COST TU1402: Quantifying the Value of Structural Health Monitoring
Workshop, August 24-28, 2016, DTU, Denmark

Bayesian Analysis Methods

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Outline

BAYESIAN INFERENCE

- Model selection & parameter estimation (Mode Calibration, FE model Updating, Damage Detection)
- Uncertainty propagation – updating structural reliability
- Optimal Experimental Design & Decision Analysis under Uncertainty

BAYESIAN COMPUTATIONAL TOOLS

- Asymptotic approximations + Sampling Techniques

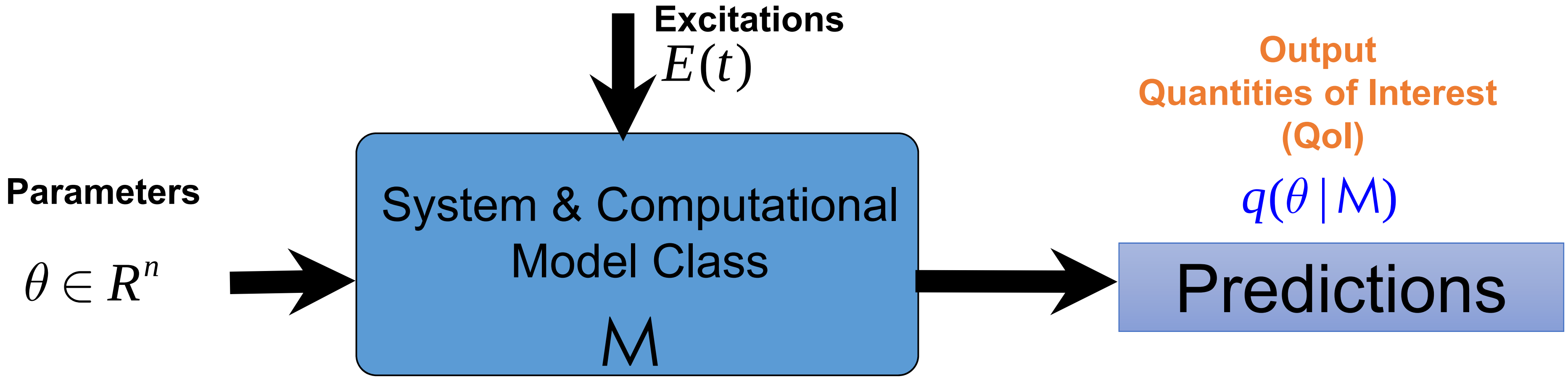
CHALLENGES IN STRUCTURAL DYNAMICS

- Theoretical, Algorithmic, Computational

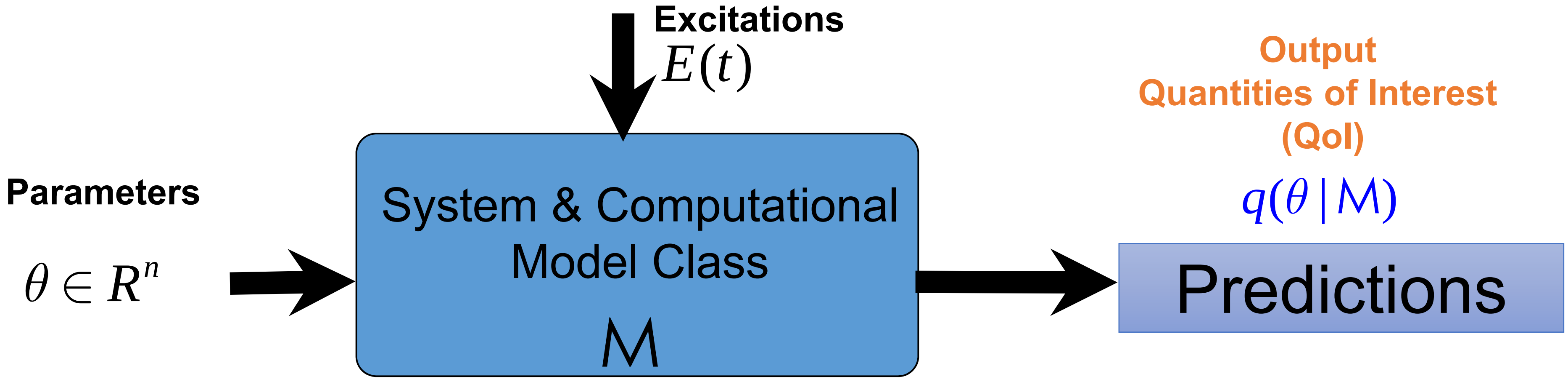
APPLICATIONS

CONCLUSIONS

Structural Dynamics Models



Structural Dynamics Models



- **Nonlinear / linear model classes – Governing Equation of Motion (e.g. FE model)**

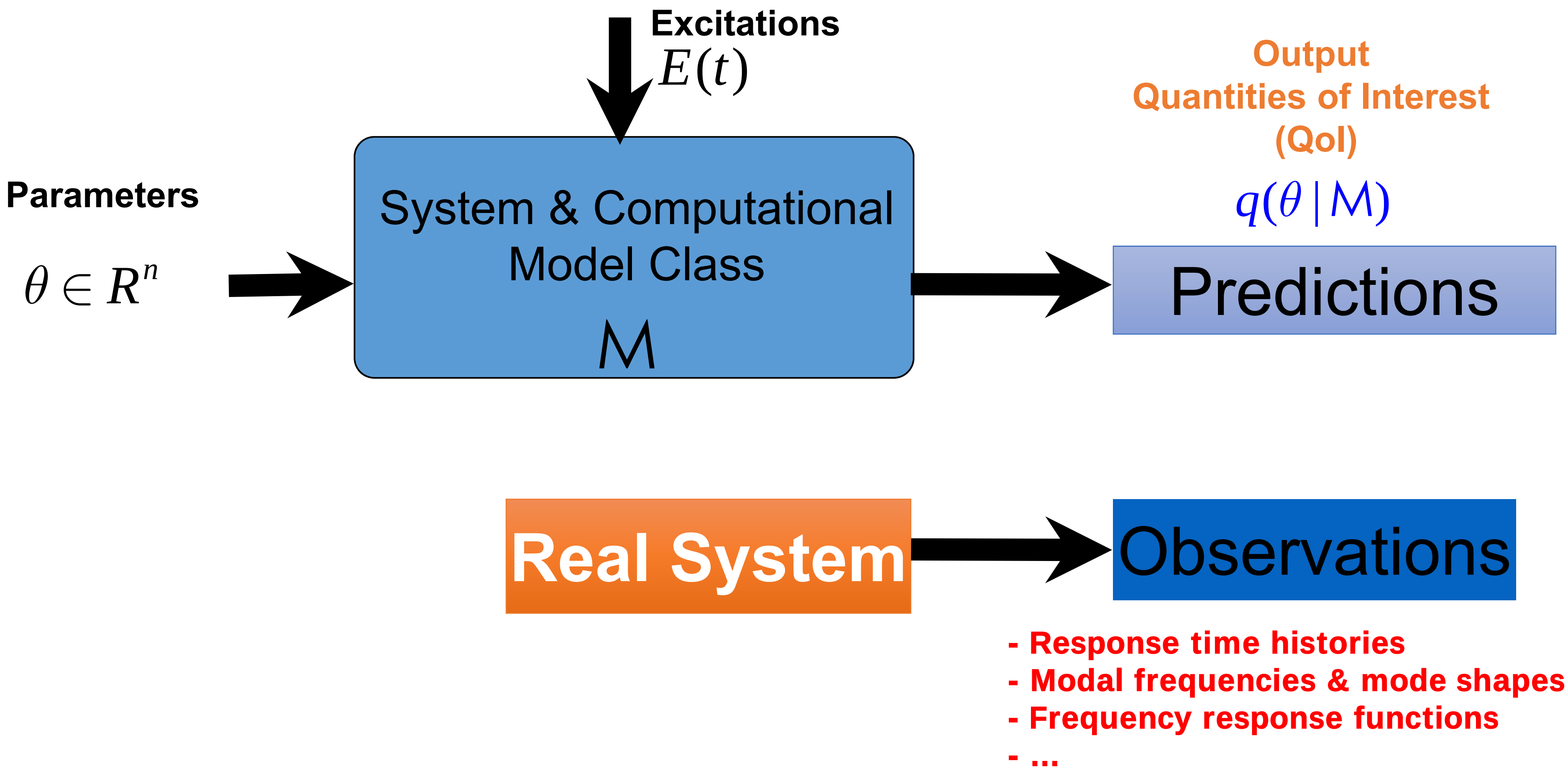
$$M(\theta)\ddot{u}(t) + g(u(t), \dot{u}(t); \theta) = L(\theta)\ E(t)$$

