Optimal inspection strategies in structural systems

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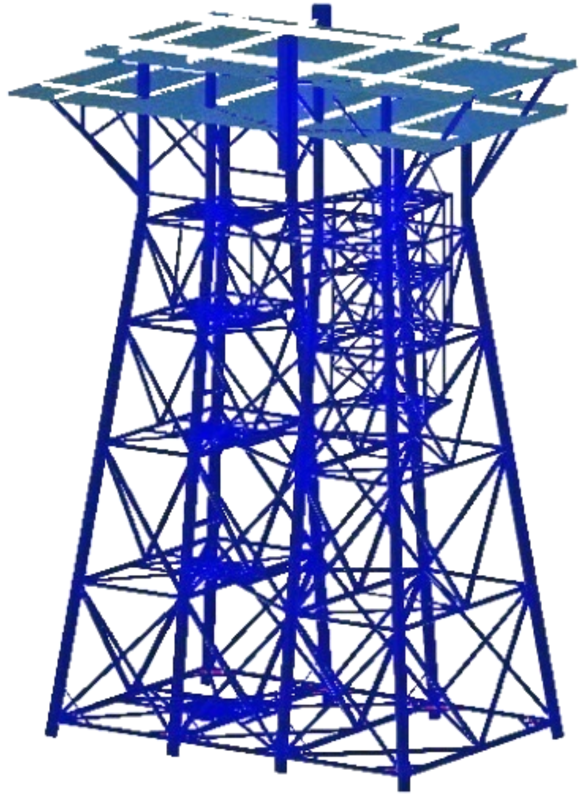
Optimal inspection planning – a system problem

Objective: Minimize total expected lifetime cost and risk of the system

Inspection parameters:

- When?
- Where?
- What?
- How?



Optimal inspection planning – a system problem

Objective: Minimize total expected lifetime cost and risk of the system

Inspection parameters

- When?
- Where?

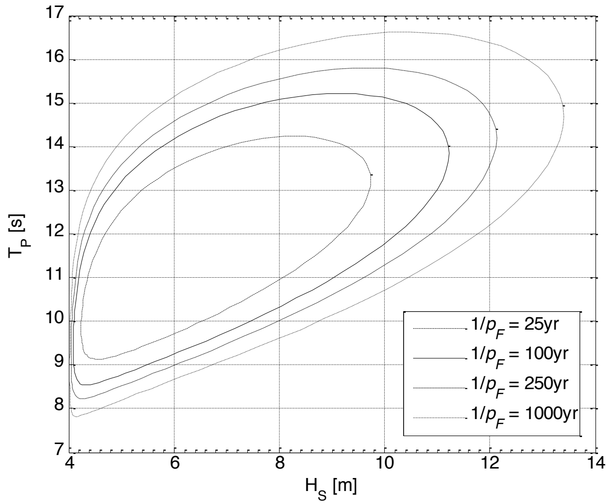
A high-dimensional optimization problem
Ideally solved quantitatively

- What?
- How?

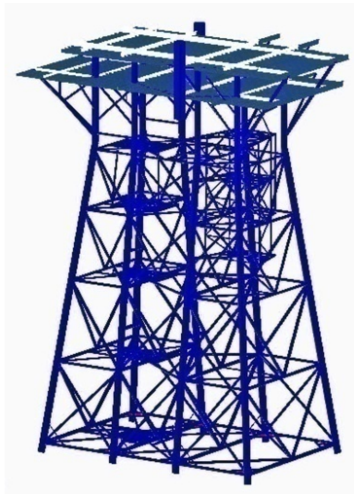
A few deterioration mechanisms and possible inspection methods
Identified by expert assessment

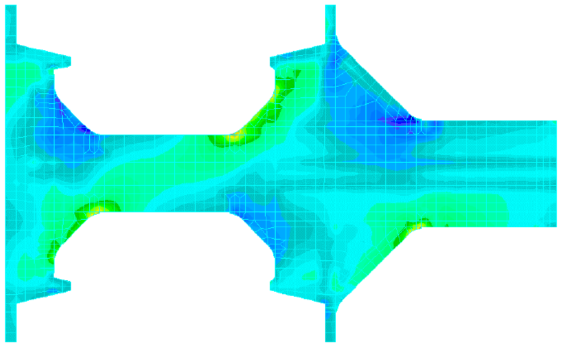
Goal: Planning based on detailed models

<u>Fatigue loads</u>

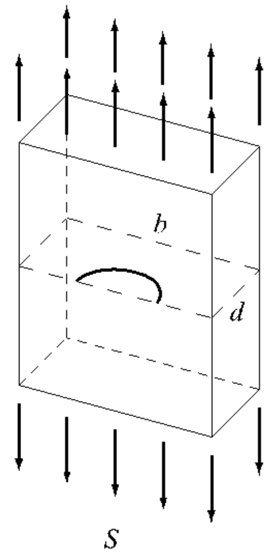


<u>Structural response</u>





<u>Crack growth</u>



$$\frac{da}{dN} = C_{P,a}\left(\quad K_a(a,c)\right)^{m_{fm}}$$

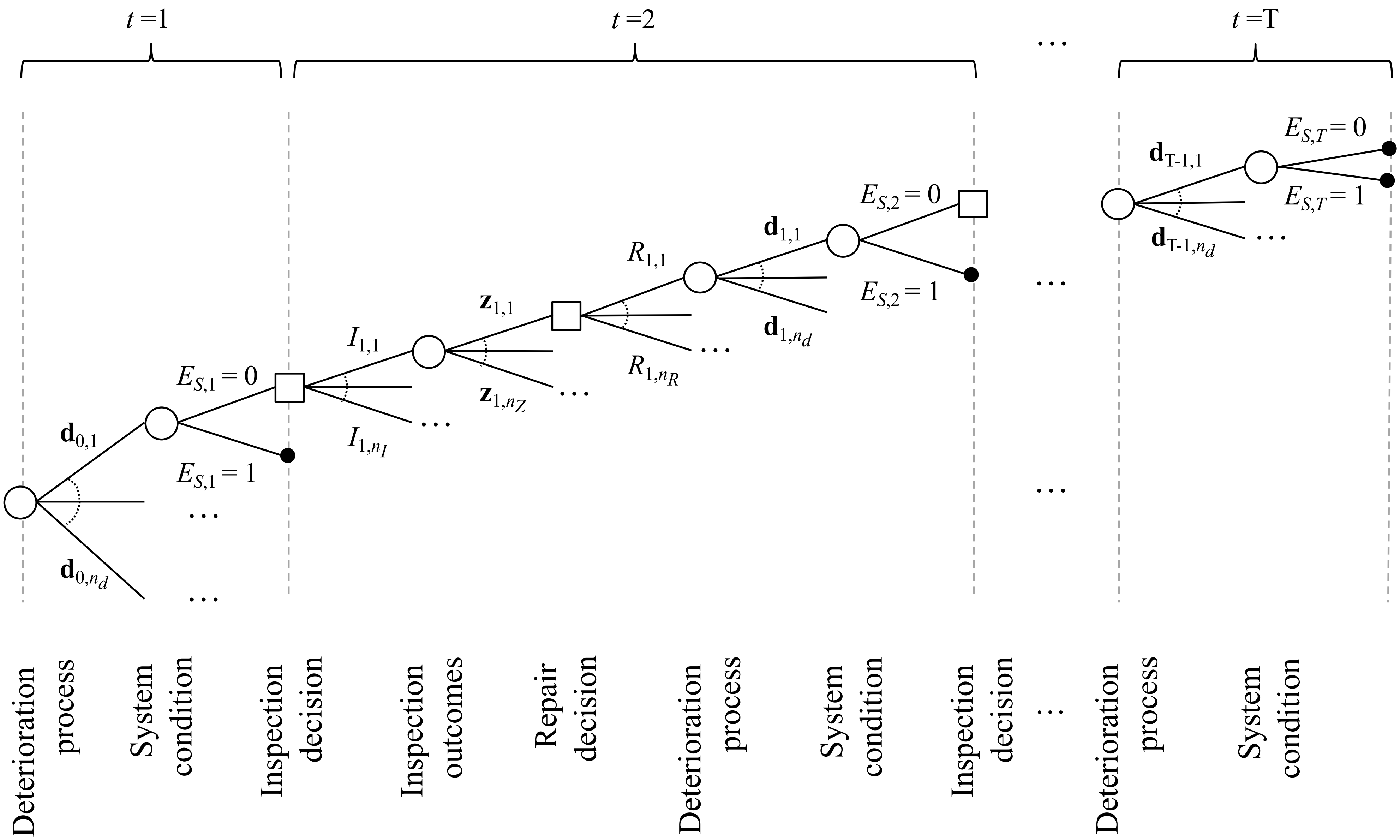
$$\frac{dc}{dN} = C_{P,c}\left(\quad K_c(a,c)\right)^{m_{fm}}$$

