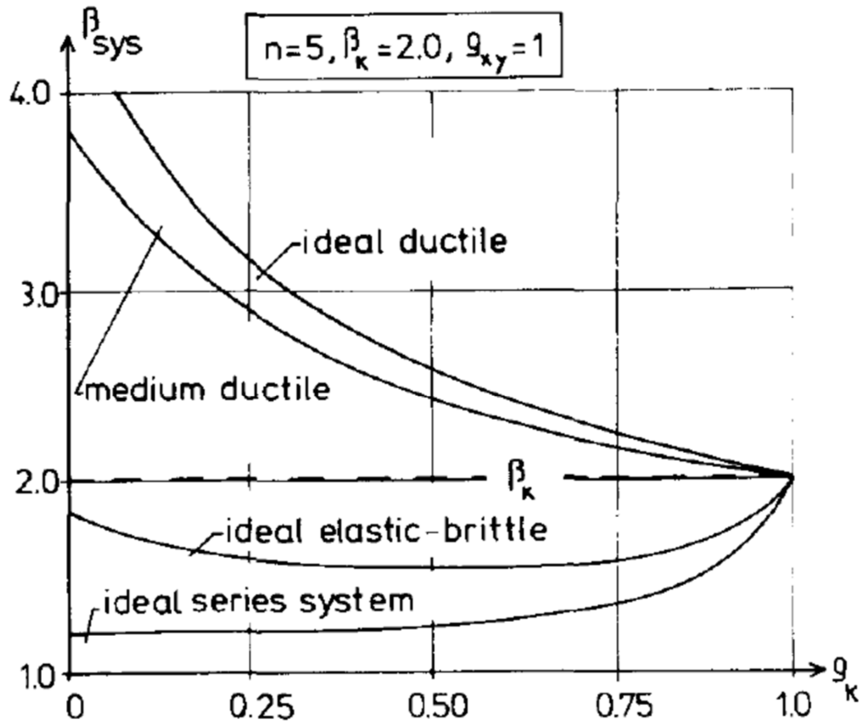


General Daniels systems

Gollwitzer, S. & R. Rackwitz: On the reliability of Daniels systems. Structural Safety, Vol. 7, 1990, pp. 229-243.




ideal parallel system
ideal ductile
medium ductile
medium brittle
brittle
ideal elastic - brittle
ideal series system
β_{sys}
β_k
$\beta_k = 2.0$
$\rho_{xy}=1$, $\rho_k=0$
7.0
6.0
5.0
4.0
3.0
2.0
1.0
1
3
5
10
15
n



System reliability index versus correlation ρ_k of strength between components.

Probability of failure for Parallel system

A parallel system with n failure elements:

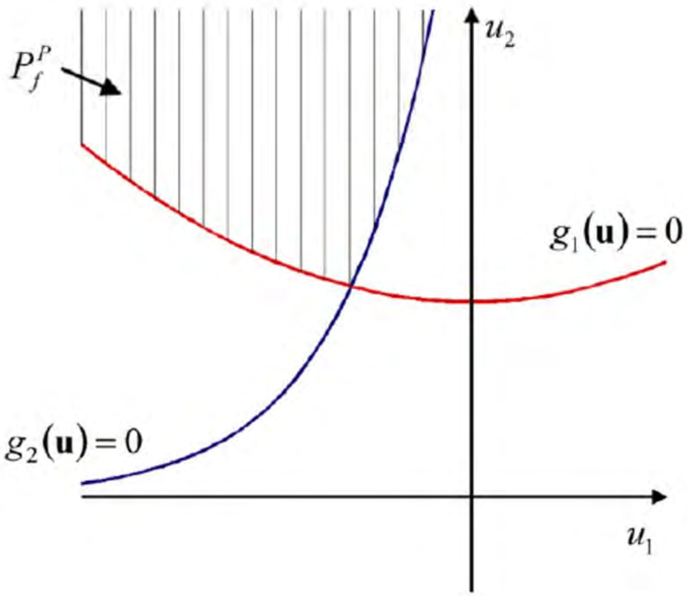
$$M_i = g_i(\mathbf{X}), \quad i = 1, 2, \ldots, n$$