$$q(\theta \mid \mathcal{M}) = Q(u, \dot{u}, \ddot{u}, E, \theta)$$

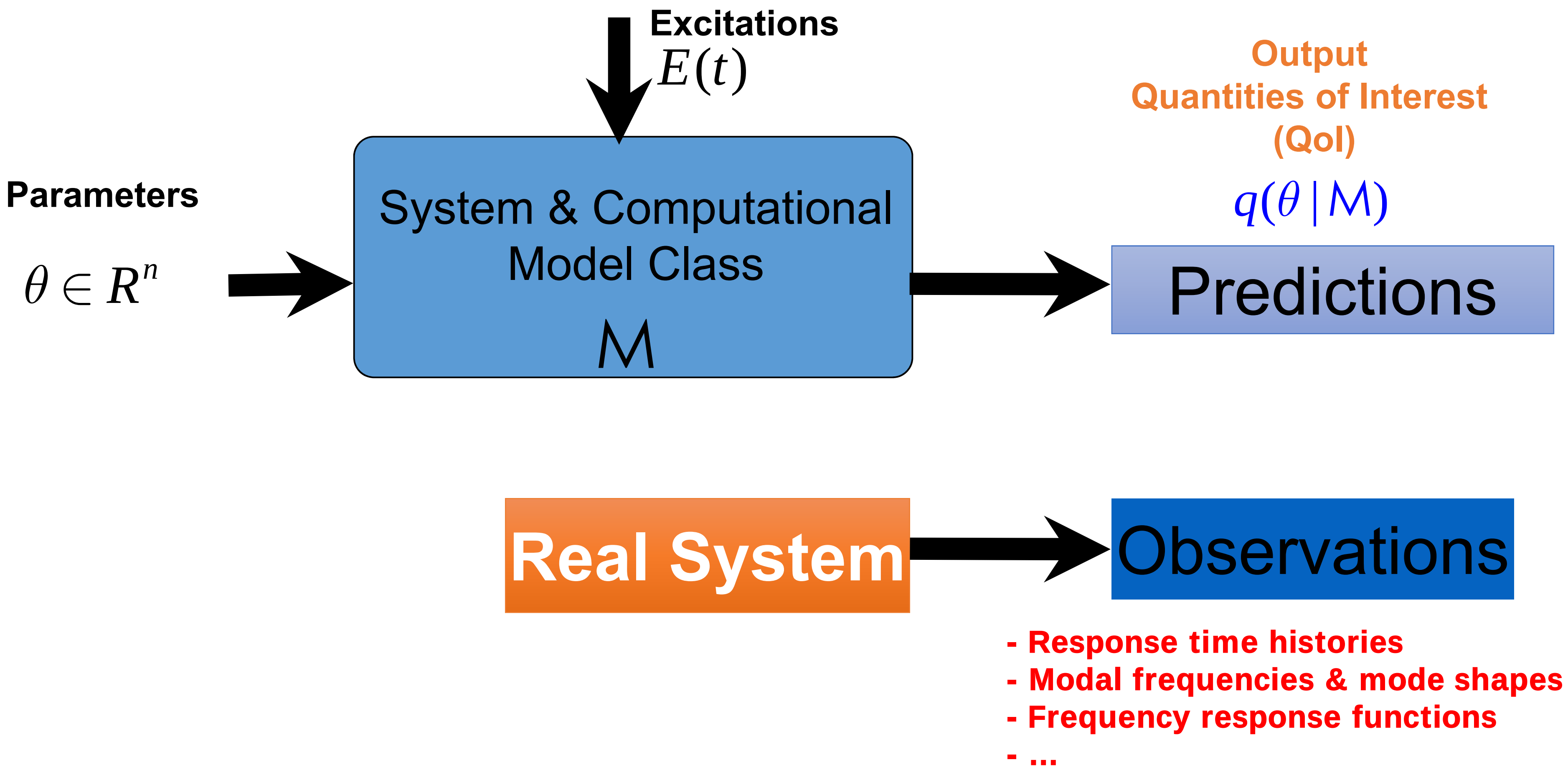
- **Linear model classes – Eigenproblem**

$$[K(\theta) - \omega^2 M(\theta)]\phi = 0$$

Bayesian UQ+P



Bayesian UQ+P



<u>Probability models</u> quantify uncertainties & model the missing/incomplete information.
<u>Cox interpretation of probability</u>, representing the degree of belief or plausibility of a proposition based on available information.
<u>Calculus of probability</u> is used for consistent plausible reasoning.

Bayesian Inference

Parameter Estimation

Posterior PDF — Likelihood — Prior PDF

$$p(\theta \mid y, M) = \frac{p(y \mid \theta, M) \;\; \pi(\theta \mid M)}{p(y \mid M)}$$

Evidence

Bayesian Inference

Parameter Estimation

Posterior PDF — Likelihood — Prior PDF

$$p(\theta \mid y, \mathrm{M}) = \frac{p(y \mid \theta, \mathrm{M}) \ \pi(\theta \mid \mathrm{M})}{p(y \mid \mathrm{M})}$$

Evidence

Model Prediction Error Equation

$$\text{Data} \ = \ \text{Model Predictions} + \text{Error}$$

$$y \ = \ q(\theta \mid \mathrm{M}) \ + \ e$$

Bayesian Inference

Parameter Estimation

Posterior PDF — Likelihood — Prior PDF

$$p(\theta \mid y, \mathrm{M}) = \frac{p(y \mid \theta, \mathrm{M}) \;\; \pi(\theta \mid \mathrm{M})}{p(y \mid \mathrm{M})}$$

Evidence

Model Prediction Error Equation

Data $=$ Model Predictions + Error

$$y \;=\; q(\theta \mid \mathrm{M}) \;+\; e \qquad e = e^{e} + e^{m} \sim N(0, \Sigma)$$

Bayesian Inference

Parameter Estimation

Likelihood   Prior PDF

Posterior PDF

$$p(\theta \mid y, M) = \frac{p(y \mid \theta, M) \; \pi(\theta \mid M)}{p(y \mid M)}$$

Evidence

Model Prediction Error Equation

Data = Model Predictions + Error

$$y = q(\theta \mid M) + e \qquad e = e^e + e^m \sim N(0, \Sigma)$$

Likelihood

$$p(y \mid \theta, M) = \frac{|\Sigma(\theta)|^{-1/2}}{(2\pi)^n} \exp\left[-\frac{1}{2}[y - q(\theta \mid M)]^T \Sigma^{-1}(\theta)[y - q(\theta \mid M)]\right]$$

Bayesian Inference

Parameter Estimation

Posterior PDF — Likelihood — Prior PDF

$$p(\theta \mid y, \mathrm{M}) = \frac{p(y \mid \theta, \mathrm{M}) \;\; \pi(\theta \mid \mathrm{M})}{p(y \mid \mathrm{M})}$$

Evidence

Model Prediction Error Equation

$$\text{Data} \;=\; \text{Model Predictions + Error}$$

$$y \;=\; q(\theta \mid \mathrm{M}) \;+\; e \qquad\qquad e = e^{e} + e^{m} \sim N(0, \Sigma)$$

Likelihood

$$p(y \mid \theta, \mathrm{M}) = \frac{|\Sigma(\theta)|^{-1/2}}{(2\pi)^{n}} \exp\left[-\frac{1}{2}[y - q(\theta \mid \mathrm{M})]^{T} \Sigma^{-1}(\theta)[y - q(\theta \mid \mathrm{M})]\right]$$

Model Class Selection

$$\Pr(\mathrm{M}_i \mid y) = \frac{p(y \mid \mathrm{M}_i) \;\; \Pr(\mathrm{M}_i)}{f(y)} \qquad\qquad p(y \mid \mathrm{M}_i) = \int p(y \mid \theta_i, \mathrm{M}_i) \; \pi(\theta_i \mid \mathrm{M}_i) \; d\theta_i$$

Evidence

Bayesian Inference

Parameter Estimation

Posterior PDF — Likelihood — Prior PDF — Evidence

$$p(\theta \mid y, M) = \frac{p(y \mid \theta, M)\ \pi(\theta \mid M)}{p(y \mid M)}$$

Model Prediction Error Equation

Data = Model Predictions + Error

$$y = q(\theta \mid M) + e \qquad e = e^e + e^m \sim N(0, \Sigma)$$

Likelihood

$$p(y \mid \theta, M) = \frac{|\Sigma(\theta)|^{-1/2}}{(2\pi)^n} \exp\left[-\frac{1}{2} [y - q(\theta \mid M)]^T \Sigma^{-1}(\theta) [y - q(\theta \mid M)] \right]$$

Model Class Selection

$$\Pr(M_i \mid y) = \frac{p(y \mid M_i)\ \Pr(M_i)}{f(y)} \qquad p(y \mid M_i) = \int p(y \mid \theta_i, M_i)\ \pi(\theta_i \mid M_i)\ d\theta_i$$

Evidence

Prediction Error Correlation

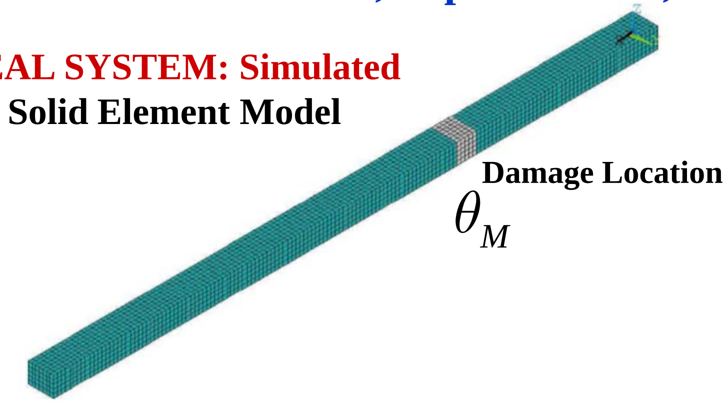
OFTEN USED CASE: Spatially and temporally UNCORELLATED prediction errors

Bayesian Model Class Selection to select among a range of possible prediction error correlation classes and estimate parameters