Optimal inspection planning
– a sequential decision problem

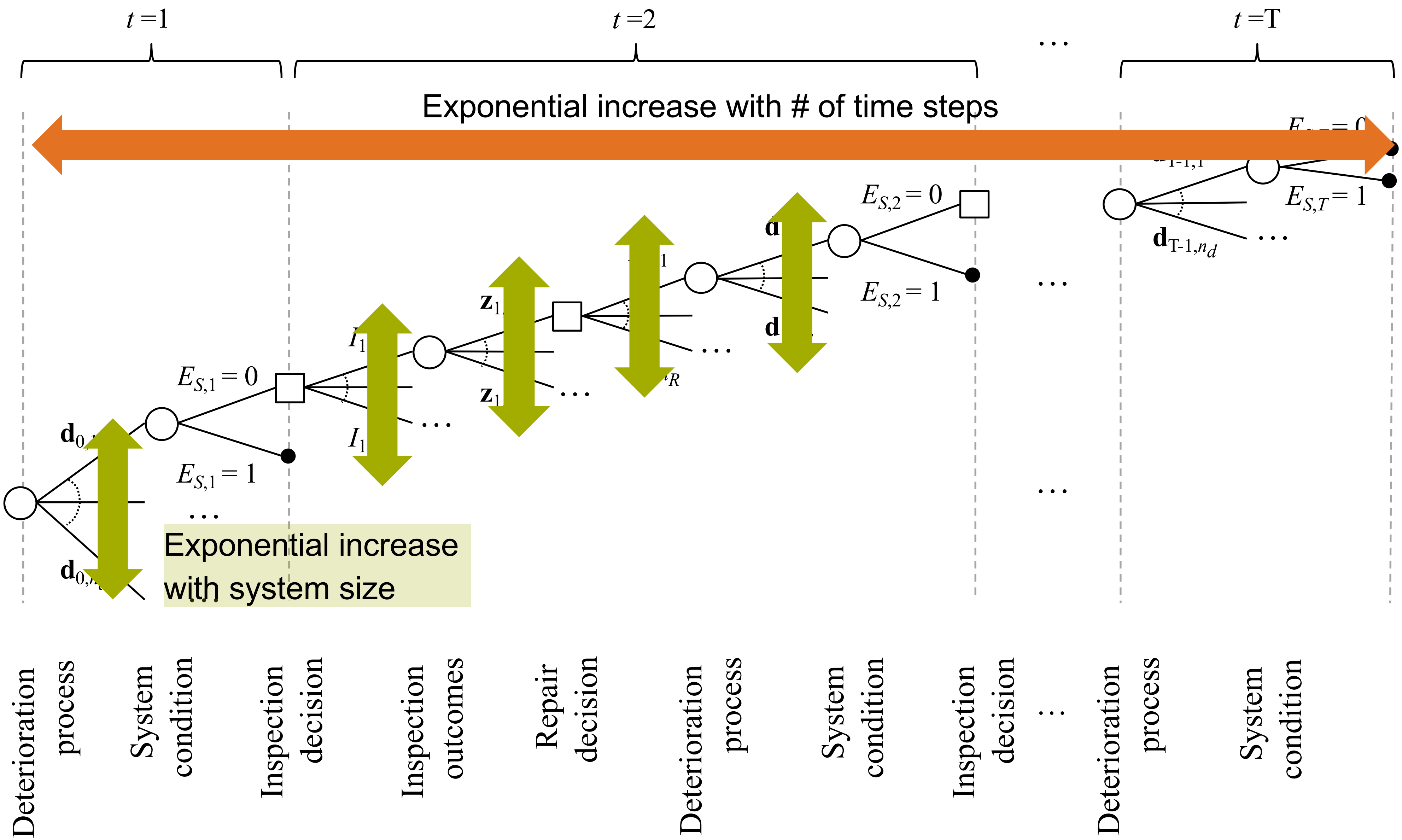
- Decisions are made at multiple times.
- Future inspection outcomes have an effect on the optimality of previous decisions

  → exponential complexity

A generic decision tree

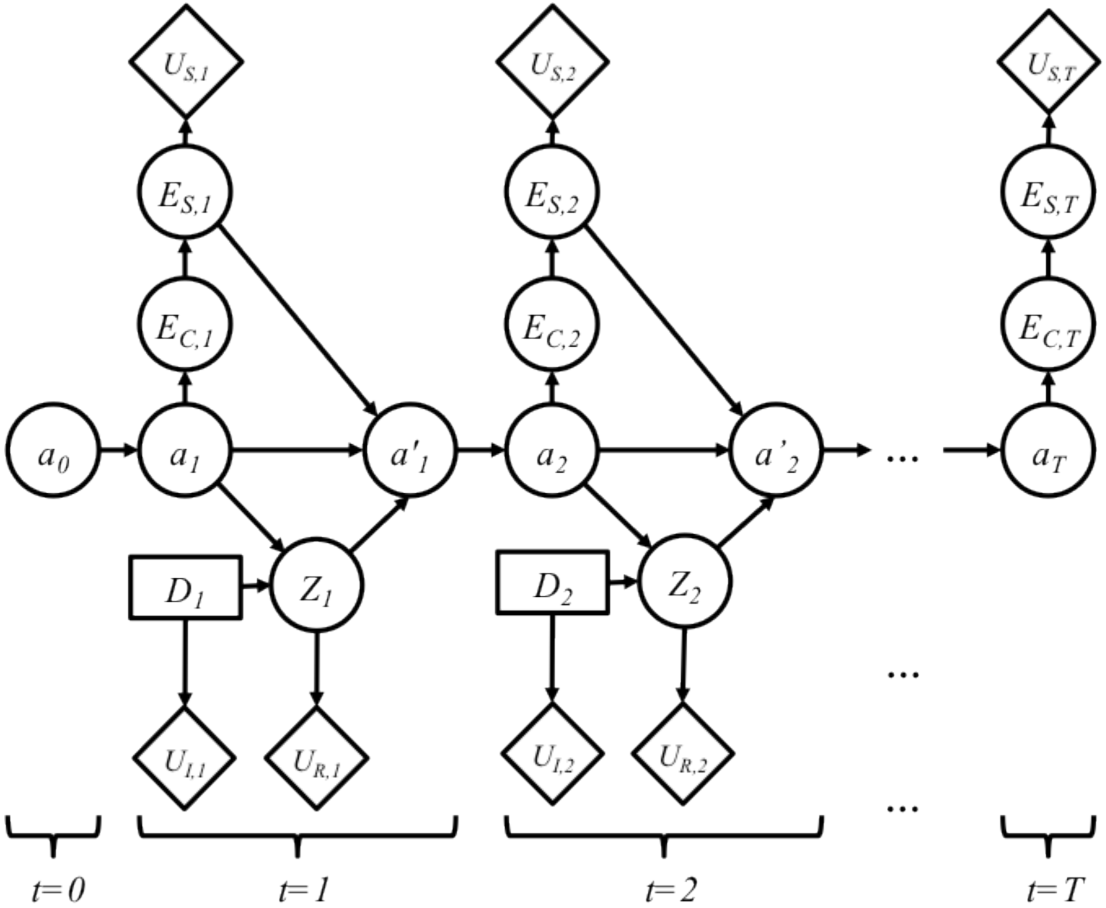


A generic decision tree



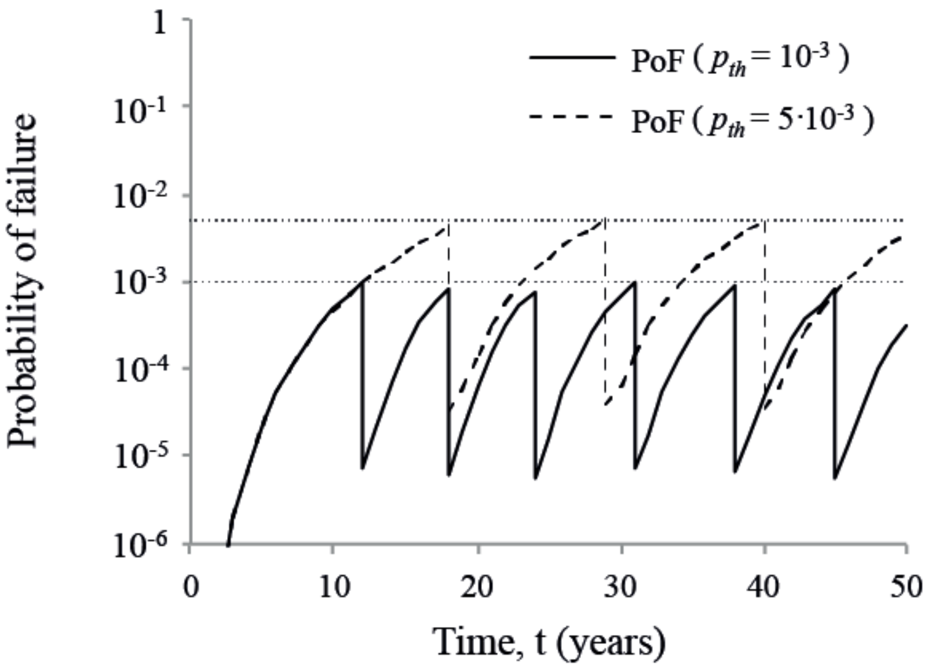
Optimal inspection planning
– a sequential decision problem

- Optimal solutions can be found with **POMDP** or **LIMID**
- These approaches at present are limited to single components and/or simple models