Transformation from basic variables X to standard normal distributed variables, U: $\mathbf{X} = \mathbf{T}(\mathbf{U})$

Failure of parallel system if all elements fail:

$$P_f^P = P(M_1 \le 0 \cap \ldots \cap M_n \le 0) = P\left(\bigcap_{i=1}^{n} \{M_i \le 0\}\right)$$

$$= P\left(\bigcap_{i=1}^{n} \{g_i(\mathbf{X}) \le 0\}\right) = P\left(\bigcap_{i=1}^{n} \{g_i(\mathbf{T}(\mathbf{U})) \le 0\}\right)$$



FORM approximation (crude solution)

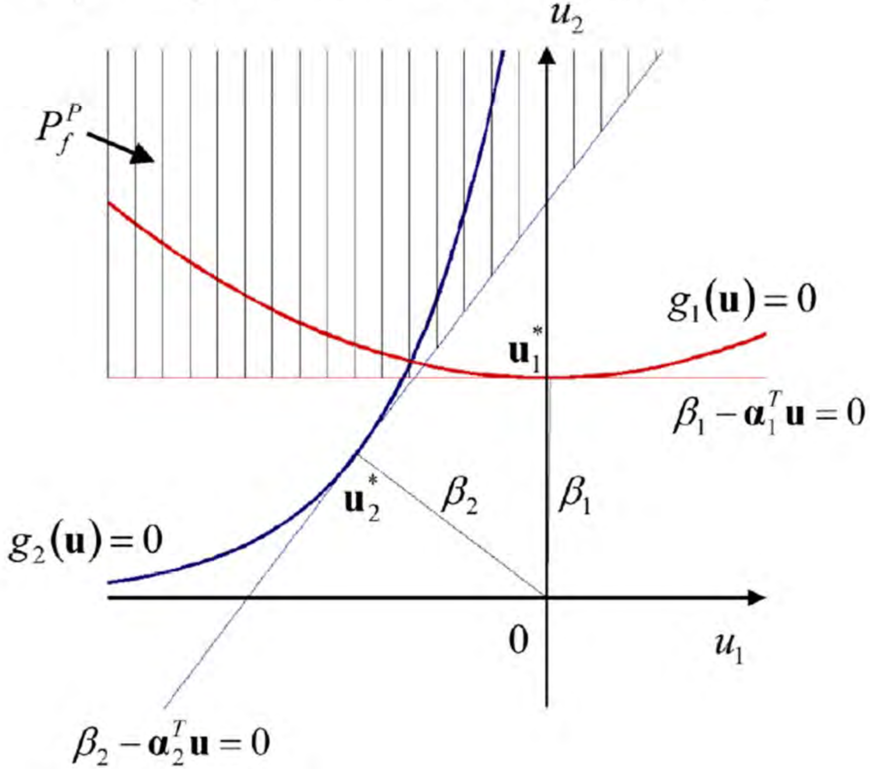
First approach: linearization of failure surfaces in individual β-points:

$$P_f^P \approx P\left(\bigcap_{i=1}^{n}\left\{\beta_i - \boldsymbol{\alpha}_i^T \mathbf{u} \le 0\right\}\right) = P\left(\bigcap_{i=1}^{n}\left\{-\boldsymbol{\alpha}_i^T \mathbf{u} \le -\beta_i\right\}\right)$$