Example: Damaged R/C Beam

Simoen, Papadimitriou, Lombaert, Journal of Sound and Vibration, 2013

REAL SYSTEM: Simulated
3D Solid Element Model



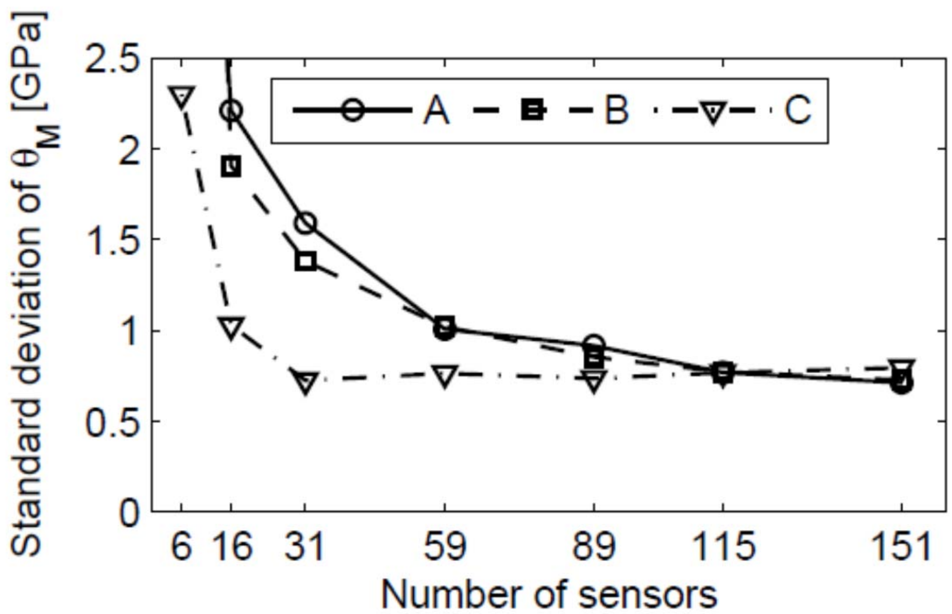
Prediction Error Correlation Models

A. Uncorrelated: 3 parameters
B. Exponentially Corr: 7 parameter
C. Exponential Damped Cosine Corr: 11 parameters

Correlation length depends on mode shape

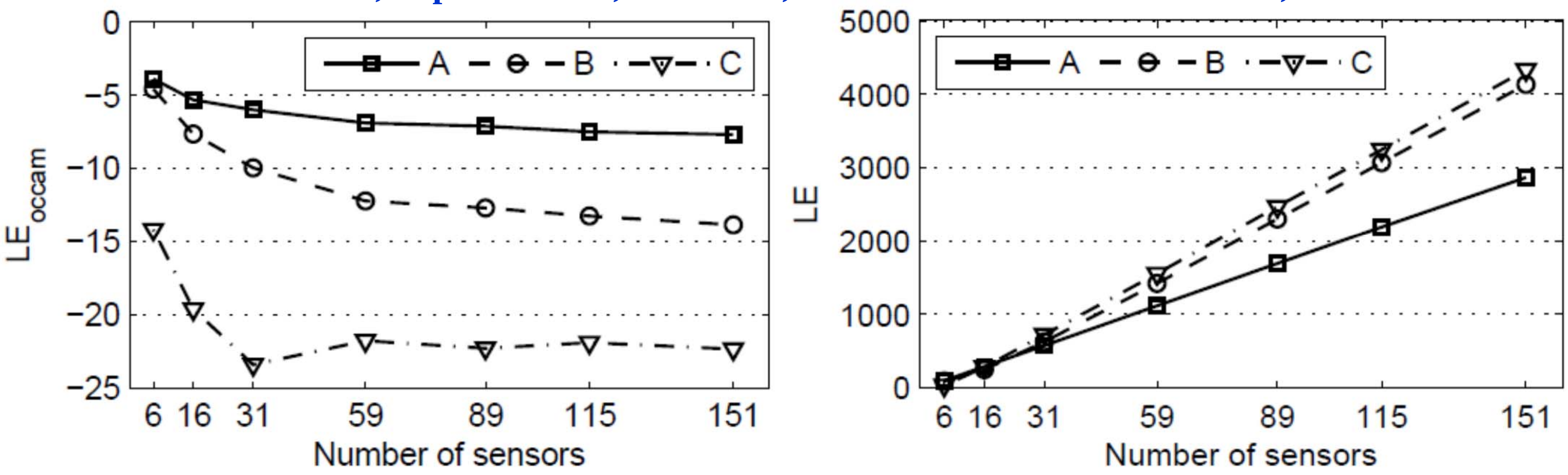
MODEL CLASS
2-D Beam Element Model
150 Elements, 151 nodes

Damage identification
Data: Lowest 4 modal frequencies and mode shapes



Demonstration: Beam model

Simoen, Papadimitriou, Lombaert, Journal of Sound and Vibration, 2013



$$\text{Log Evidence} = \text{LE} = E[\log f(D\,|\,\theta, M)] - E\left[\log \frac{f(\theta\,|\,D, M)}{\pi(\theta\,|\,M)}\right]$$

$$= \text{LE}_{data} + D_{KL}$$

$$= \text{LE}_{data} + \text{LE}_{occam}$$

Average data fit

Information gained from observations

Bayesian Computational Tools

Parameter Estimation

Posterior PDF — Prior — Likelihood — Evidence

$$p(\theta \mid y, M) = \frac{\pi(\theta \mid M)}{p(y \mid M)} \frac{|\Sigma(\theta)|^{-1/2}}{(2\pi)^n} \exp\left[-\frac{1}{2}[y - q(\theta \mid M)]^T \Sigma^{-1}(\theta)[y - q(\theta \mid M)]\right]$$

ASYMPTOTIC APPROXIMATION

Gaussian Posterior PDF

Most Probable Model:

$$\hat{\theta} = \arg\min_{\theta}[-\ln p(\theta \mid y, M)]$$

Covariance:

$H(\theta)$ **is Hessian of** $-\ln p(\theta \mid y, M)$

Optimization problem

- Gradient-based [Adjoint, Model intrusive, Sequential]
- Stochastic: CMA-ES [Gradient-free, Model non-intrusive, Parallel]

CHALLENGES

- Multimodal PDF, Unidentifiable cases
- Moderate to high computational effort due to repeated system analyses

Bayesian Computational Tools

Parameter Estimation

Posterior PDF — Prior — Likelihood — Evidence

$$p(\theta \mid y, M) = \frac{\pi(\theta \mid M)}{p(y \mid M)} \frac{|\Sigma(\theta)|^{-1/2}}{(2\pi)^n} \exp\left[-\frac{1}{2}[y - q(\theta \mid M)]^T \Sigma^{-1}(\theta)[y - q(\theta \mid M)]\right]$$

SAMPLING ALGORITHMS (gradient free, model non-intrusive)

Samples drawn from Posterior PDF (e.g. using variants of MCMC) $\theta^{(i)} \sim p(\theta \mid y, M)$

CHALLENGES

- **C1 (Computational): model complexity (large # of DOFs, Nonlinearities)**
- **C2 (Computational): single chain MCMC algorithms are sequential**
- **A1 (Algorithmic): multi-modal PDF, unidentifiable cases, peaked posteriors**

EFFECTIVE ALGORITHMS: Transitional TMCMC (Annealing property)

Ching & Chen, ASCE Journal of Engineering Mechanics 2007

BUS (independent of number of uncertain parameters)

Straub and Papaioannou ASCE Journal of Engineering Mechanics, 2015

Bayesian Computational Tools

Parameter Estimation

Posterior PDF **Prior** **Likelihood**

$$p(\theta \mid y, \mathcal{M}) = \frac{\pi(\theta \mid \mathcal{M})}{p(y \mid \mathcal{M})} \frac{|\Sigma(\theta)|^{-1/2}}{(2\pi)^n} \exp\left[-\frac{1}{2}[y - q(\theta \mid \mathcal{M})]^T \Sigma^{-1}(\theta)[y - q(\theta \mid \mathcal{M})]\right]$$

Evidence

SAMPLING ALGORITHMS (gradient free, model non-intrusive)

Samples drawn from Posterior PDF (e.g. using variants of MCMC) $\theta^{(i)} \sim p(\theta \mid y, \mathcal{M})$

PARALLELIZATION – SURROGATES - SOFTWARE

- **Multi-Chain, highly parallelized TMCMC, handle various parallelization levels**

  Angelikopoulos, Papadimitriou & Koumoutsakos, J Chemical Physics 2012

- **Surrogate techniques: Adaptive kriging**
- **Improved with Langevin adjusted proposals (adjoint, model intrusive)**

  X-TMCMC: **Angelikopoulos, Papadimitriou & Koumoutsakos, CMAME 2015**

Bayesian Computational Tools

Parameter Estimation

Posterior PDF **Prior** **Likelihood**

$$p(\theta \mid y, \mathcal{M}) = \frac{\pi(\theta \mid \mathcal{M})}{p(y \mid \mathcal{M})} \frac{|\Sigma(\theta)|^{-1/2}}{(2\pi)^n} \exp\left[-\frac{1}{2} [y - q(\theta \mid \mathcal{M})]^T \Sigma^{-1}(\theta) [y - q(\theta \mid \mathcal{M})] \right]$$

Evidence

SAMPLING ALGORITHMS (gradient free, model non-intrusive)

Samples drawn from Posterior PDF (e.g. using variants of MCMC) $\theta^{(i)} \sim p(\theta \mid y, \mathcal{M})$

FURTHER IMPROVEMENTS IN TMCMC

- **Betz, Papaioannou & Straub, ASCE J. Engineering Mechanics, 2016**
- **BASIS – Bayesian Annealing Sequential Importance Sampling**

  Wu, Angelikopoulos, Papadimitriou & Koumoutsakos, ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part B: Mechanical Engineering, 2016 (under review)

Bayesian Computational Tools

Parameter Estimation

Posterior PDF Prior Likelihood

$$p(\theta \mid y, M) = \frac{\pi(\theta \mid M)}{p(y \mid M)} \frac{|\Sigma(\theta)|^{-1/2}}{(2\pi)^n} \exp\left[-\frac{1}{2}[y - q(\theta \mid M)]^T \Sigma^{-1}(\theta)[y - q(\theta \mid M)]\right]$$

Evidence

ASYMPTOTIC APPROXIMATIONS

SAMPLING ALGORITHMS

[gradient free, model non-intrusive]

SOFTWARE

- **Π4U, <u>HPC: Parallel Implementation</u> to optimally distribute the chains and repeated system simulations in a multi-host configuration of complete heterogeneous computer workers.**

  http://www.cse-lab.ethz.ch/software/Pi4U

  Hadjidoukas, Angelikopoulos, Papadimitriou & Koumoutsakos, J Computational Physics 2015

Bayesian Computational Tools

Parameter Estimation

Posterior PDF　Prior　Likelihood

$$p(\theta \mid y, M) = \frac{\pi(\theta \mid M)}{p(y \mid M)} \frac{|\Sigma(\theta)|^{-1/2}}{(2\pi)^n} \exp\left[-\frac{1}{2}[y - q(\theta \mid M)]^T \Sigma^{-1}(\theta)[y - q(\theta \mid M)]\right]$$

Evidence

ASYMPTOTIC APPROXIMATIONS

SAMPLING ALGORITHMS

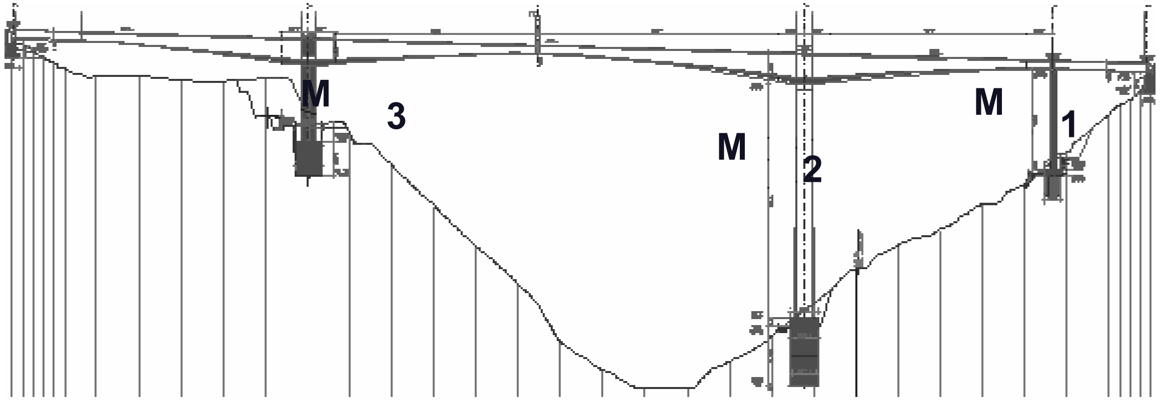
[gradient free, model non-intrusive]

