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Role of SHM
in the Context of Service Life Integrity Management

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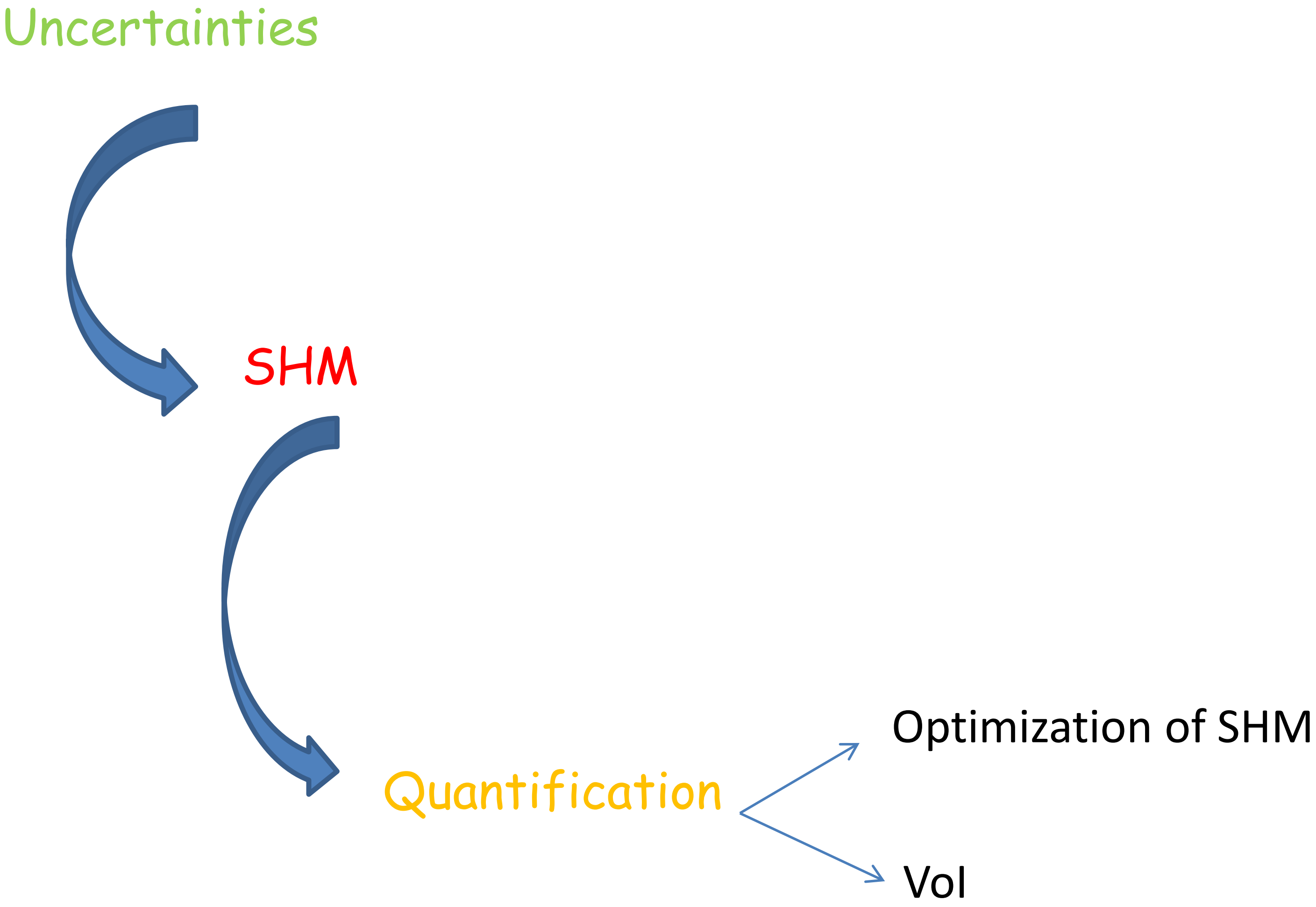
Motivation

Motivation


Uncertainties
SHM
Quantification
Optimization of SHM
VoI

Motivation

What we have:

Assessment of service-life costs

Probabilistic model of structural performance

Prior

annual oberservations

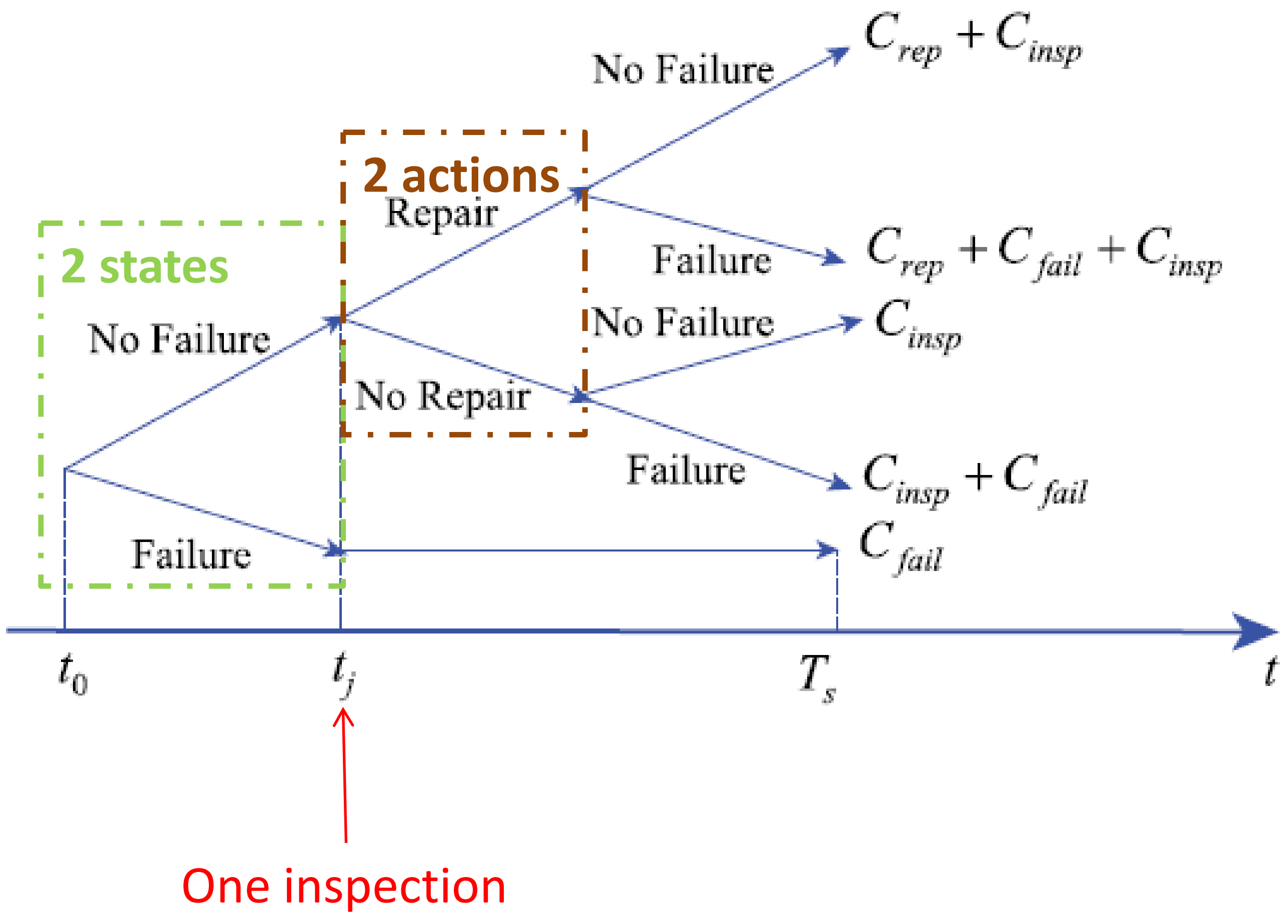
Posterior

Bayesian pre-posterior analysis

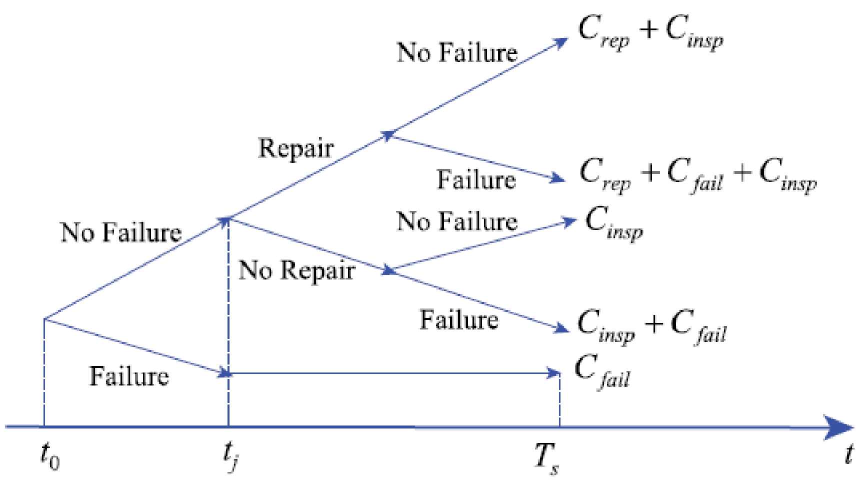
Outline

- Service-life cost assessment
- Probabilistic analysis of structural performance
- Assessment of VoI from annual oberservations of the deterioration
- Example
- Conclusion

Service-life cost assessment



Service-life cost assessment



$$C_{SL}(t_j) =$$