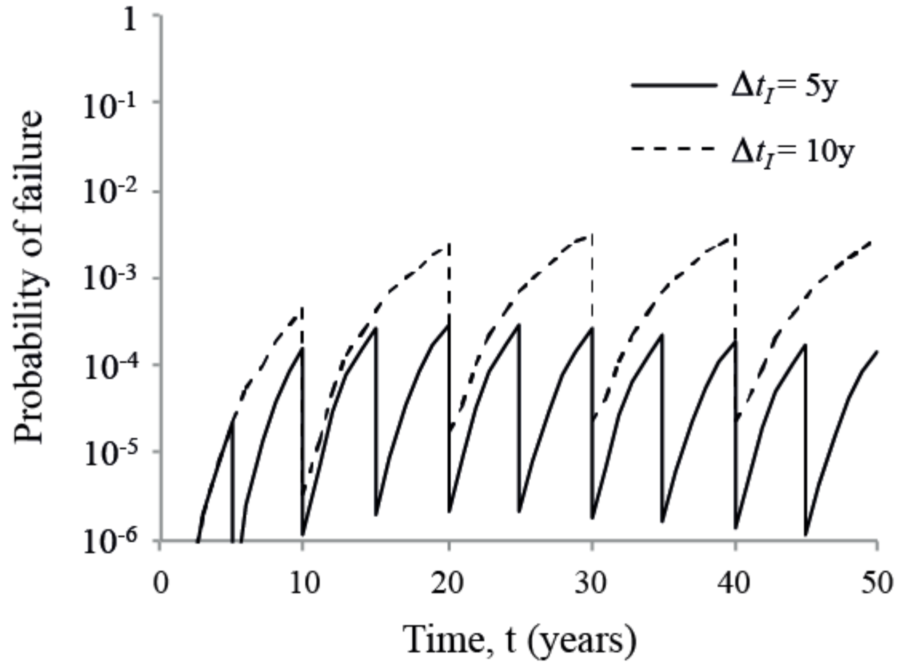


Heuristic approaches at the component level

- Threshold approach

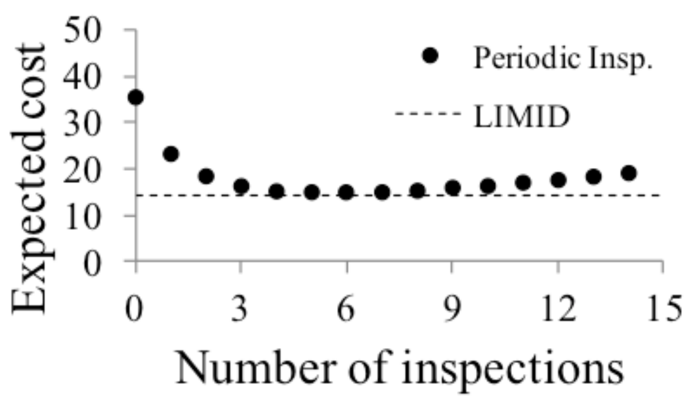


- Constant inspection intervals

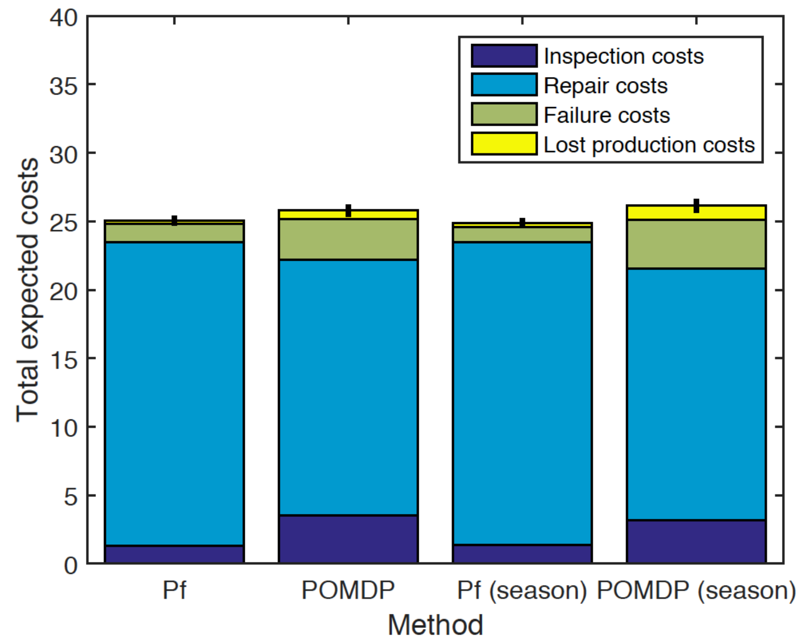


Optimal inspection planning at the component level

- Simple heuristics have shown to give results comparable to POMDP/LIMID



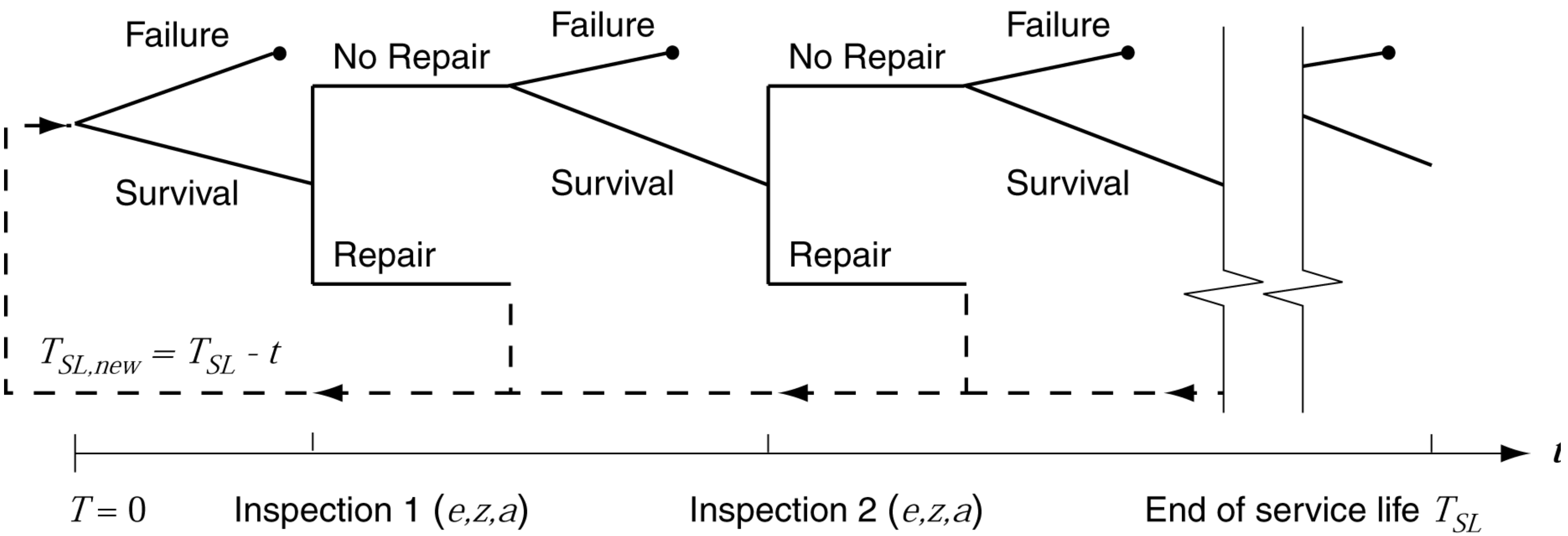
Luque & Straub (2013). Proc. IPW



Nielsen & Sorensen (2015). Proc. ICASP

Optimal inspection planning at the component level

- The heuristics lead to ”simple” problems

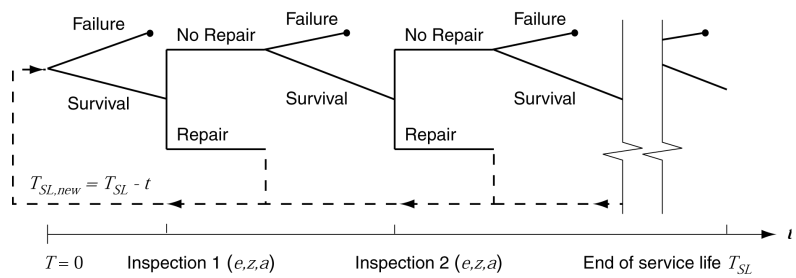


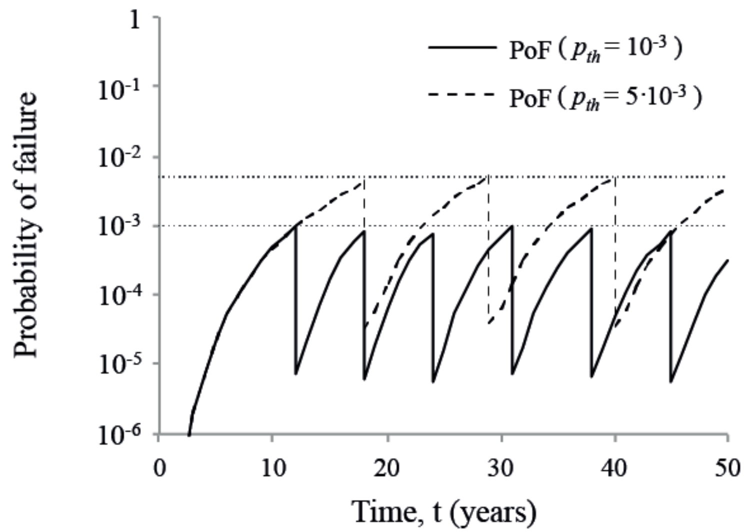
Component level solutions cannot be directly extended to the system level

because

1. Computational efforts to compute $E[C_T]$ increase drastically

2. Heuristics are more difficult to define





Optimal inspection planning
– a sequential system decision problem

The combination of a sequential decision problem with a system analysis leads to problems whose **computation is more than challenging**