MODEL REDUCTION

- Component mode synthesis (consistent with parameterization scheme) to drastically reduce the system & computational effort without sacrificing accuracy
  - **Papadimitriou & Papadioti, Computer & Structures 2013**
  - **Jensen, Milas, Kusanovic, Papadimitriou, CMAME 2014**
  - **Jensen, Munoz, Papadimitriou, CMAME 2016**

Application: Metsovo Bridge

R/C Bridge at Metsovo, Greece



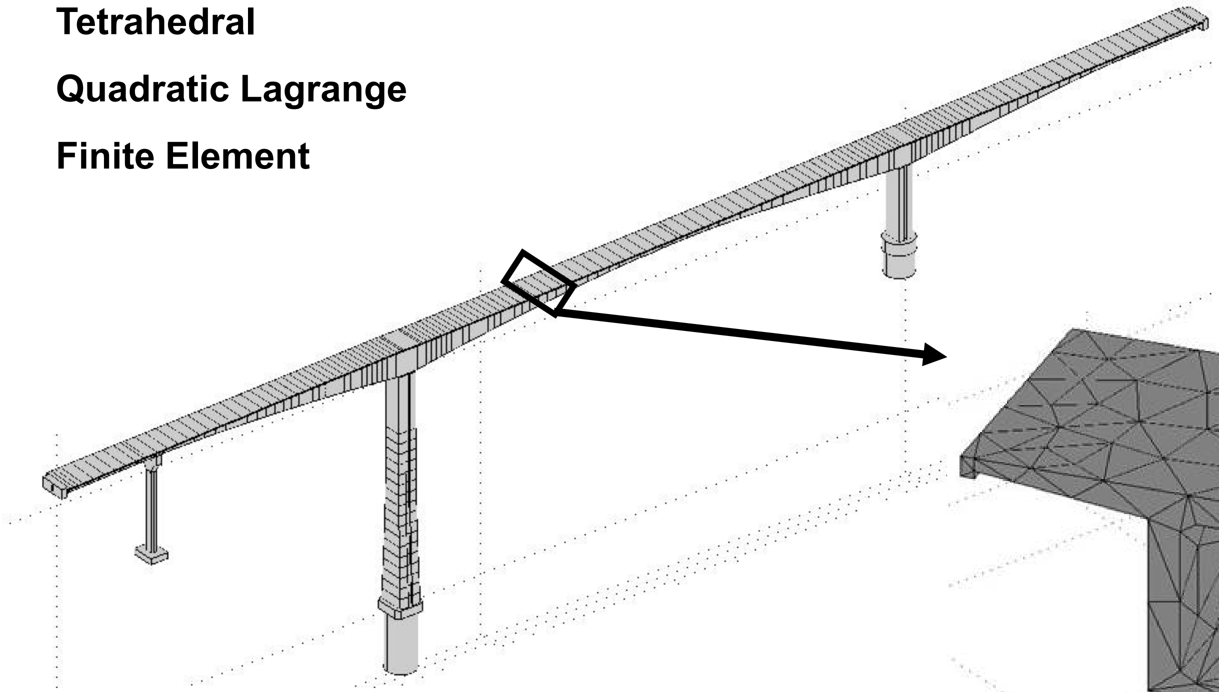
- **Total length: 537m long**
- Deck width: 14m
- M1 height: 45m tall
- **M2 height:110m tall**
- M3 height: 35m tall
- **Central span length: 235m long**

FE Model of Metsovo Bridge

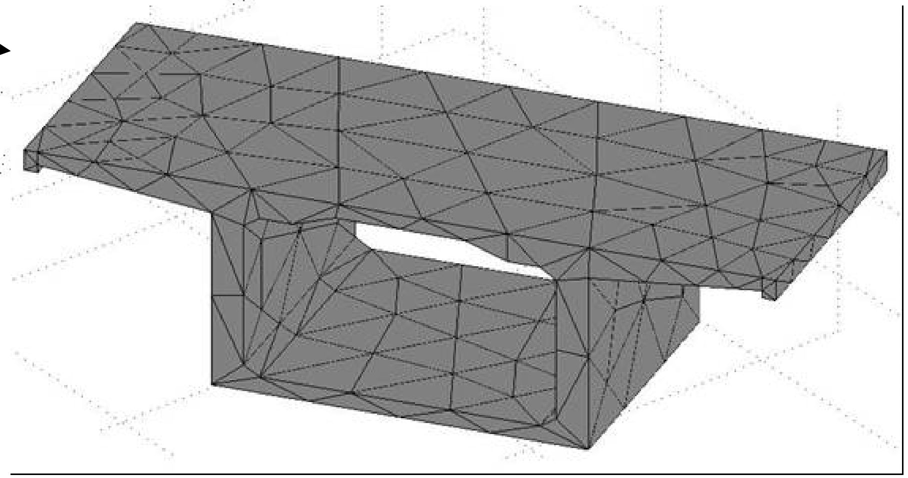
Tetrahedral

Quadratic Lagrange

Finite Element



Extra coarse mesh



DOF: ~ 800,000 - 1,000,000

Largest FE size is limited by

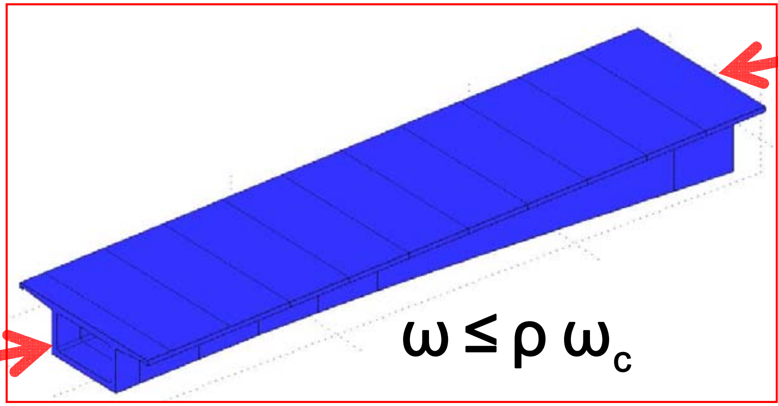
box cross-section thickness.

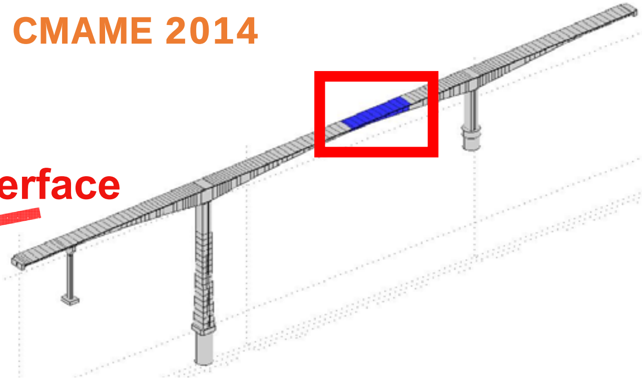
FE size < 0.4m

Length scale (20 modes) > 50m

Component Mode Synthesis (CMS)

Papadimitriou & Papadioti, Computer & Structures 2013
Jensen, Milas, Kusanovic, Papadimitriou, CMAME 2014





ω_c

Cut-off Frequency

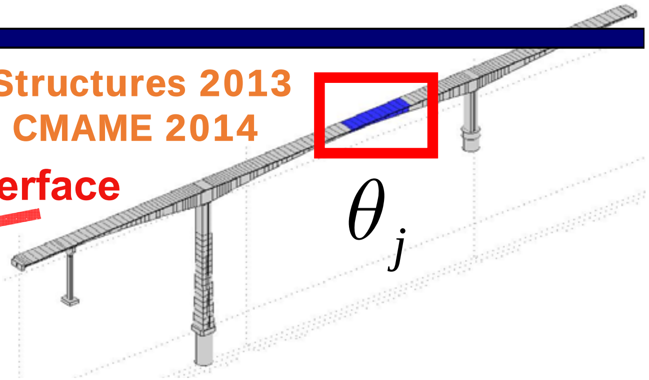
- COMPONENT/SUBSTRUCTURE LEVEL
- Expand the solution within the component in a reduced base
  - Fixed-interface normal modes
    - Retain a small number of eigenvectors (form the reduced basis) and eigenvalues,

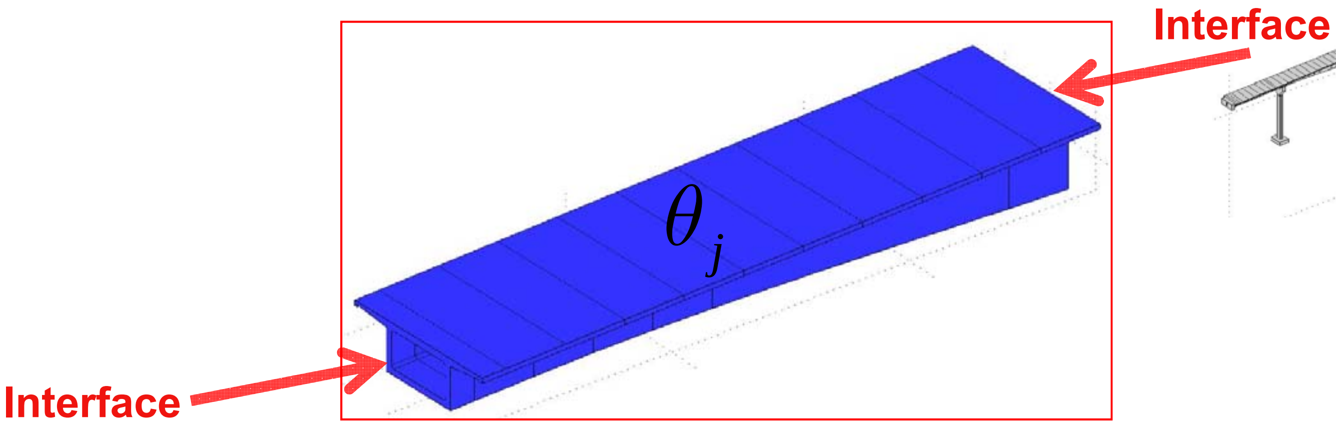
$$\omega \leq \rho\, \omega_c, \qquad \rho = 5$$