$$= \Phi_n(-\boldsymbol{\beta}; \boldsymbol{\rho})$$

Φ_n n-dimensional standard normal distribution

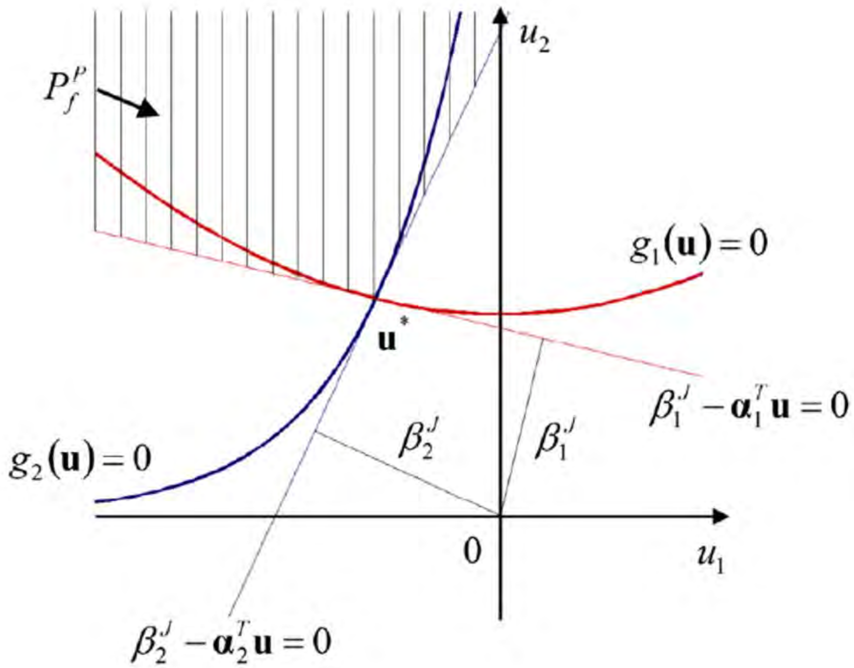
$\rho_{ij} = \boldsymbol{\alpha}_i^T \boldsymbol{\alpha}_j$ correlation between failure surfaces



FORM approximation (good solution)

Failure surfaces: linearization in "joint β-point", $\mathbf{u}^*$
$\mathbf{u}^*$: solution to optimization problem:

$$\min_{\mathbf{u}} \quad \gamma = \tfrac{1}{2}\mathbf{u}^T\mathbf{u}$$
$$\text{s.t.} \quad g_i(\mathbf{u}) \le 0 \quad , \quad i = 1, 2, \ldots, n$$



Joint β-point, $\mathbf{u}^*$

- Determined by optimization techniques
- Graphically (for 1 or 2 dimensional problems

Remark: In joint β-point: only $n_A \le n$ elements active

Determination of probability of failure for Parallel system (FORM)

Based on joint β-point $\mathbf{u}^*$, linear safety margins are established for each of the n_A active elements

$$M_i = \beta_i^J - \boldsymbol{\alpha}_i^T \mathbf{U}, \quad i = 1, 2, \ldots, n_A$$

where

$$\boldsymbol{\alpha}_i = \frac{-\nabla_u g_i(\mathbf{T}(\mathbf{u}^*))}{\left|\nabla_u g_i(\mathbf{T}(\mathbf{u}^*))\right|}, \qquad \beta_i^J = \boldsymbol{\alpha}_i^T \mathbf{u}^*$$

$$\boldsymbol{\beta}^J = (\beta_1^J, \beta_2^J, \ldots, \beta_{n_A}^J).$$

Probability of failure for parallel system approximatively:

$$P_f^P \approx P\left(\bigcap_{i=1}^{n_A} \left\{\beta_i^J - \boldsymbol{\alpha}_i^T \mathbf{U} \le 0\right\}\right) = P\left(\bigcap_{i=1}^{n_A} \left\{-\boldsymbol{\alpha}_i^T \mathbf{U} \le -\beta_i^J\right\}\right)$$
$$= \Phi_{n_A}(-\boldsymbol{\beta}^J; \boldsymbol{\rho})$$

Φ_{n_A}	n_A dimensional standard normal distribution
$\rho_{ij} = \boldsymbol{\alpha}_i^T \boldsymbol{\alpha}_j$	correlation coefficients between failure surfaces