$$P_{\mathbf{S}_{t_j}} C_{insp} \frac{1}{(1+r)^{t_j}} + \sum_{i=1}^{t_j} P_{\mathbf{F}_i} C_{fail} \frac{1}{(1+r)^i} + P_{\mathbf{IR}_{t_j}} C_{rep} \frac{1}{(1+r)^{t_j}} + \sum_{i=t_j+1}^{T_S} P_{\mathbf{F}_i,\mathbf{IR}_{t_j}} C_{fail} \frac{1}{(1+r)^i} + \sum_{i=t_j+1}^{T_S} P_{\mathbf{F}_i,\overline{\mathbf{IR}}_{t_j}} C_{fail} \frac{1}{(1+r)^i}$$

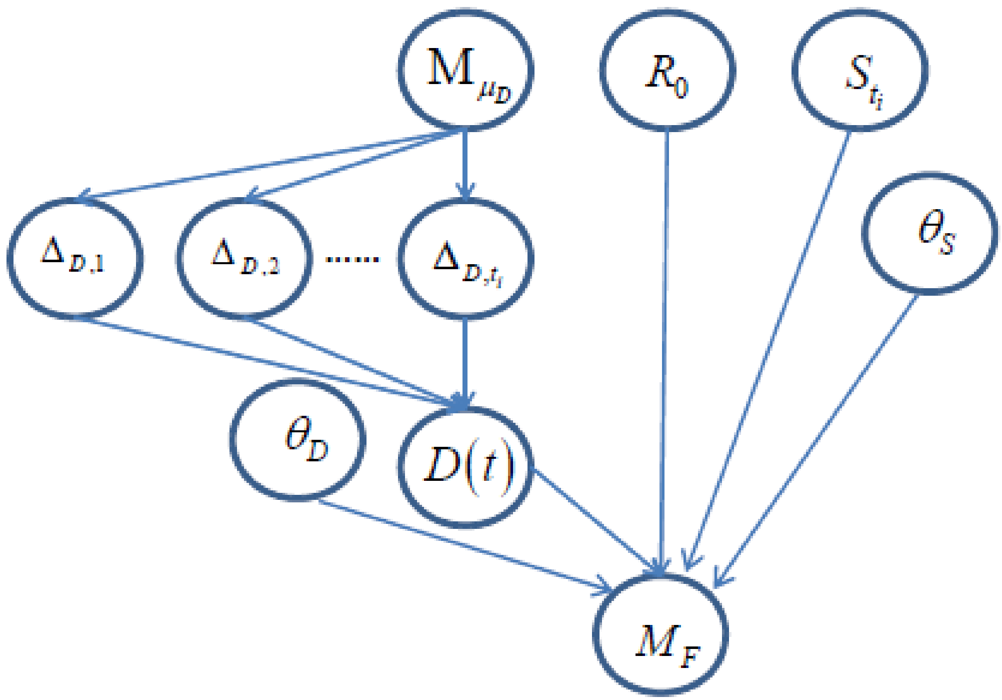
Probabilistic analysis of structural performance

a time dependent ultimate limit state function

$$g\left(\mathbf{X},t\right)=R_0\theta_D\left(1-D\left(t\right)\right)-z\theta_S\mathbf{S}_t$$