  - Interface constrained modes

Model Updating using CMS

Papadimitriou & Papadioti, Computer & Structures 2013
Jensen, Milas, Kusanovic, Papadimitriou, CMAME 2014





- **COMPONENT PARAMETERIZATION**

$$K = \bar{K}\ h(\theta_j) \qquad\qquad M = \bar{M}\ g(\theta_j)$$

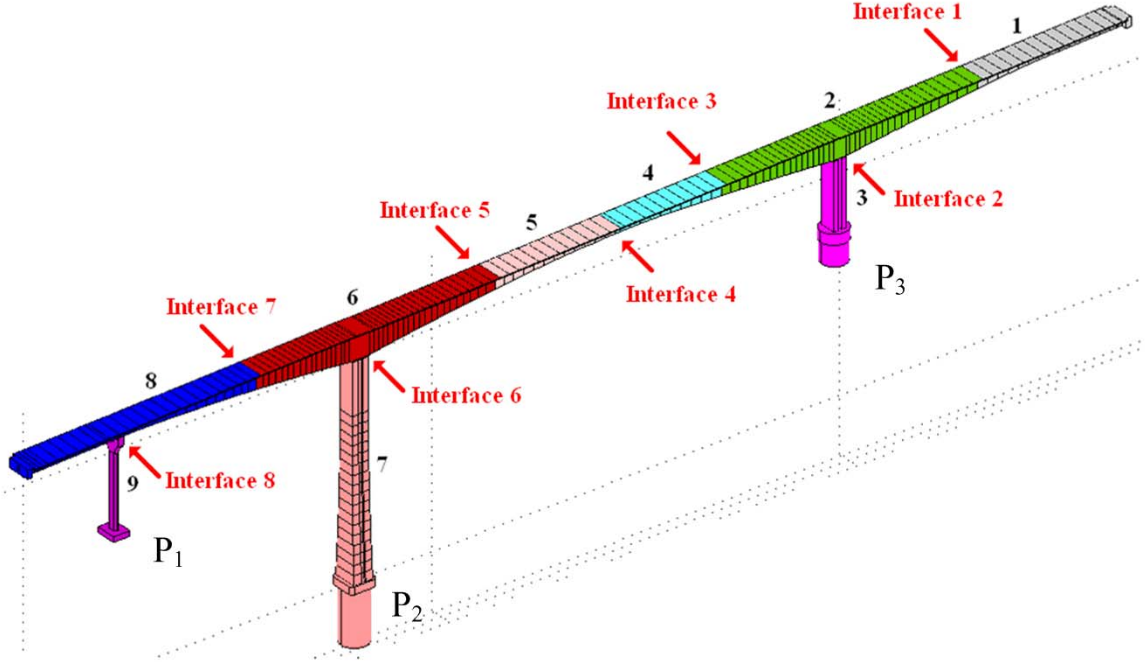
- **Reduced mass and stiffness matrices at system level**

$$K^{CB} = K_0^{CB} + \sum_{j=1}^{N_\theta} K_{1,j}^{CB}\ \frac{h(\theta_j)}{g(\theta_j)} + K_{2,j}^{CB}\ h(\theta_j)$$

$$M^{CB} = M_0^{CB} + \sum_{j=1}^{N_\theta} M_{1,j}^{CB}\ \sqrt{g(\theta_j)} + M_{2,j}^{CB}\ g(\theta_j)$$

Model Updating using CMS

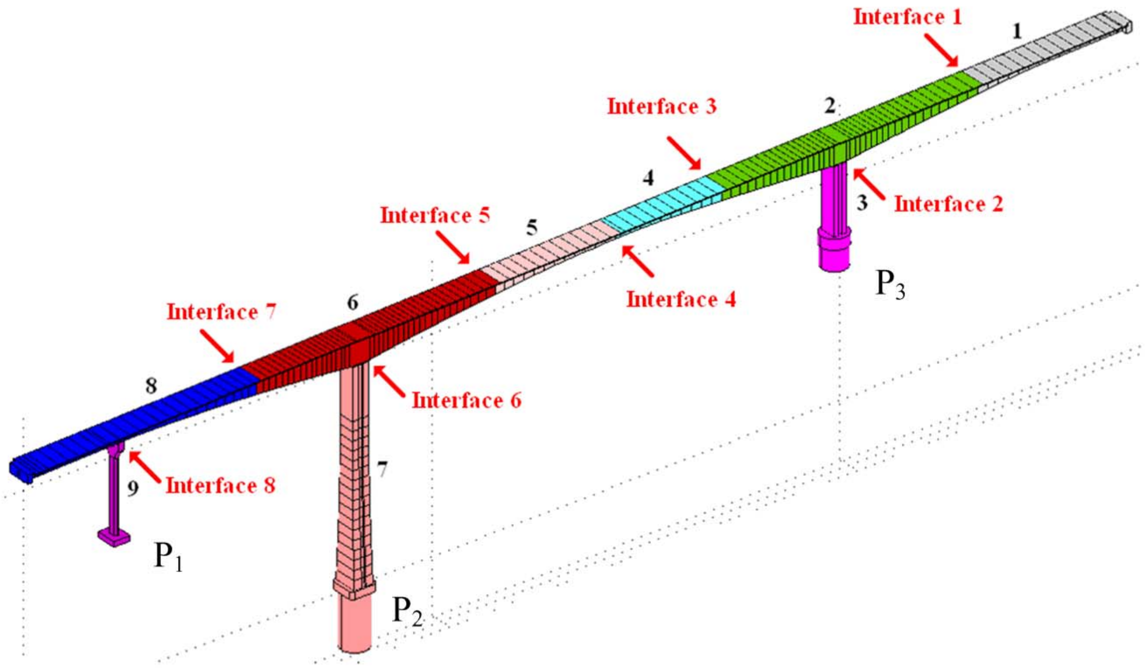
Papadimitriou & Papadioti, Computer & Structures 2013
Jensen, Milas, Kusanovic, Papadimitriou, CMAME 2014



- **Reduction of interface DOFs using characteristic interface modes**

  Castanier et al., AIAA 2001

Model Updating using CMS

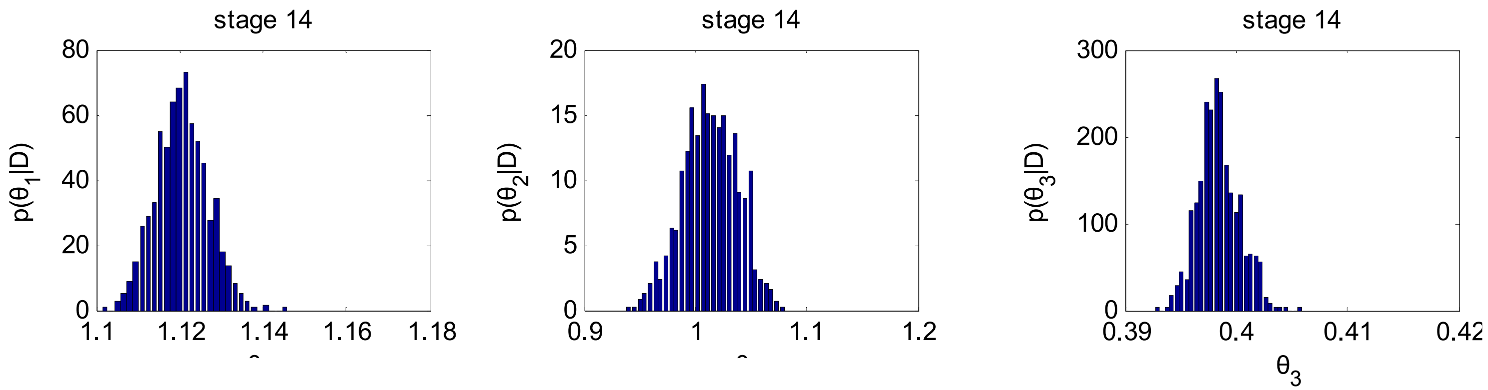


FE model: ~ 1,000,000 DOF
Reduced model: ~ 500 DOF

Three orders of magnitude reduction of DOF for <0.1% accuracy

20 modes retained in analysis

Parameter Estimation using Vibration Data



1. Deck stiffness **2. Pier stiffness** **3. Soil stiffness**

Prediction error models: Spatially correlated

Bayesian Computational Tools

Model Class Selection

$$\Pr(M_i \mid y) = \frac{p(y \mid M_i)\ \Pr(M_i)}{f(y)}$$

Evidence

$$p(y \mid M_i) = \int p(y \mid \theta_i, M_i)\ \pi(\theta_i \mid M_i)\ d\theta_i$$

SAMPLING ALGORITHMS

- **Estimation of EVIDENCE requires special algorithms and extra system simulations**
- **<u>Transitional MCMC (TMCMC)</u>: EVIDENCE does not require extra system simulations**