Proposed solution:

1. Use of heuristics for defining inspection-repair strategies at the system level
2. Bayesian network to compute system failure probability conditional on inspection results
3. Monte-Carlo approach to integrate over future inspection results

Mathematical formulation of the optimization

Total life-time risk:

$$R_F(\mathcal{S}, \mathbf{Z}) = \sum_{t=1}^{T} c_F(t) \cdot \Pr(F_t | \mathcal{S}, \mathbf{Z}_{0:t-1})$$

Bayesian network

Total life-time cost and risk:

$$C_T(\mathcal{S}, \mathbf{Z}) = C_I(\mathcal{S}, \mathbf{Z}) + C_R(\mathcal{S}, \mathbf{Z}) + R_F(\mathcal{S}, \mathbf{Z})$$

Expected total life-time cost and risk:

$$\mathrm{E}[C_T] = \mathrm{E}_{\mathbf{Z}}[C_T(\mathcal{S}, \mathbf{Z})] = \int_{\Omega_{\mathbf{Z}(\mathcal{S})}} C_T(\mathcal{S}, \mathbf{z})\, f_{\mathbf{Z}}(\mathbf{z})\; d\mathbf{z}$$

Monte Carlo

Optimal inspection-repair strategy:

$$\mathcal{S}^* = \arg\min_{\mathcal{S}} \mathrm{E}_{\mathbf{Z}}[C_T(\mathcal{S}, \mathbf{Z})]$$

Heuristics with
few parameters

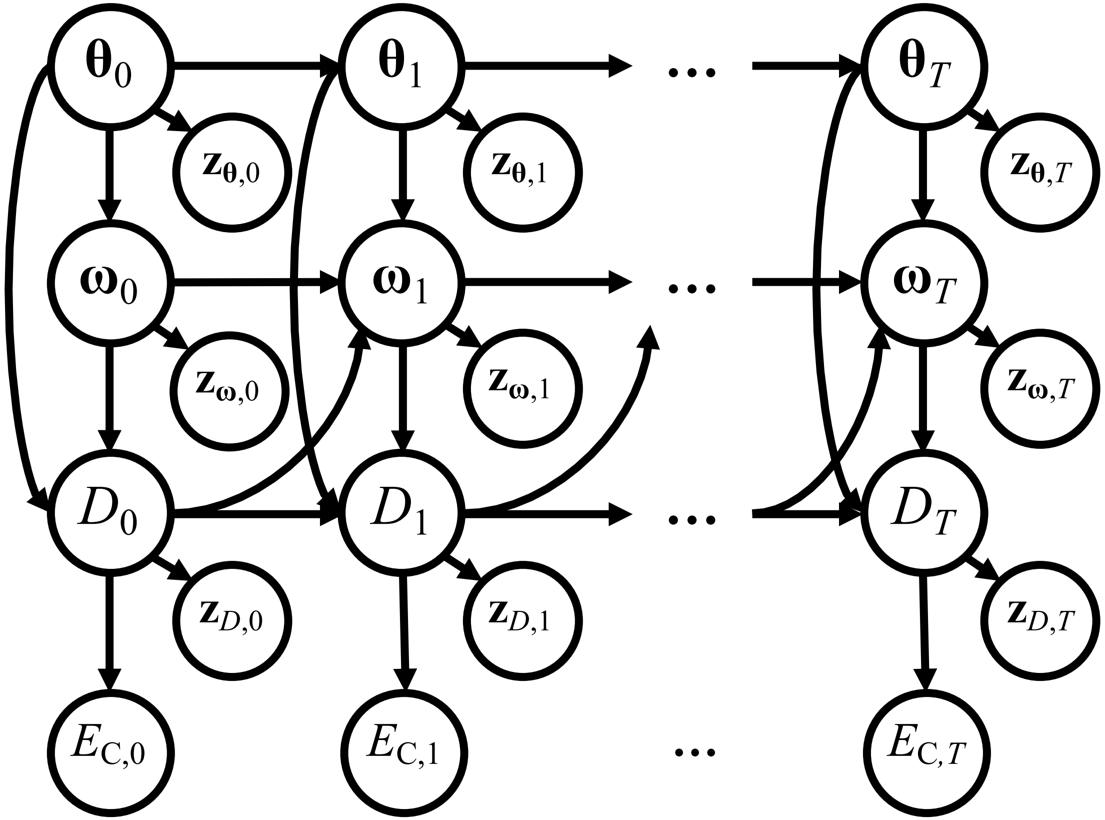
DBN model (component level)

Time-invariant parameters

Time-variant parameters

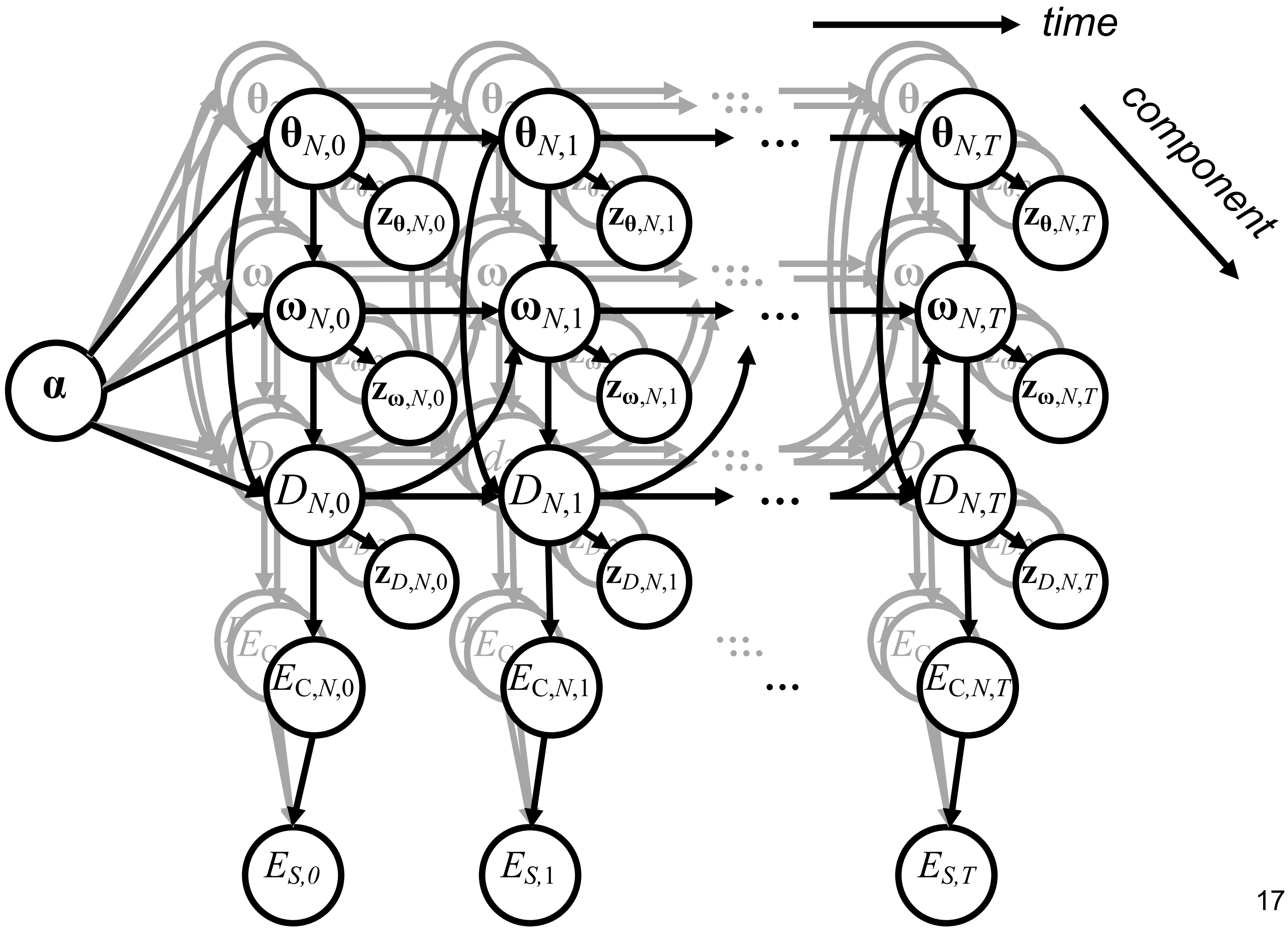
Deterioration function

Component condition



Observations

Hierarchical DBN model (system level)