Deterioration $D\left(t\right)=\sum_{i=1}^{t}\Delta_{D,i}$

The event of failure: $\mathbf{F}_{t_i}=$

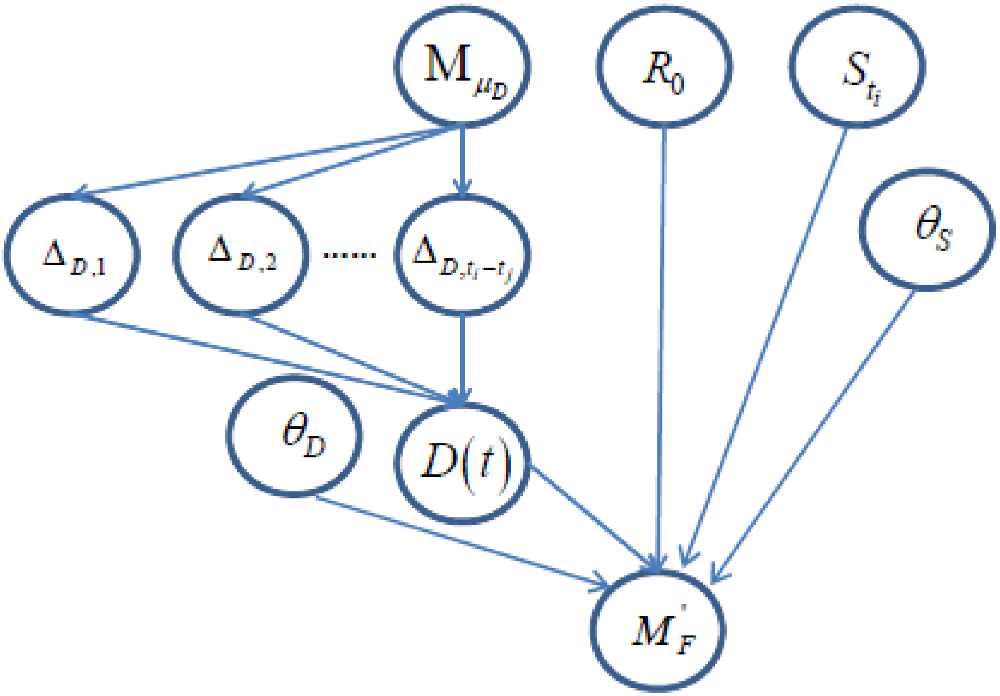


Probabilistic analysis of structural performance

One inspection at t_j

The event of detection and repair: $\mathbf{IR}_{t_j} = \left\{ \sum_{k=1}^{t_j} \Delta_{D,k} \geq D_{IR} \right\}$

The event of failure: $\mathbf{F}_{t_i,\mathbf{IR}_{t_j}} = \left\{ R_0 \theta_D \left(1 - \sum_{k=1}^{t_i - t_j} \Delta_{D,k} \right) - z \theta_S S_{t_i} < 0 \right\}$



$$P_{S_{t_i}} = P\left(\bigcap_{k=1}^{t_i} \overline{\mathbf{F}_k} \right)$$

$$P_{F_{t_i}} = P\left(\bigcap_{k=1}^{t_i - 1} \overline{\mathbf{F}_k} \cap \mathbf{F}_{t_i} \right)$$

$$P_{IR_{t_i}} = P\left(\bigcap_{k=1}^{t_i - 1} \overline{\mathbf{F}_k} \cap \mathbf{IR}_{t_i} \right)$$

$$P_{F_{t_i},\mathbf{IR}_{t_j}} = P\left(\bigcap_{k=1}^{t_j - 1} \overline{\mathbf{F}_k} \cap \mathbf{IR}_{t_j} \bigcap_{l=t_j}^{t_i - 1} \overline{\mathbf{F}_l} \cap \mathbf{F}_{t_i,\mathbf{IR}_{t_j}} \right)$$

$$P_{F_{t_i},\overline{\mathbf{IR}}_{t_j}} = P\left(\bigcap_{k=1}^{t_j - 1} \overline{\mathbf{F}_k} \cap \overline{\mathbf{IR}}_{t_j} \bigcap_{l=t_j}^{t_i - 1} \overline{\mathbf{F}_l} \cap \mathbf{F}_{t_i} \right)$$