Applications

- **SHM and damage identification**

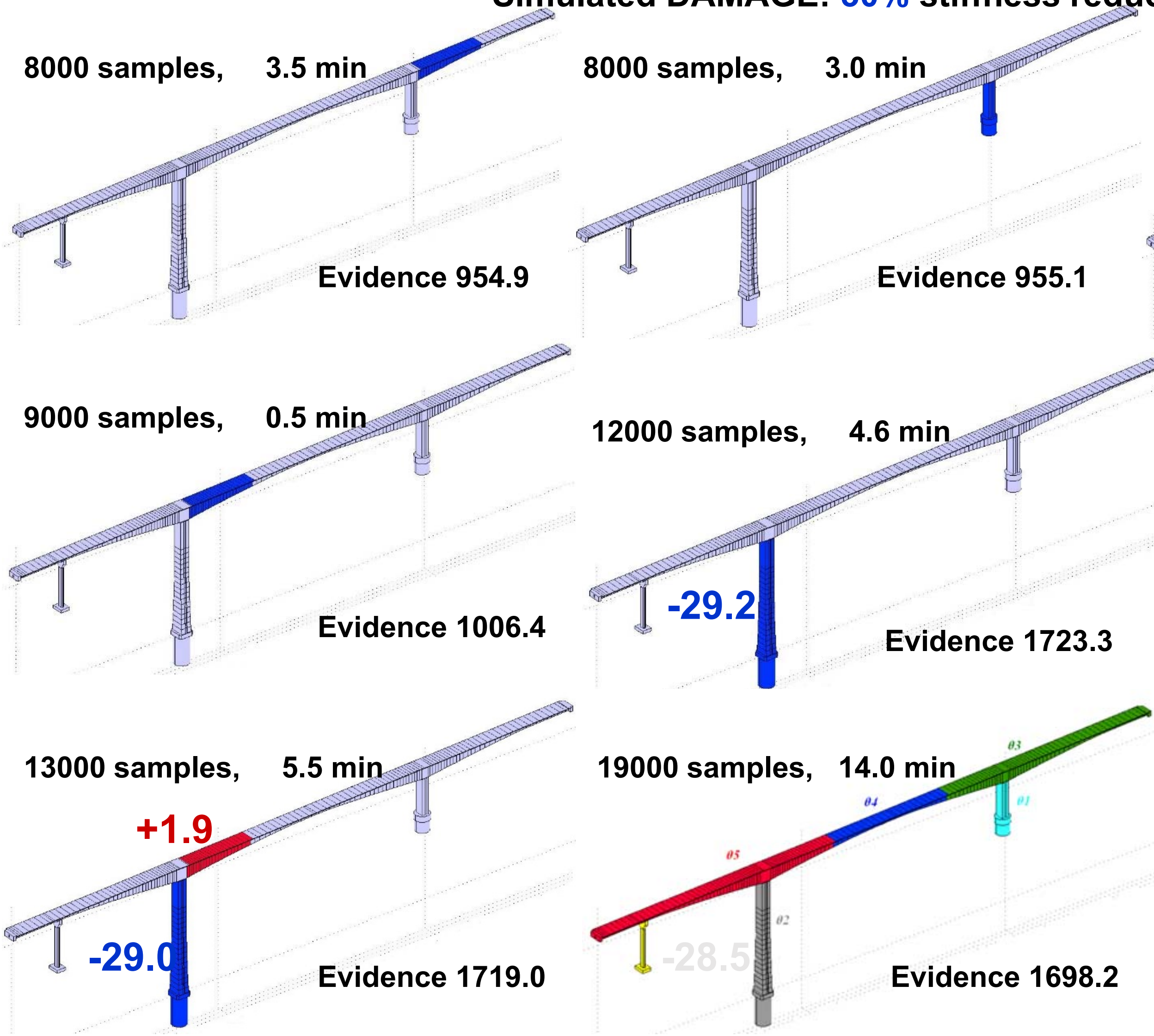
SHM of Bridge: **Computational Effort**

TMCMC

Papadimitriou & Papadioti, Computers & Structures, 2013
Ntotsios, Papadimitriou et al., Bull. Earthquake Eng., 2009

1000 samples/stage

Simulated DAMAGE: 30% stiffness reduction in highest pier



Model Reduction

TIME: 40 minutes

Full Model (no reduction)

TIME: 1 month & 8 days

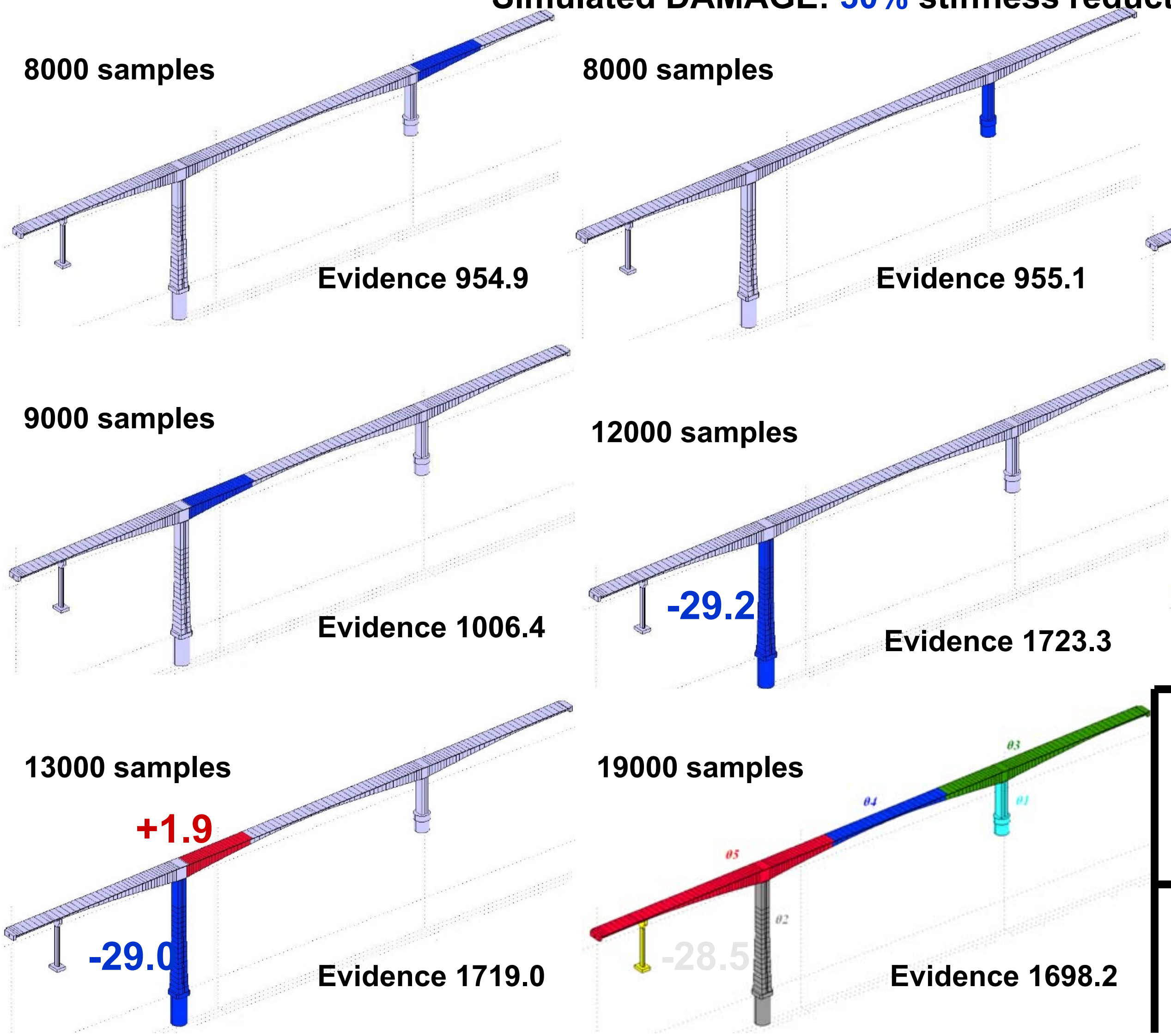
SHM of Bridge: **Computational Effort**

TMCMC

Papadimitriou & Papadioti, Computers & Structures, 2013
Ntotsios, Papadimitriou et al., Bull. Earthquake Eng., 2009

1000 samples/stage

Simulated DAMAGE: 30% stiffness reduction in highest pier



Surrogate Models

TIME: 5 minutes

Full Model (no reduction)

TIME: 1 month & 8 days

SHM of Bridge: **Computational Effort**

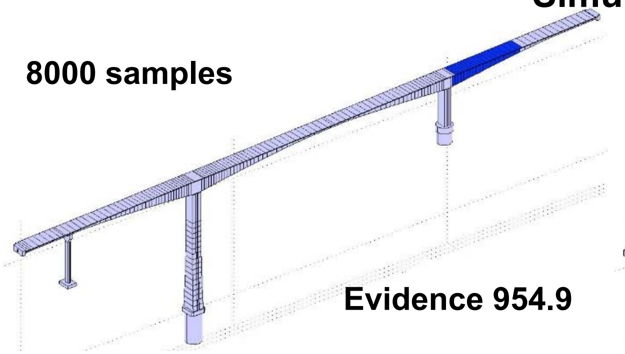
TMCMC

1000 samples/stage

Papadimitriou & Papadioti, Computers & Structures, 2013
Ntotsios, Papadimitriou et al., Bull. Earthquake Eng., 2009

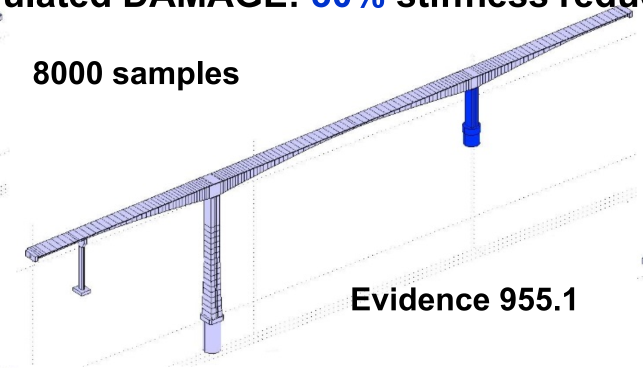
Simulated DAMAGE: 30% stiffness reduction in highest pier