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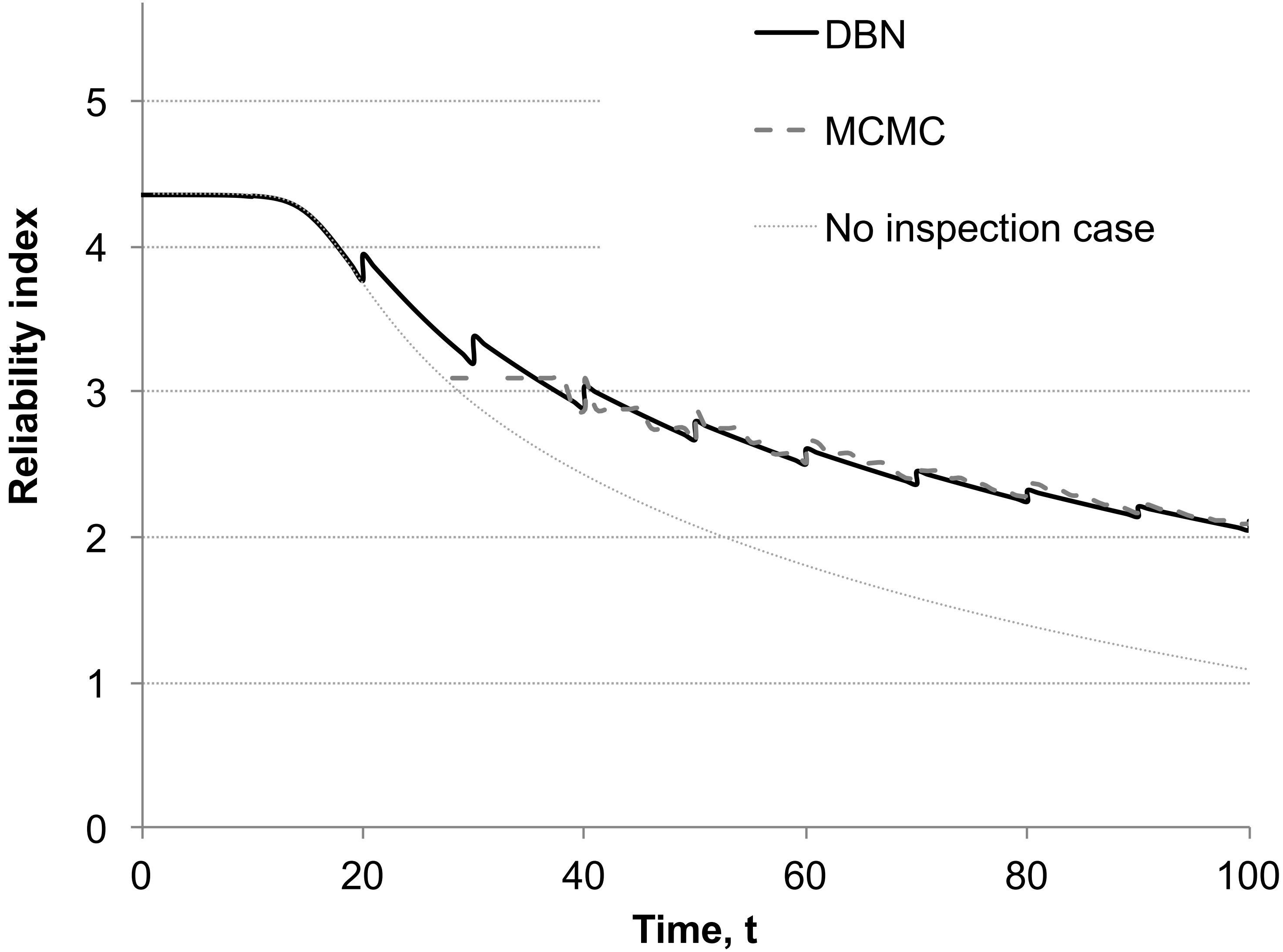
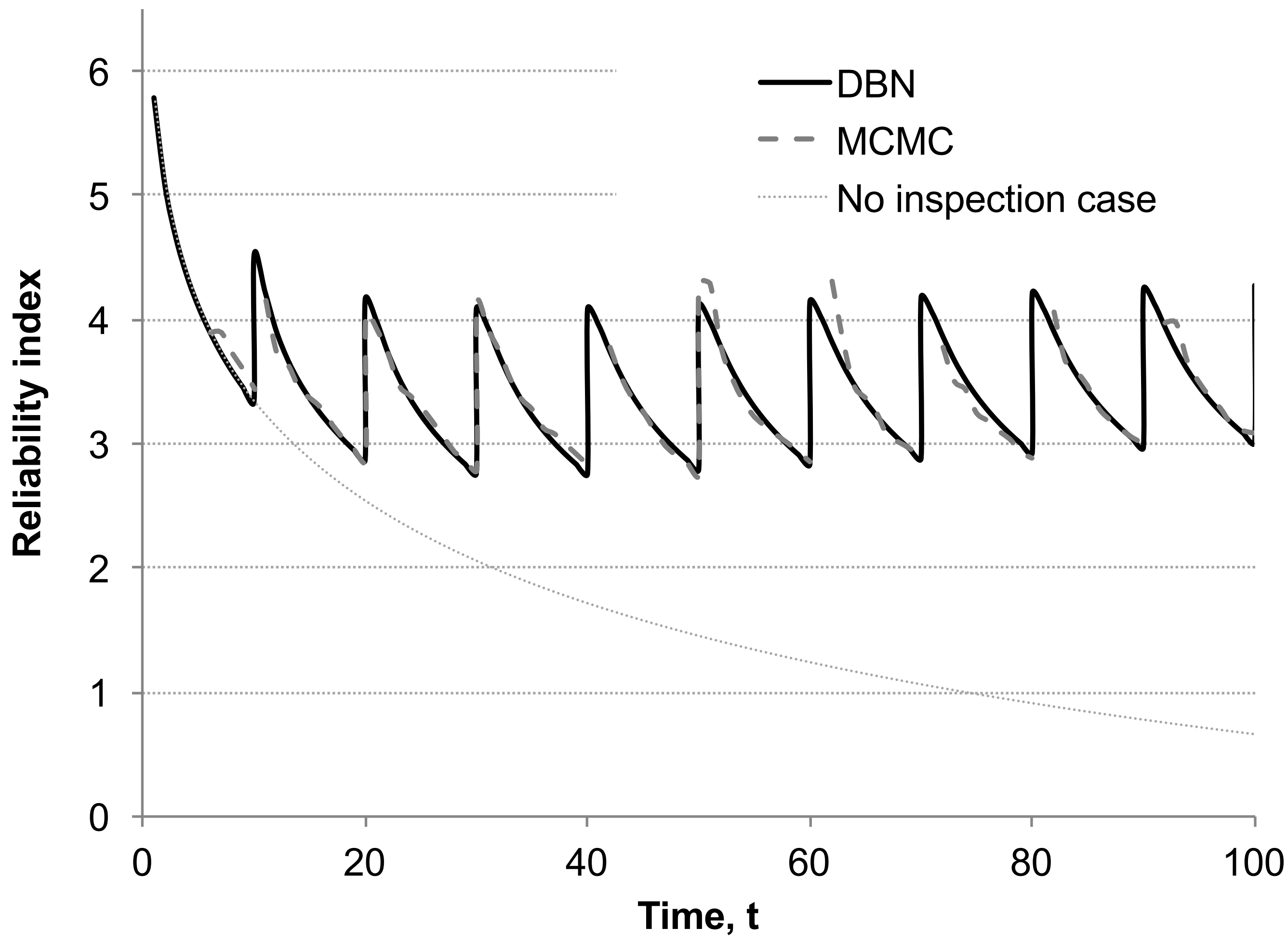
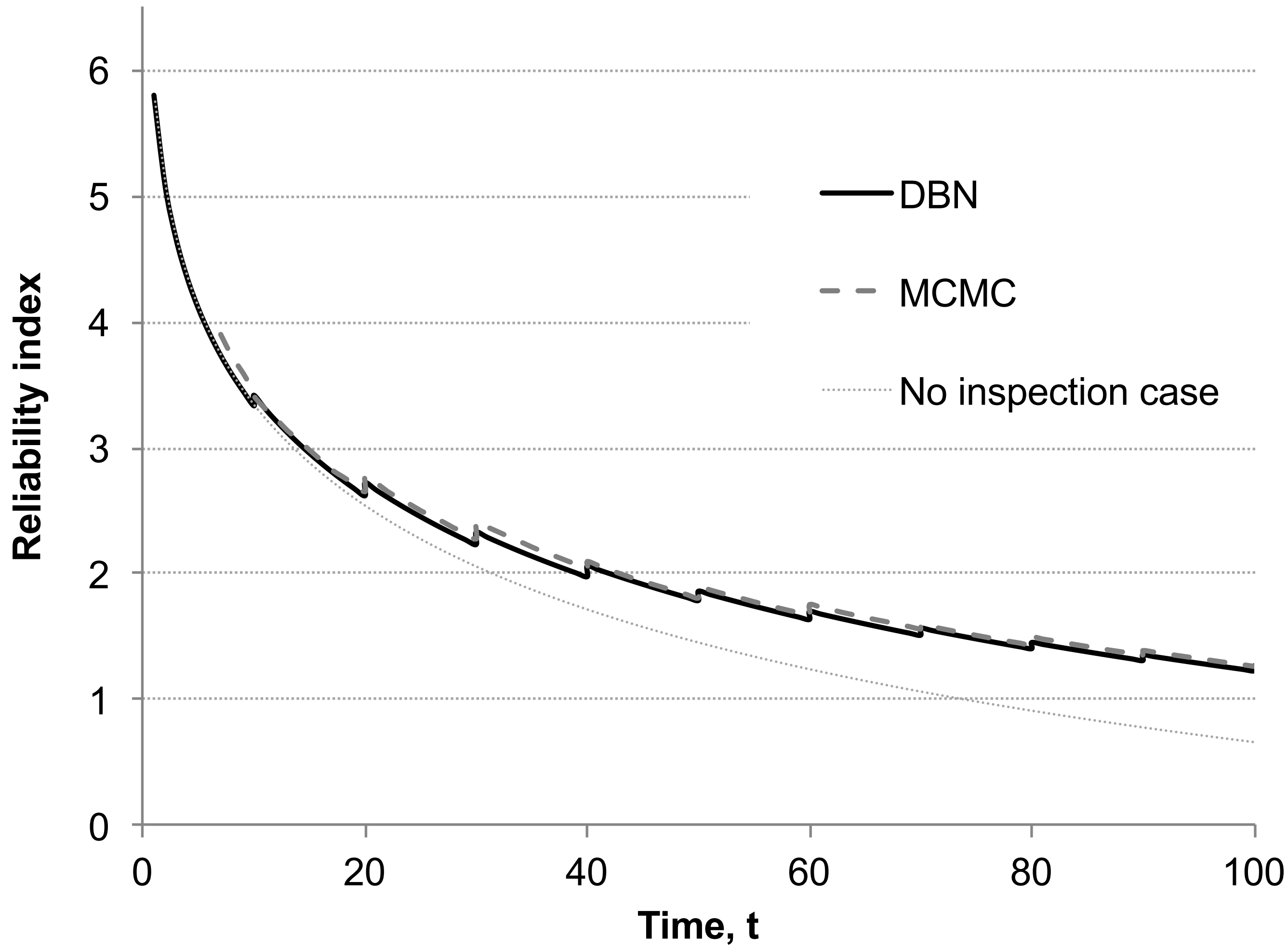
Updated system reliability