Bounds for Parallel system

Simple Bounds:

$$0 \le P_f^P \le \min_{i=1}^{n_A}\left(P(M_i^J \le 0)\right)$$

Upper bound exact: if all n_A elements are full correlated: $\rho_{ij} = 1$

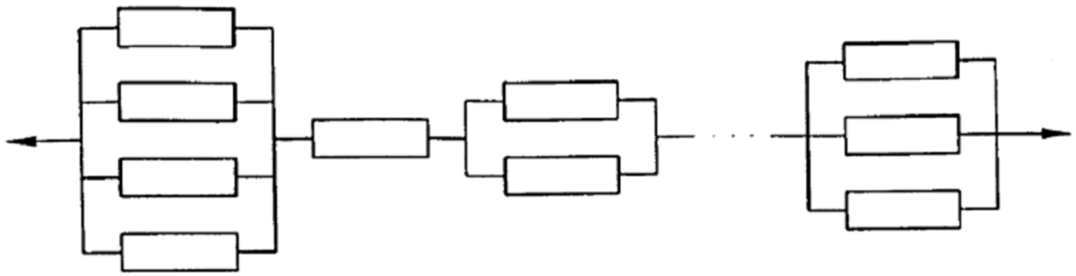
If all correlation coefficients, $\rho_{ij} \ge 0$:

$$P_f^P \ge \prod_{i=1}^{n_A} P(M_i^J \le 0)$$

Second Order upper bound:

$$P_f^P \le \min_{i,j=1}^{n_A} P(M_i^J \le 0 \bigcap M_j^J \le 0)$$

RELIABILITY OF GENERAL SYSTEM



Probability of failure for a series system consisting of n_P parallel systems, each with n_{A_i}, $i = 1, 2, \ldots, n_P$ elements:

$$P_f^S = P\left(\bigcup_{i=1}^{n_P} \bigcap_{j=1}^{n_{A_i}} \left\{ g_{ij}(\mathbf{X}) \le 0 \right\}\right)$$

g_{ij} failure function for element j in parallel system i.

Generalised system reliability index:

$$\beta^S = -\Phi^{-1}\left(1 - \Phi_{n_P}(\boldsymbol{\beta}^P; \boldsymbol{\rho}^P)\right)$$

Structural system reliability analysis

- Load and resistance modelling
- Logical systems, Daniels systems
- Target reliabilities

Reliability level

- ISO 2394:2015: General principles for reliability of structures
  - Decision making / Design:
    1. Risk-informed decision making
       - → acceptable and target reliability level for probabilistic design
    2. Reliability-based decision making – probabilistic design
       - → partial safety factors for design by e.g. IEC 61400-1
    3. Semi-probabilistic method – partial safety factor method

- JCSS: Joint Committee on Structural Safety: Probabilistic Model Code

- …

Reliability level – ISO 2394

Risk-based decision making involves

- Optimization

  Maximization of utility function (e.g. cost-benefit function)

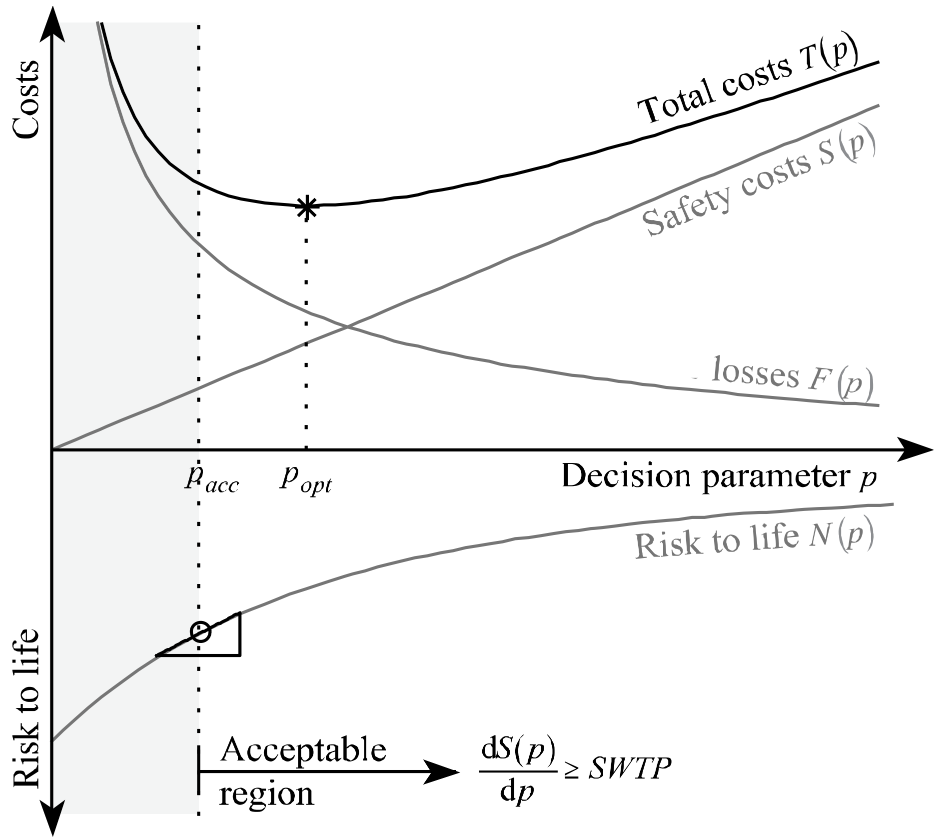
  → target (nominal) reliability level, P_{opt}