Assessment of VoI from annual oberservations of the deterioration

To monitor in t_{mon} years (starting from $t_{mon,st}$) ending at the of the service life

$$\mu''_{M_{\mu_D}}(t_{mon}, \Delta_{D,t_{mon}}) = \frac{\left(\dfrac{\mu_{M_{\mu_D}}}{t_{mon}} + \dfrac{\dfrac{1}{t_{mon}} \sum_{k=1}^{t_{mon}} \Delta_{D,k}}{n'} \right)}{\left(\dfrac{1}{t_{mon}} + \dfrac{1}{n'} \right)}$$

$$\sigma''_{M_{\mu_D}}(t_{mon}, \Delta_{D,t_{mon}}) = \sqrt{\frac{\dfrac{\sigma^2_{\Delta_{D,t_{mon}}}}{n'} \times \dfrac{\sigma^2_{M_{\mu_D}}}{t_{mon}}}{\dfrac{\sigma^2_{M_{\mu_D}}}{n'} + \dfrac{\sigma^2_{\Delta_{D,t_{mon}}}}{t_{mon}}}}$$

$$n' = \frac{\sigma^2_{\Delta_{D,t_{mon}}}}{\sigma^2_{M_{\mu_D}}}$$

Assessment of VoI from annual oberservations of the deterioration

The service-life costs can be written as a function of the outcomes of the monitoring measurements:

$$C_{SL,mon}(t_{mon}, \boldsymbol{\Delta}_{D,t_{mon}}, t_j) =$$

$$\sum_{i=1}^{t_{mon,st}} P_{\mathbf{F}_i} C_{fail} \frac{1}{(1+r)^i} + (1 - \sum_{i=1}^{t_{mon,st}} P_{\mathbf{F}_i}) \times$$

$$\left(P''_{\mathbf{S}_{t_j}} C_{insp} \frac{1}{(1+r)^{t_j}} + \sum_{i=t_{mon,st}}^{t_j} P''_{\mathbf{F}_i} C_{fail} \frac{1}{(1+r)^i} + P''_{\mathbf{IR}_{t_j}} C_{rep} \frac{1}{(1+r)^{t_j}} + \sum_{i=t_j+1}^{T_S} P''_{F_i,\mathbf{IR}_{t_j}} C_{fail} \frac{1}{(1+r)^i} + \sum_{i=t_j+1}^{T_S} P''_{\mathbf{F}_i,\overline{\mathbf{IR}}_{t_j}} C_{fail} \frac{1}{(1+r)^i} \right)$$

Assessment of VoI from annual oberservations of the deterioration

Probabilities:

$$P''_{S_{t_i}} = P\left(\bigcap_{k=t_{mon,st}+1}^{t_i} \overline{\mathbf{F}''_k} \,\middle|\, \bigcap_{l=1}^{t_{mon,st}} \overline{\mathbf{F}_l} \right)$$

$$P''_{F_{t_i}} = P\left(\bigcap_{k=t_{mon,st}+1}^{t_i-1} \overline{\mathbf{F}''_k} \cap \mathbf{F}''_{t_i} \,\middle|\, \bigcap_{l=1}^{t_{mon,st}} \overline{\mathbf{F}_l} \right)$$

$$P''_{IR_{t_i}} = P\left(\bigcap_{k=t_{mon,st}+1}^{t_i-1} \overline{\mathbf{F}''_k} \cap \mathbf{IR}''_{t_i} \,\middle|\, \bigcap_{l=1}^{t_{mon,st}} \overline{\mathbf{F}_l} \right)$$

$$P''_{F_{t_i},\mathbf{IR}_{t_j}} = P\left(\bigcap_{k=t_{mon,st}+1}^{t_j-1} \overline{\mathbf{F}''_k} \cap \mathbf{IR}''_{t_j} \bigcap_{m=t_j}^{t_i-1} \overline{\mathbf{F}''_m} \cap \mathbf{F}''_{t_i,\mathbf{IR}_{t_j}} \,\middle|\, \bigcap_{t_l=1}^{t_{mon,st}} \overline{\mathbf{F}_{t_l}} \right)$$

$$P''_{F_{t_i},\overline{\mathbf{IR}}_{t_j}} = P\left(\bigcap_{k=t_{mon,st}+1}^{t_j-1} \overline{\mathbf{F}''_k} \cap \overline{\mathbf{IR}''}_{t_j} \bigcap_{m=t_j}^{t_i-1} \overline{\mathbf{F}''_m} \cap \mathbf{F}''_{t_i} \,\middle|\, \bigcap_{l=1}^{t_{mon,st}} \overline{\mathbf{F}_l} \right)$$