Updated reliability of an inspected component



Updated reliability of a non-inspected component



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Bayesian network

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Monte Carlo

Optimal inspection-repair strategy:

$$\mathcal{S}^* = \arg\min_{\mathcal{S}} \mathrm{E}_{\mathbf{Z}}[C_T(\mathcal{S},\mathbf{Z})]$$

Heuristics with few parameters

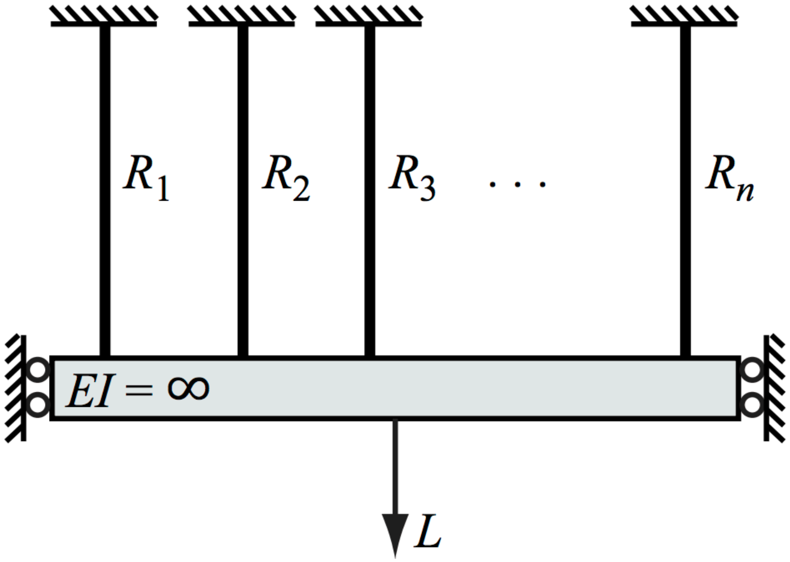
Heuristic at the system level

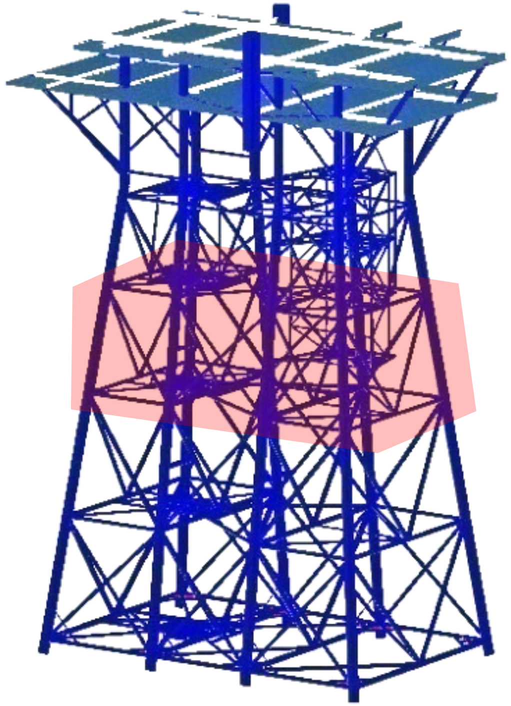
1. Inspection campaigns are performed at fixed intervals ΔT
2. The number of inspected components in each campaign is n_I
3. Components are selected for inspection following their Value of Information (or a proxy thereof)
4. If a threshold on the system reliability p_{th} is exceeded, an additional inspection campaign is carried out
5. Repairs of components are carried out if observed damages exceed a repair criterion d_R

Resulting optimization variables:

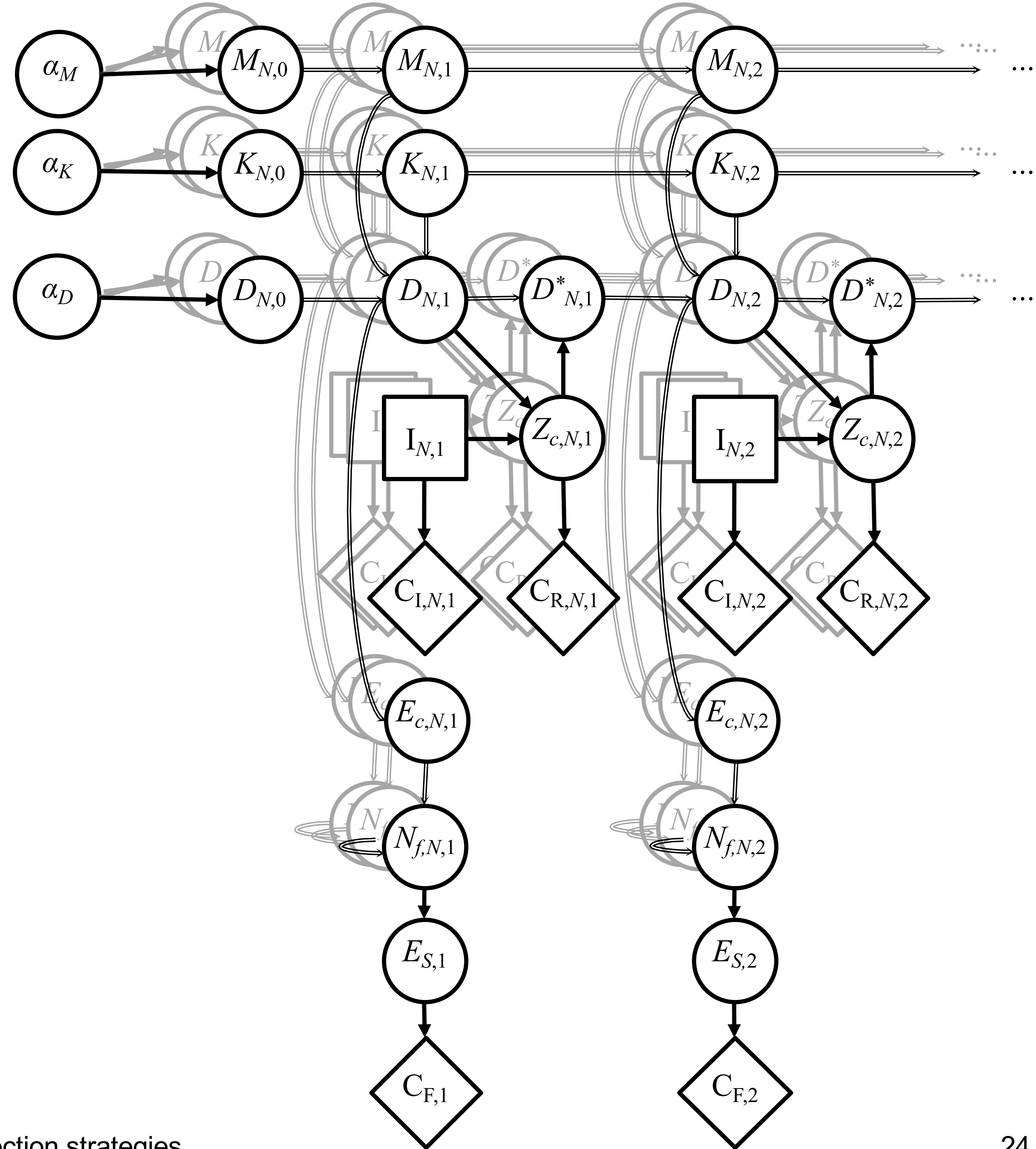
- ΔT
- n_I
- p_{th}
- d_R

Case study - Daniels system





DBN model



α_M
α_K
α_D
$M_{N,0}$
$K_{N,0}$
$D_{N,0}$
$M_{N,1}$
$K_{N,1}$
$D_{N,1}$
$D^*_{N,1}$
$\mathrm{I}_{N,1}$
$Z_{c,N,1}$
$\mathrm{C}_{\mathrm{I},N,1}$
$\mathrm{C}_{\mathrm{R},N,1}$
$E_{c,N,1}$
$N_{f,N,1}$
$E_{S,1}$
$\mathrm{C}_{\mathrm{F},1}$
$M_{N,2}$
$K_{N,2}$
$D_{N,2}$
$D^*_{N,2}$
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$Z_{c,N,2}$
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$\mathrm{C}_{\mathrm{R},N,2}$
$E_{c,N,2}$
$N_{f,N,2}$
$E_{S,2}$
$\mathrm{C}_{\mathrm{F},2}$
…

Probabilistic model

Random variable	Distribution	Mean	Std. deviation	Correlation
N	Deterministic	10		
T	Deterministic	40		
α_{D_0}	Normal	0	1	
α_M	Normal	0	1	
α_K	Normal	0	1	
$D_{i,0}$ [mm]	Exponential	1	1	0.5
$M_{i,0}$	Normal	3.5	0.3	0.6
$M_{i,t}$	$M_{i,t} = M_{i,t-1}$			
$\ln C_{i,t}$	$\ln C_{i,t} = -3.34 M_{i,t} - 15.84$			
ΔS_i	Weibull	K_i *(scale parameter)*	$\lambda_i = 0.8$ (shape parameter)	
$\Delta S_{e,i}$	$\Delta S_{e,i} = K_i \Gamma\left(1 + \frac{M_i}{\lambda_i}\right)^{\frac{1}{M_i}}$			
$K_{i,0}$	Lognormal	1.6	0.22	0.8
$K_{i,t}$	$K_{i,t} = K_{i,t-1}$			
d_C [mm]	Deterministic	50		
ξ [mm]	Deterministic	10		

Cost model

Cost	Case 1 (offshore structure)	Case 2 (bridge structure)
Inspection campaign, c_I	1	1
Component inspection, c_{Ic}	0.1	0.5
Component repair, c_{Rc}	0.3	10
System failure, c	10^4	10^3
Discount rate, r	0.02	0.02

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Monte Carlo

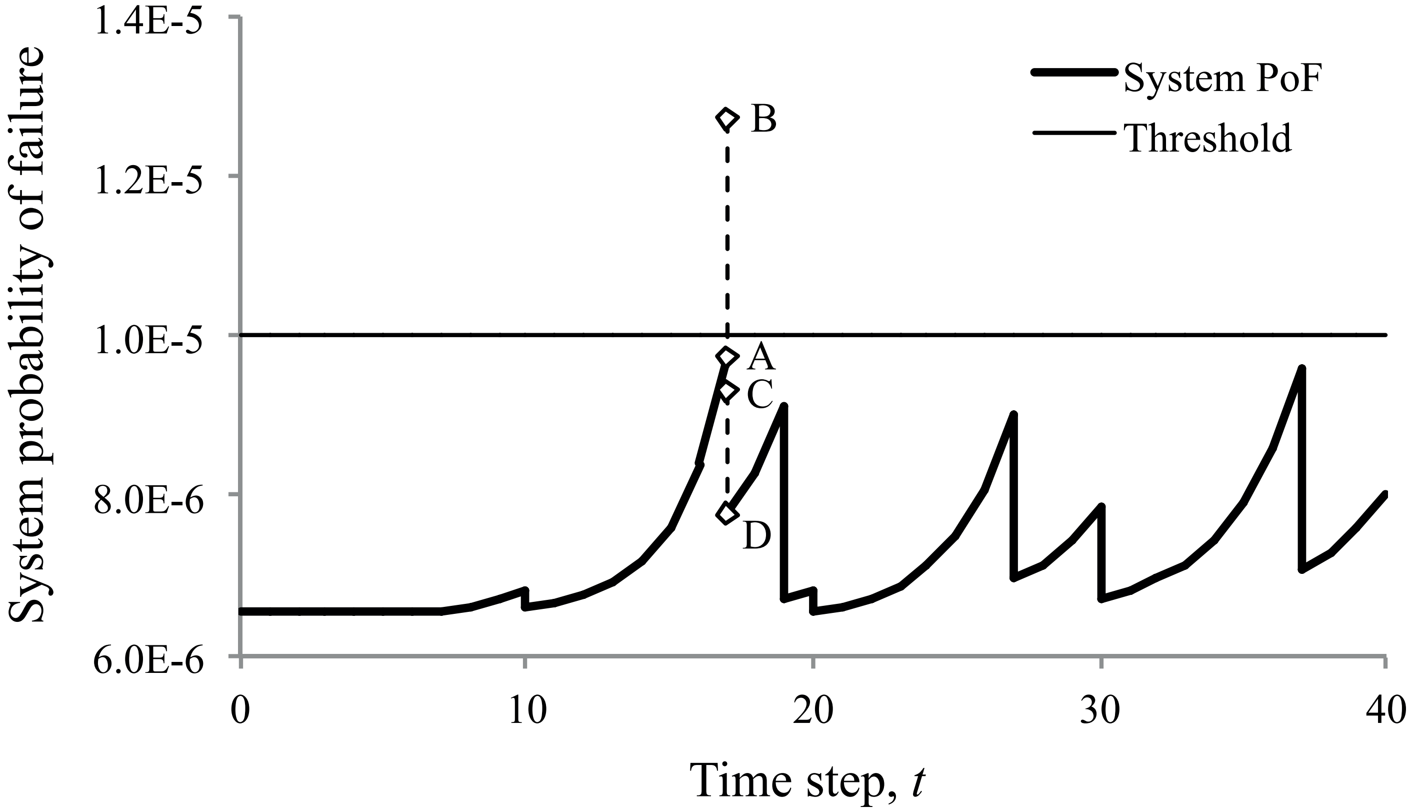
Optimal inspection-repair strategy:

$$\mathcal{S}^* = \arg\min_{\mathcal{S}} \mathrm{E}_{\mathbf{Z}}[C_T(\mathcal{S}, \mathbf{Z})]$$

Heuristics with few parameters

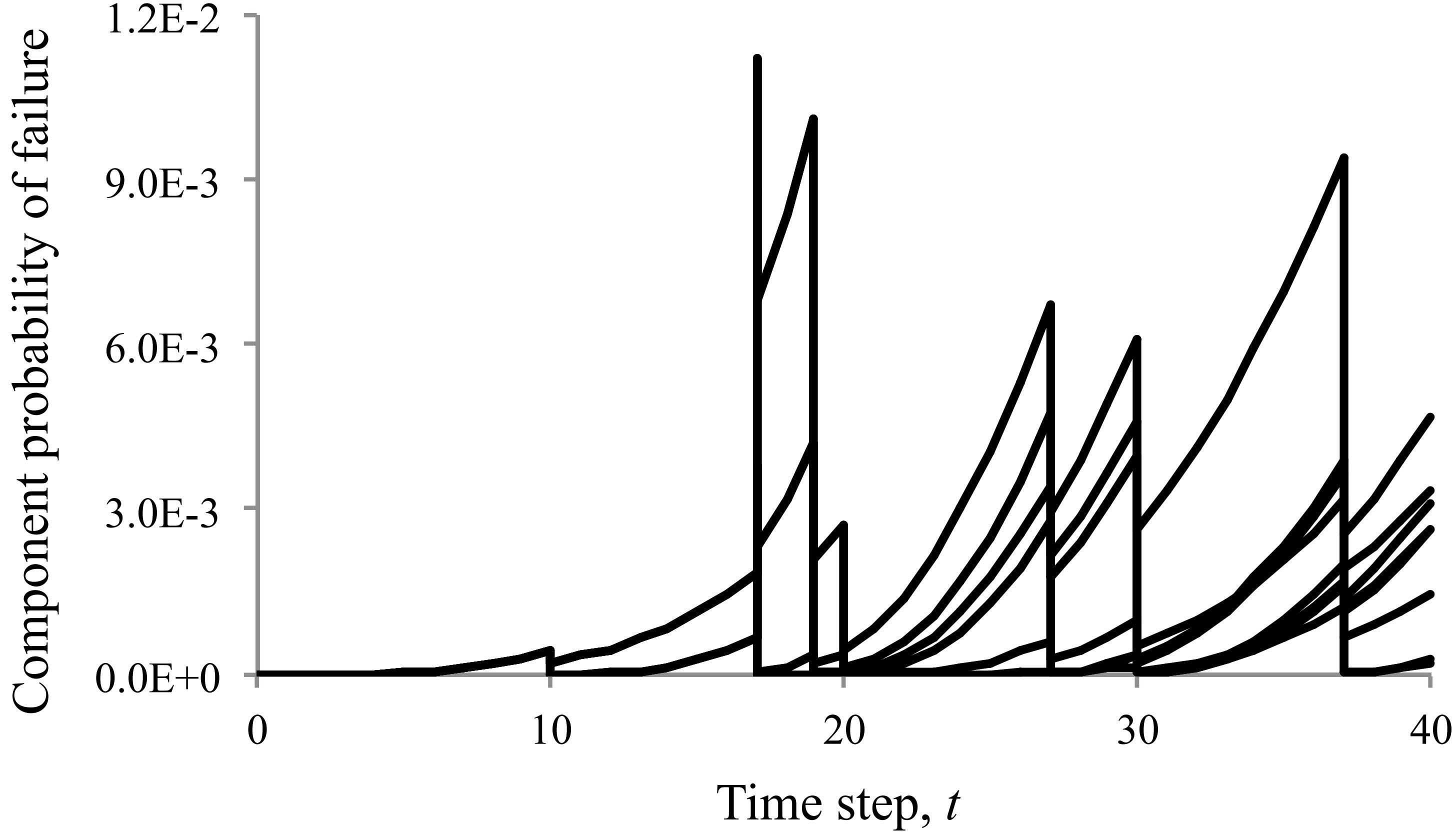
Updating with simulated inspection results