- Assessment of Acceptability

  Is the decision acceptable from a societal perspective?

  Marginal Life Saving Cost - MLSC

  → minimum acceptable reliability level, P_{acc} (wrt. risk to life)



Reliability based code calibration

Optimality and Target Reliabilities – Civil engineering structures

- Acceptance criteria may be established on the basis of
  - cost benefit considerations → economic optimum reliability level
  - LQI (Life Quality Index) → lower limit on reliability level
- JCSS and ISO2394 target reliabilities for ULS verification (1 year reference)

Relative cost of safety measure	Minor consequences of failure	Moderate consequences of failure	Large consequences of failure
High	3.1	3.3	3.7
Normal	3.7	4.2	4.4
Low	4.2	4.4	4.7

Relative cost of safety measure	Target index (irreversible SLS)
High	1.3
Normal	1.7
Low	2.3

:

Limit state	Reference period 50 years	Reference period 1 year
Ultimate	3.8	4.7
Fatigue	1.5 - 3.8	
Serviceability (irreversible)	1.5	3.0

Target reliability index / maximum probability of failure

<u>NKB, 1978:</u>

Failure type I:	Ductile failures with an extra carrying capacity beyond the defined resistance.
Failure type II:	Ductile failures without an extra carrying capacity.
Failure type III:	Failures such as brittle failure and instability failure.
Safety classes:	
Less serious:	1- and 2-storey buildings, which only occasionally hold persons
Serious:	Buildings of more than two stories which only occasionally hold people
Very serious:	Buildings of more than two stories and stages which often hold many persons

Safety class	Failure type I	Failure type II	Failure type III
Less serious	10^{-3}	10^{-4}	10^{-5}
Serious	10^{-4}	$\boxed{10^{-5}}$	10^{-6}
Very serious	10^{-5}	10^{-6}	10^{-7}

Maximum annual probabilities of failure.

Safety class	Failure type I	Failure type II	Failure type III
Less serious	3.1	3.7	4.3
Serious	3.7	$\boxed{4.3}$	4.7
Very serious	4.3	4.7	5.2

Target (minimum) annual reliability indices β

Reliability level – Wind turbines

IEC 61400-1:2017 (FDIS)

Assumptions:

- A systematic reconstruction policy is used (a new wind turbine is erected in case of failure or expiry of lifetime).
- Consequences of a failure are 'only' economic (no fatalities and no pollution).
- Wind turbines are designed to a certain wind turbine class, i.e. not all wind turbines are 'designed to the limit'.

→ Target reliability level corresponding to an annual nominal probability of failure:

$5\ 10^{-4}$ (annual reliability index equal to 3.3)

Application of this target value assumes that the risk of human lives is negligible in case of failure of a structural element.