Assessment of VoI from annual oberservations of the deterioration

The decision problem of optimizing the monitoring strategy is again defined as the minimization of the service-life cost:

$$C^*_{mon} = \min_{t_{mon}} \; \mathrm{E}_{\mathrm{M}_{\mu_D}} \left[\min_{\substack{t_j \\ s.t.:\, t_{mon,st} \leq t_j}} C_{SL,mon}(t_{mon}, \boldsymbol{\Delta}_{D,t_{mon}}, t_j) \right]$$

VoI (Value of SHM):

$$\mathrm{VoI}_{mon} = C^* - C^*_{mon} = \min_{t_j} \mathrm{C}_{SL}(t_j) - \min_{t_{mon}} \; \mathrm{E}_{\mathrm{M}_{\mu_D}} \left[\min_{\substack{t_j \\ s.t.:\, t_{mon,st} < t_j}} C_{SL,mon}(t_{mon}, \boldsymbol{\Delta}_{D,t_{mon}}, t_j) \right]$$

Example

The structure has a service life of 50 years.
The repair criterion parameter D_{IR} is set to be 0.2.
The probabilistic characteristics of the random variables:

Variable	Distribution	Mean	Standard deviation
R_0	Lognormal	1	0.1
θ_D	Lognormal	1	0.1
θ_S	Lognormal	1	0.1
S_i	Gumbel	1	0.3
$\Delta_{D,i}$	Normal	M_{μ_D}	0.1
M_{μ_D}	Normal	0.01	0.01

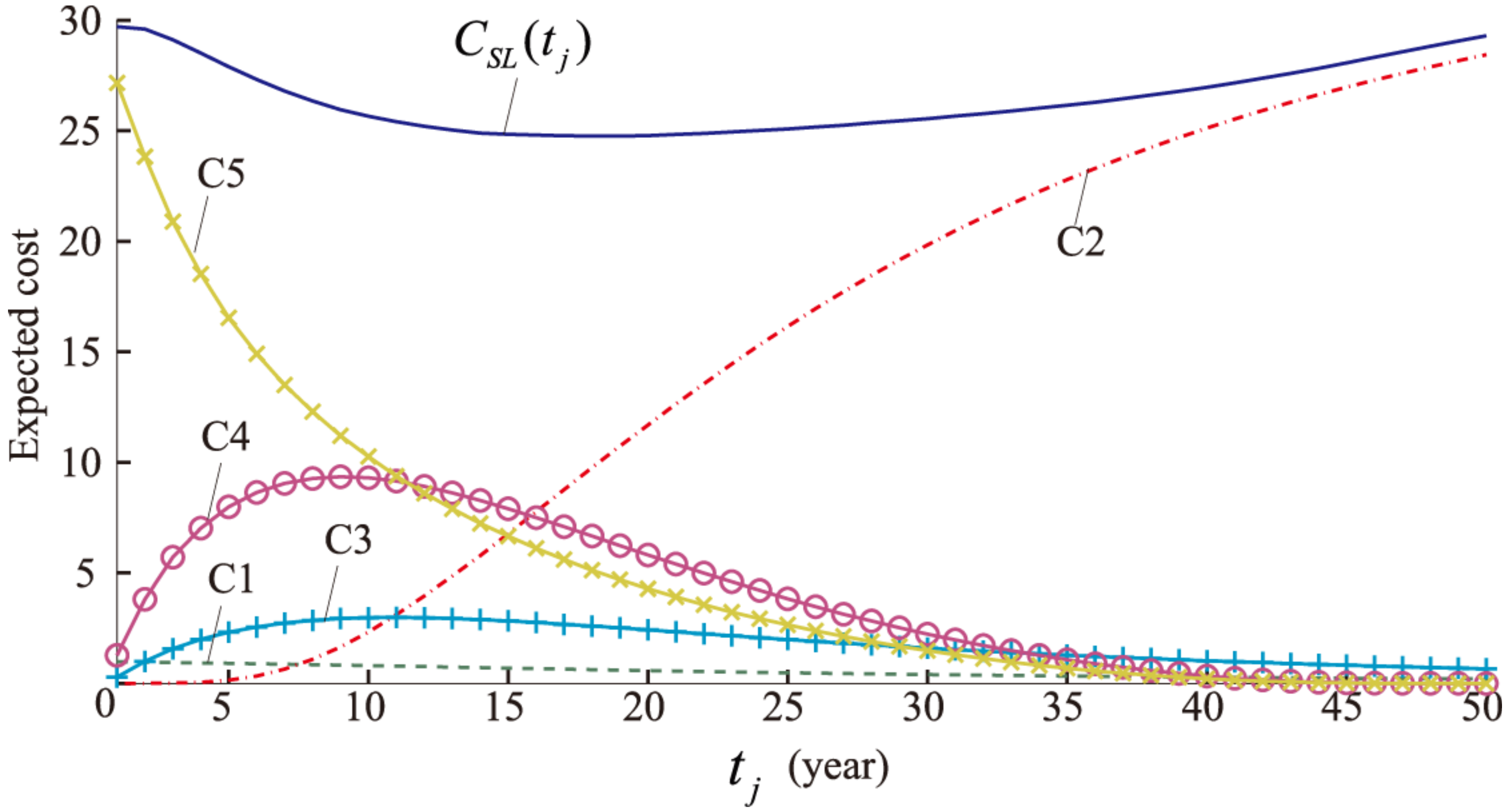
The design parameter z is set to be 0.21 which results in a failure probability at the beginning of the service of 1.1×10^{-5}.