8000 samples



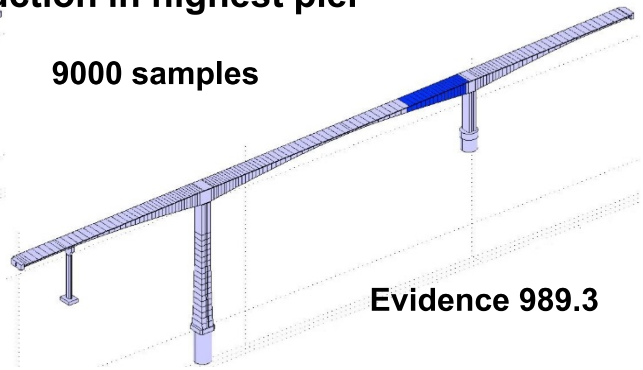
Evidence 954.9

8000 samples



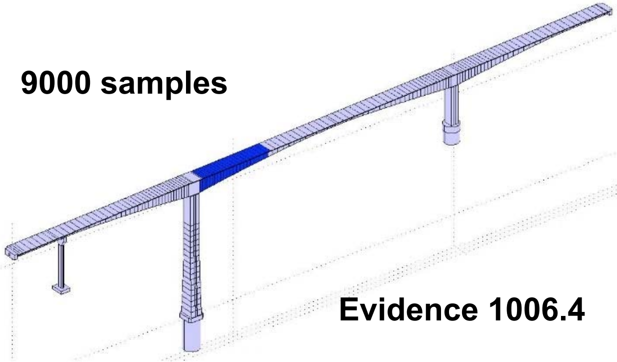
Evidence 955.1

9000 samples



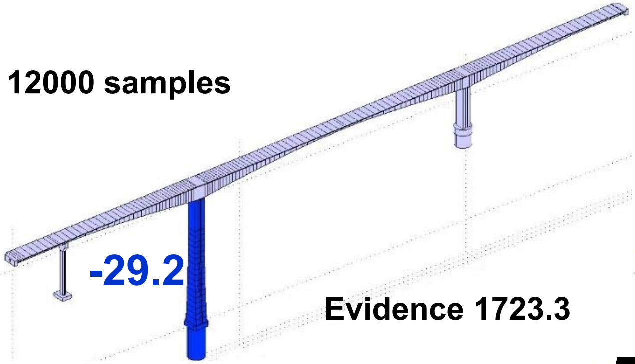
Evidence 989.3

9000 samples



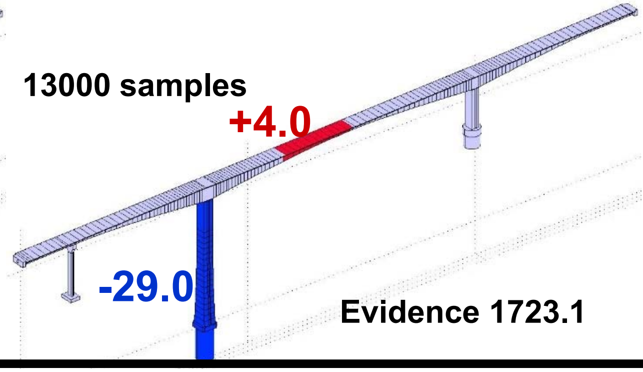
Evidence 1006.4

12000 samples



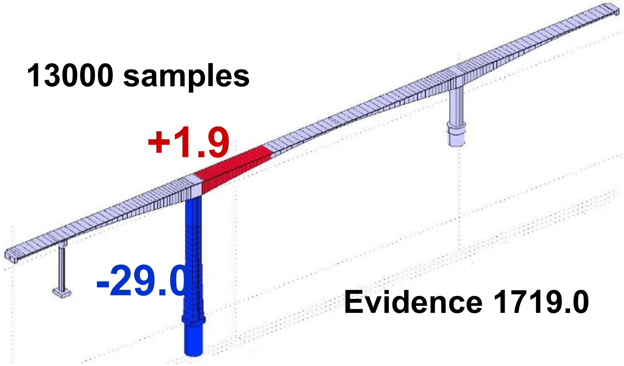
Evidence 1723.3

13000 samples



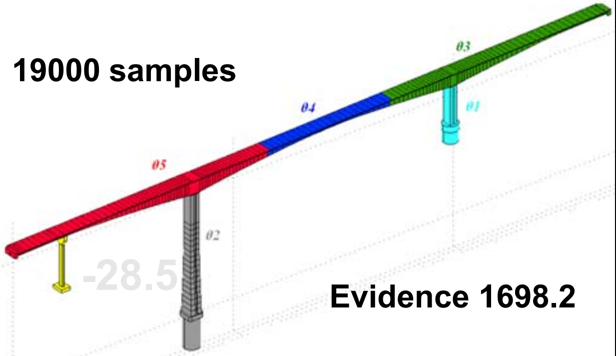
Evidence 1723.1

13000 samples



Evidence 1719.0

19000 samples



Evidence 1698.2

~1000 Computer Workers

TIME: 10 seconds

Full Model (no reduction)

TIME: 1 month & 8 days

Bayesian Uncertainty Propagation

Posterior Robust & Hyper-Robust Predictions

$$E[G_q(\theta) \mid y, M] = \int G_q(\theta \mid M) \; p(\theta \mid y, M) \; d\theta$$

Posterior PDF

- Mean
- Standard deviation
- Credible intervals
- Failure probability
- Robust optimization

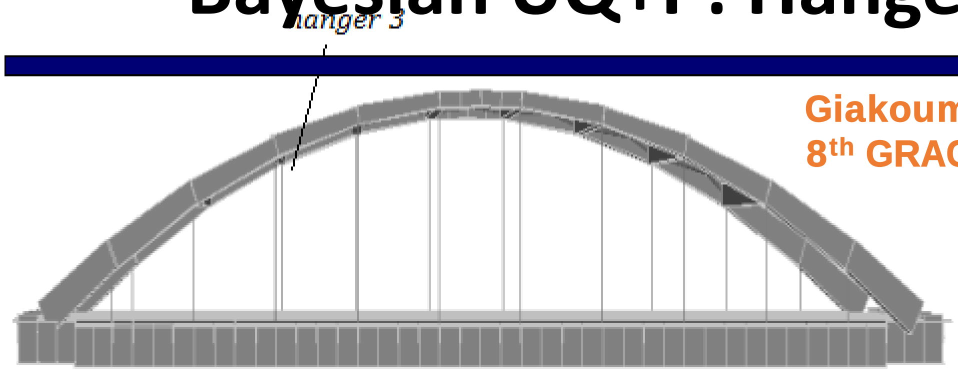
Updating Structural Reliability

$$P_F(M) = \Pr(z \in F \mid M) = \int P_F(z \mid M) \; p(z \mid M) \; dz$$

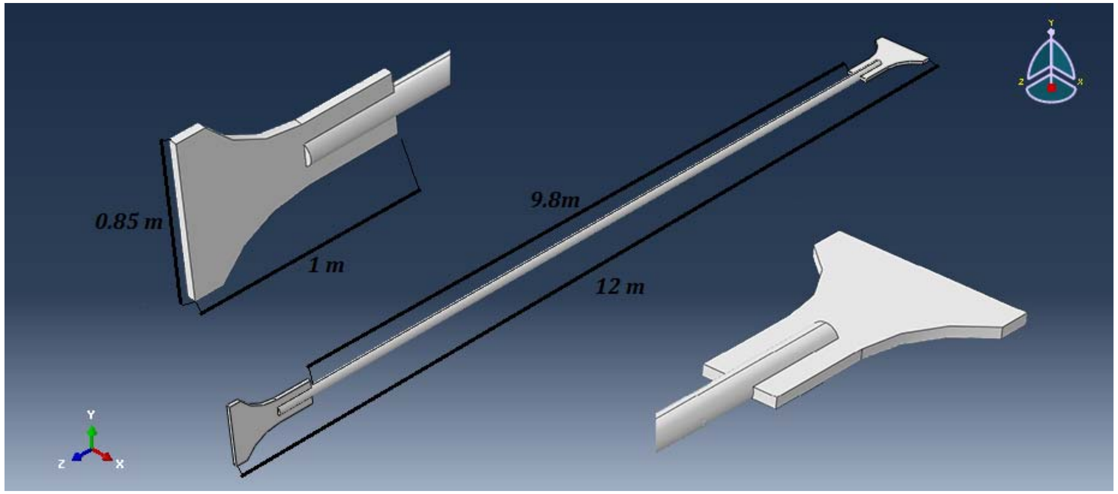
- **Stochastic excitation and uncertain model parameters**
- **PRIOR: Subset simulation**
  Au and Beck, Probabilistic Engineering Mechanics, 2001
  Papaioannou, Betz, Zwirglmaier and Straub, Probabilistic Engineering Mechanics, 2015
- **POSTERIOR: TMCMC samples are used at first stage of Subset simulation**
  Jensen, Vergara, Papadimitriou, Milas, CMAME, 2013

Bayesian UQ+P: Hanger Axial Load Estimation

Giakoumi, Papadimitriou, Argyris, Spyrou, Panetsos,
8th GRACM Int Congress on Computational Mechanics 2015



ARCH bridge with 20 hangers



HANGER geometry & support conditions

MEASURED DATA

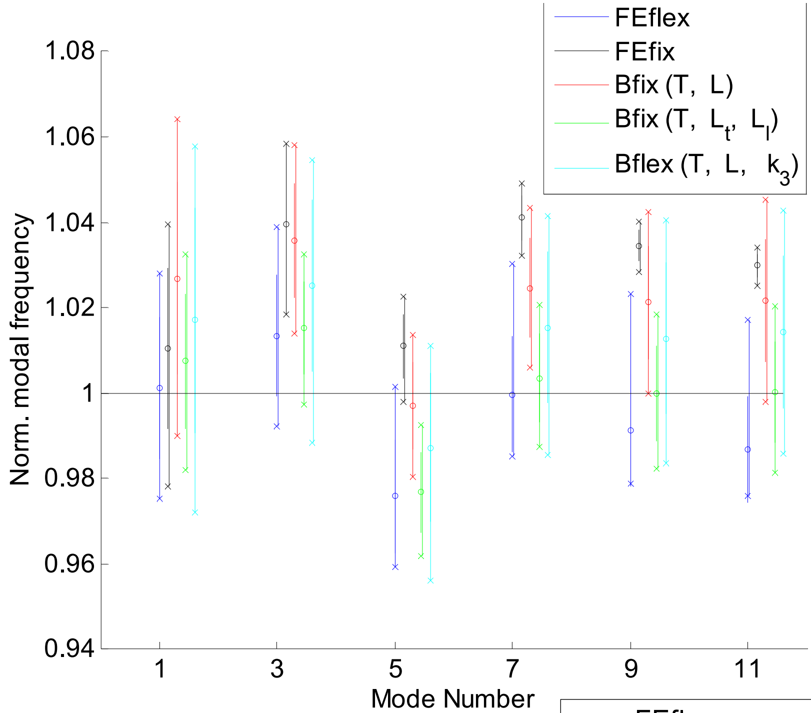
Six (6) modal frequencies per longitudinal and transverse direction