Corresponds to minor / moderate consequences of failure and moderate / high cost of safety measure (JCSS)

Exercises, self-study and reading

Read / self-study:

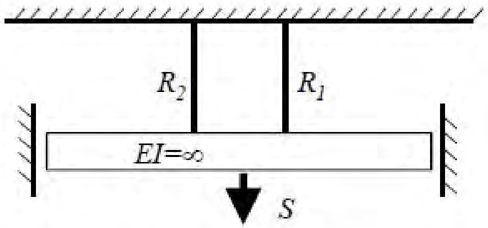
- Additional slides on Daniels systems
- Paper by Gollwitzer & Rackwitz on Daniels systems

Exercise:

- Exercise - Parallel system

Additional slides on Daniels systems
- from S Thöns, DTU

Example for structural system reliability

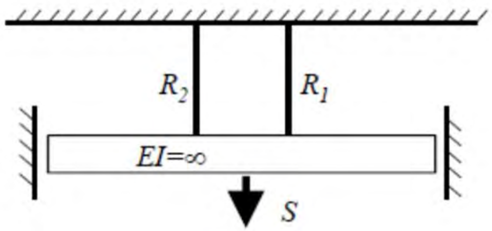


A simple system consisting of 2 components is considered. The correlation of the resistances and the component behaviour is unknown.

- Resistances are defined with mean and standard deviation: $R_1 \sim LN(1.1,0.1)$ and $R_2 \sim LN(1.1,0.1)$
- Loading is defined with mean and standard deviation : $S \sim WBL(1.5,0.2)$

Determine the bounds of system probability of failure.

Example for structural system reliability



What type of system models apply?

- Parallel system
- Ductile Daniels system
- Brittle Daniels System

Example for structural system reliability

$$\prod_{i=1}^{n} P(F_i) \leq P_{F_S} \leq \min_{i=1}^{n}\{P(F_i)\}$$

No correlation Full correlation

- Simple bounds (no and full correlation) for a parallel system

$$P_{F_S} = P\left(\sum_{i=1}^{n} R_i - S \leq 0\right)$$

- Ductile Daniels system for no and full correlation

$$P_{F_S} = \prod_{i=1}^{n} P\left((n-i+1)\hat{R}_i - S \leq 0\right)$$

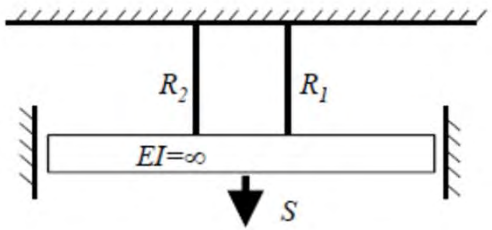
$$\hat{R}_1 \leq \hat{R}_2 \leq \ldots \leq \hat{R}_n$$

- Brittle Daniels system for full and no correlation

What solution methods can be applied?

How to account for the unknown correlation?

Example for structural system reliability



A simple system consisting of 2 components is considered. The correlation of the resistances and component behaviour is unknown.

Results:

- Simple bounds (no and full correlation) for a parallel system

$$6.3\cdot10^{-6} \leq P_{F_S} \leq 2.5\cdot10^{-3}$$

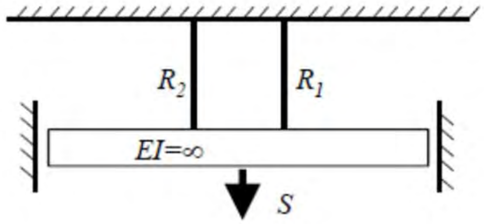
- Ductile Daniels system for no and full correlation

$$3.4\cdot10^{-4} \leq P_{F_S} \leq 2.5\cdot10^{-3}$$

- Brittle Daniels system for full and no correlation

$$2.5\cdot10^{-3} \leq P_{F_S} \leq 4.6\cdot10^{-3}$$

Example for structural system reliability



A simple system consisting of 2 components is considered. The correlation of the resistances and component behaviour is unknown.

Results:

- Bounds considering all system modelling options

Simple bounds, no correlation

$$6.3\cdot 10^{-6} \le P_{F_S} \le 4.6\cdot 10^{-3}$$

Brittle Daniels system, no correlation

- Bounds considering only Daniels system modelling

Ductile Daniels system, no correlation

$$3.4\cdot 10^{-4} \le P_{F_S} \le 4.6\cdot 10^{-3}$$

Brittle Daniels system, no correlation