The values of the interest rate, the inspection cost and other cost parameters:

Variable	R	C_{insp}	C_{rep}	C_{fail}
Value	0.02	1	10	100

Example

MC simulation results for the variation of different costs with the inspection time:

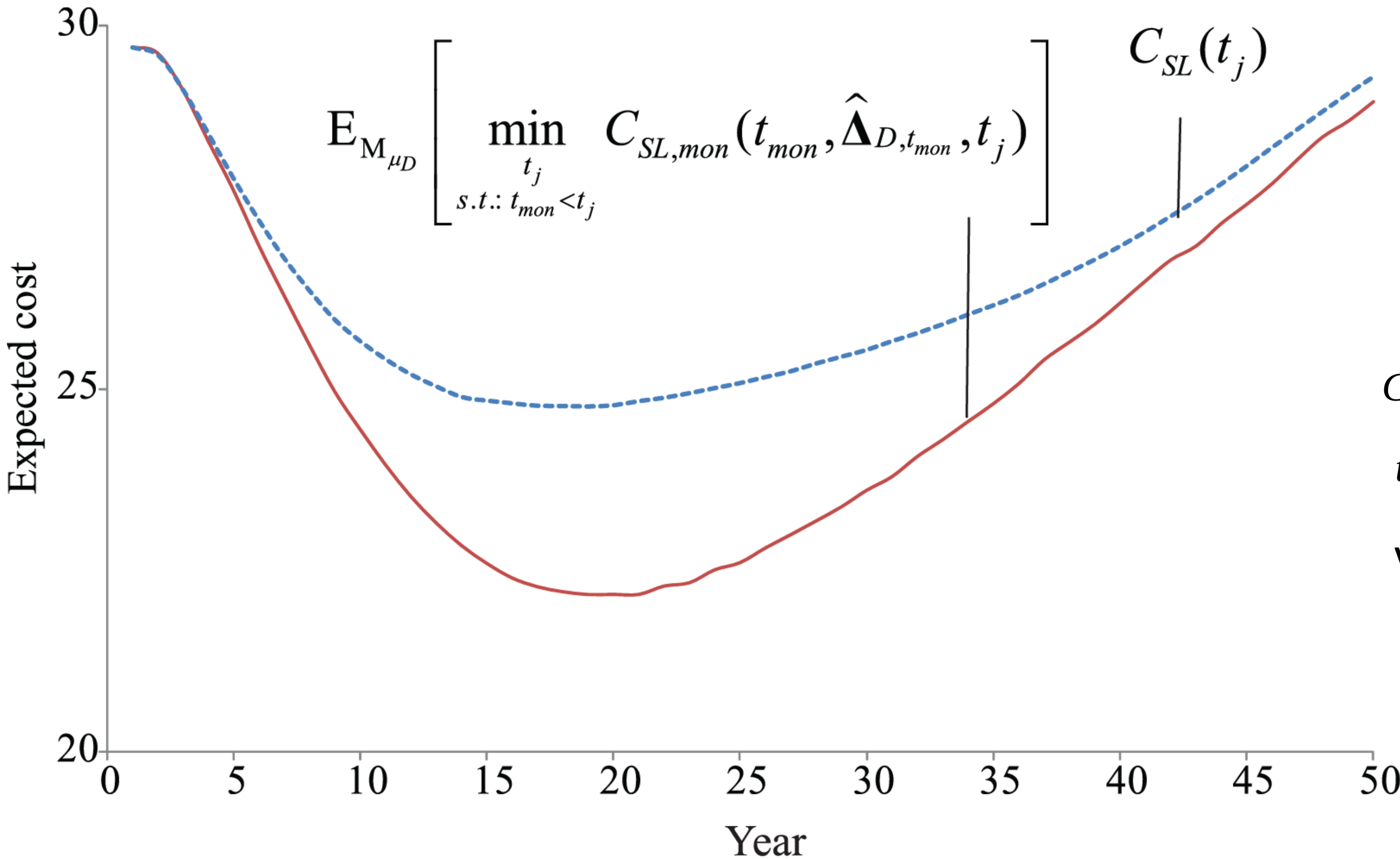


$$C_{SL}(t_j) = C_1 + C_2 + C_3 + C_4 + C_5 =$$

$$P_{\mathbf{S}_{t_j}} C_{insp} \frac{1}{(1+r)^{t_j}} + \sum_{i=1}^{t_j} P_{\mathbf{F}_i} C_{fail} \frac{1}{(1+r)^{i}} + P_{\mathbf{IR}_{t_j}} C_{rep} \frac{1}{(1+r)^{t_j}} + \sum_{i=t_j+1}^{T_S} P_{\mathbf{F}_i,\mathbf{IR}_{t_j}} C_{fail} \frac{1}{(1+r)^{i}} + \sum_{i=t_j+1}^{T_S} P_{\mathbf{F}_i,\overline{\mathbf{IR}}_{t_j}} C_{fail} \frac{1}{(1+r)^{i}}$$

Example

Now, the annual deterioration increment is monitored and these SHM results are utilized to update $M_{\mu D}$ and thus to modify the service-life costs.