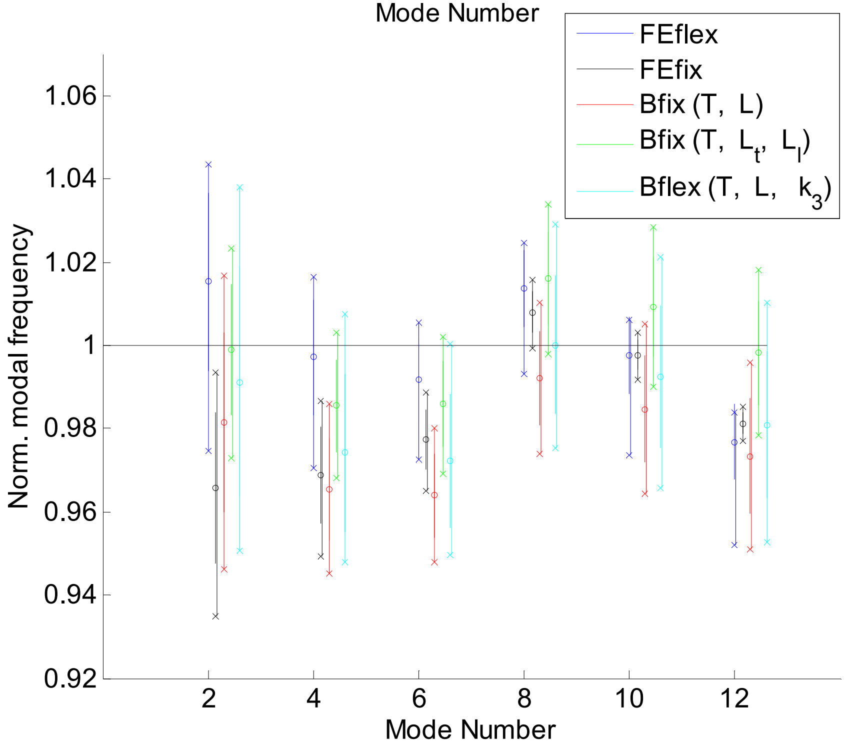
Axial Load Estimation

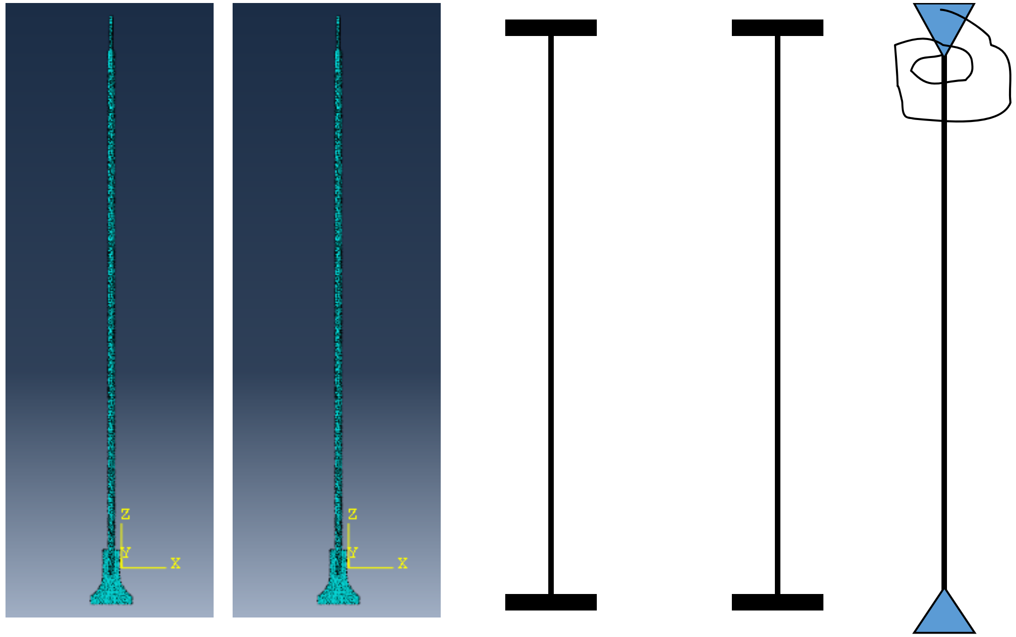
Geometric nonlinear computational mechanics

Estimation of Tangent Stiffness Matrix

Solution of Eigenvalue Problem



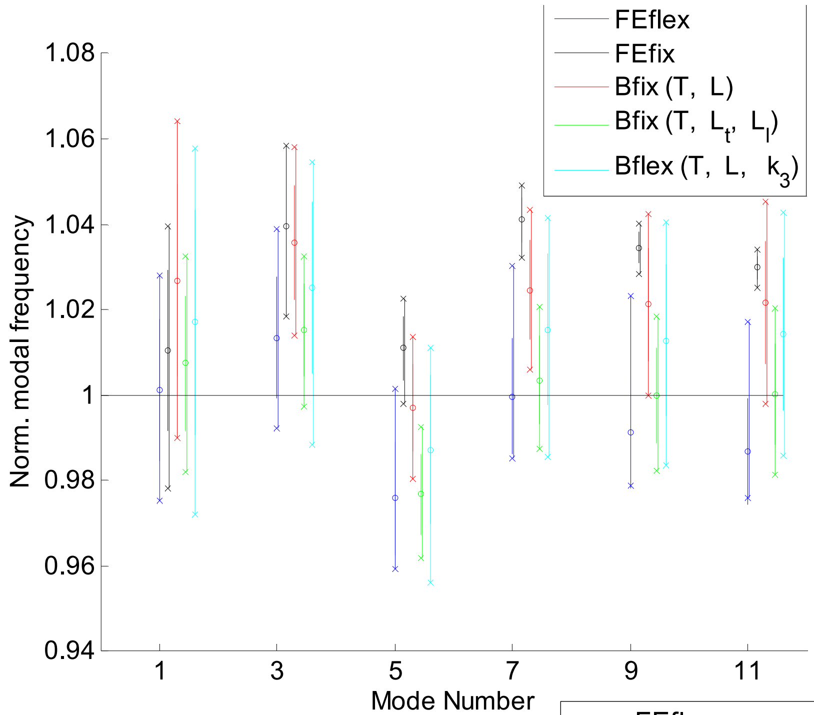


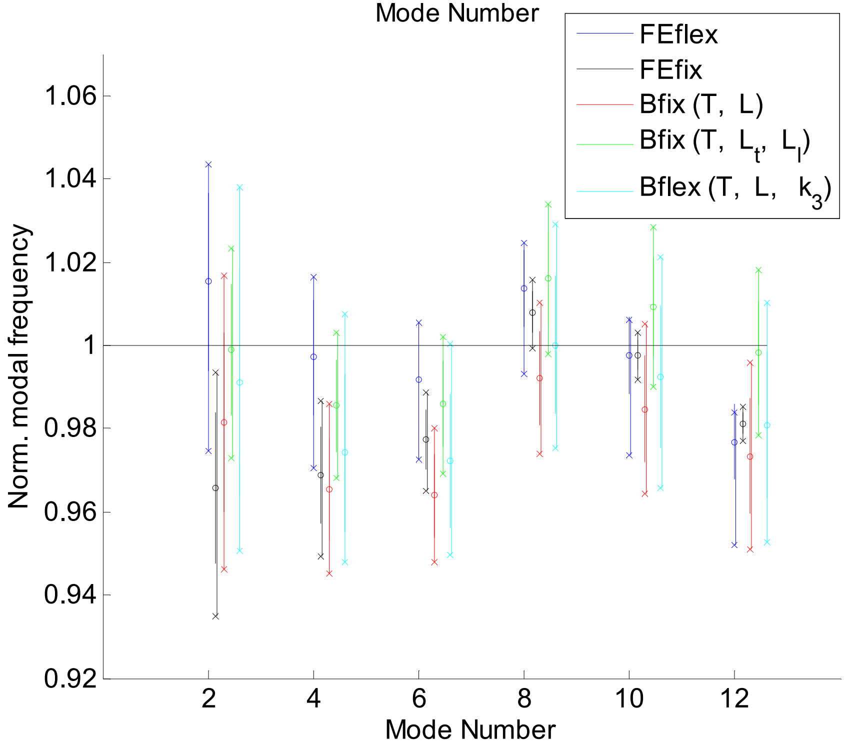


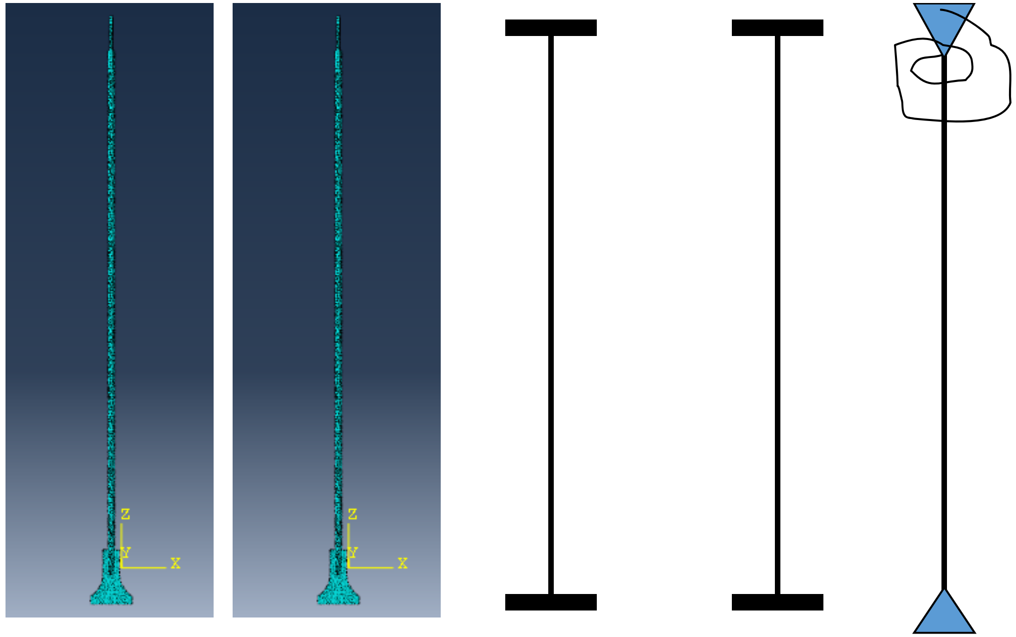
FE Fixed	FE Flex	Beam Fix L	Beam Fix Lt, Ll	Beam Flex; L
Fixed Ends	Bending Stiffness	Fixed ends	Fixed Ends	Bending Stiffness
1 param.	5 param.	2 param.	3 param	3 param.

Model Evidence

22.2	24.0	20.7	22.5	20.9







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Hanger Axial Load Estimation

0.70	0.92	0.86	0.86	0.88
+6%	+9%	+11%	+8%	+12%

Concluding Remarks

- **Unsuitable prediction error models (uncorrelated or correlated) may lead to misleading results in terms of posterior parameter uncertainty. This affects Bayesian uncertainty propagation, Bayesian optimal experimental design and decision making under uncertainty.**

- **Computational demanding operations can be drastically reduced by integrating:**
  - **Model Reduction techniques (e.g. CMS with components consistent with the parameterization schemes)**
  - **Multi-chain MCMC (e.g. TMCMC, X-TMCMC) for parallel implementation in multi-core CPUs to efficiently distribute the large number of full system simulation runs**
  - **Surrogate models (adaptive kriging) for drastically reducing the number of full system simulation runs**

THANK YOU!