$C^*_{mon} = 22.16$

$t_{mon} = 31$

$Vol_{mon}=2.6$

Conclusion

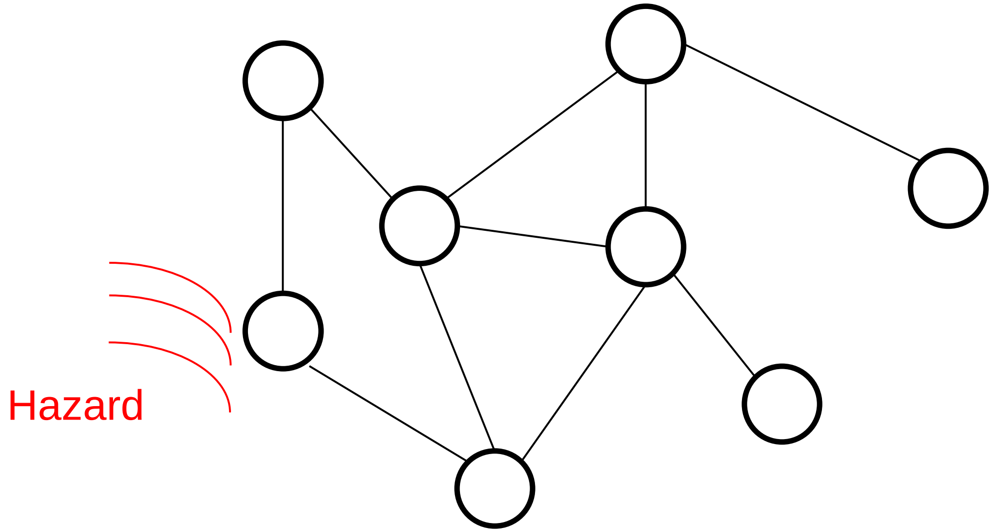
(1) An approach is introduced for the quantification of the value of SHM build upon a service-life cost assessment and a generic structural performance model in conjunction with SHM.

(2) The value of SHM is quantified in the framework of the Bayesian pre-posterior decision theory as the difference between the expected service-life costs considering an optimal structural integrity management and the expected service-life costs utilizing an optimal SHM strategy to support an optimal structural integrity management.

Outlook

Where to implement the monitoring?

Large-scale engineered structures & networked systems



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Thank you for your attention! ☺