Example $\Pr(F_t|\mathbf{Z}_{0:t-1})$:

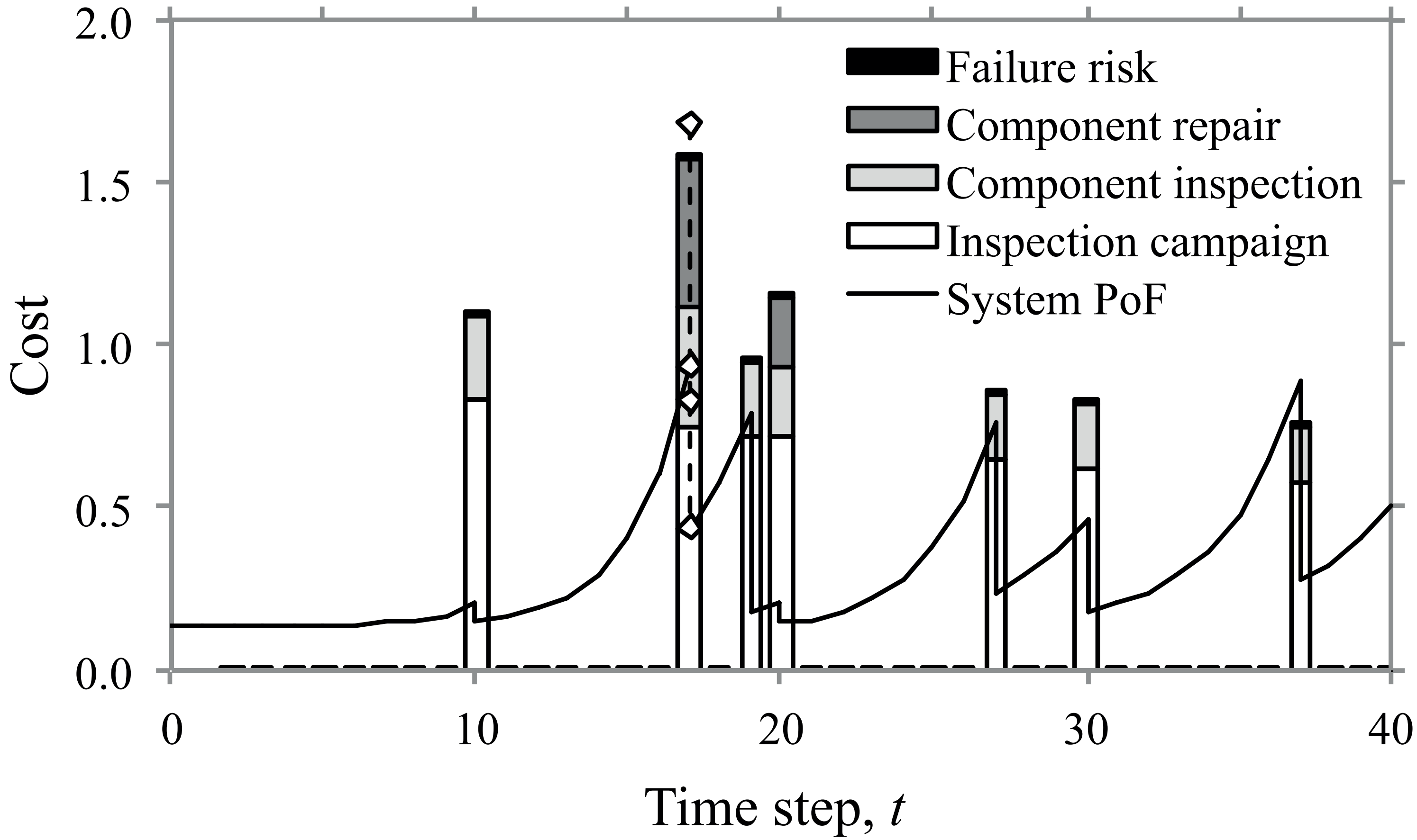


Updating with simulated inspection results

Corresponding conditional component reliabilities

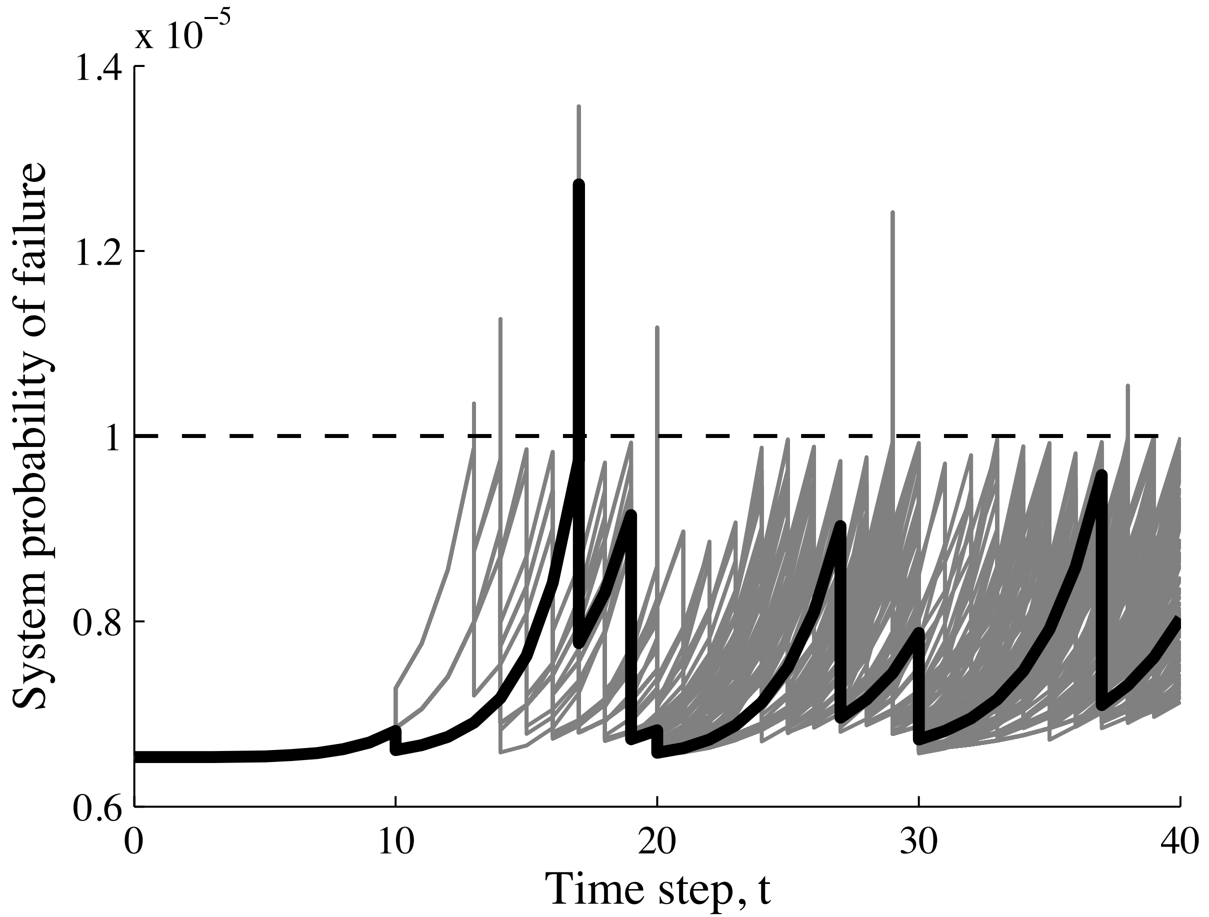


Costs associated with an inspection history



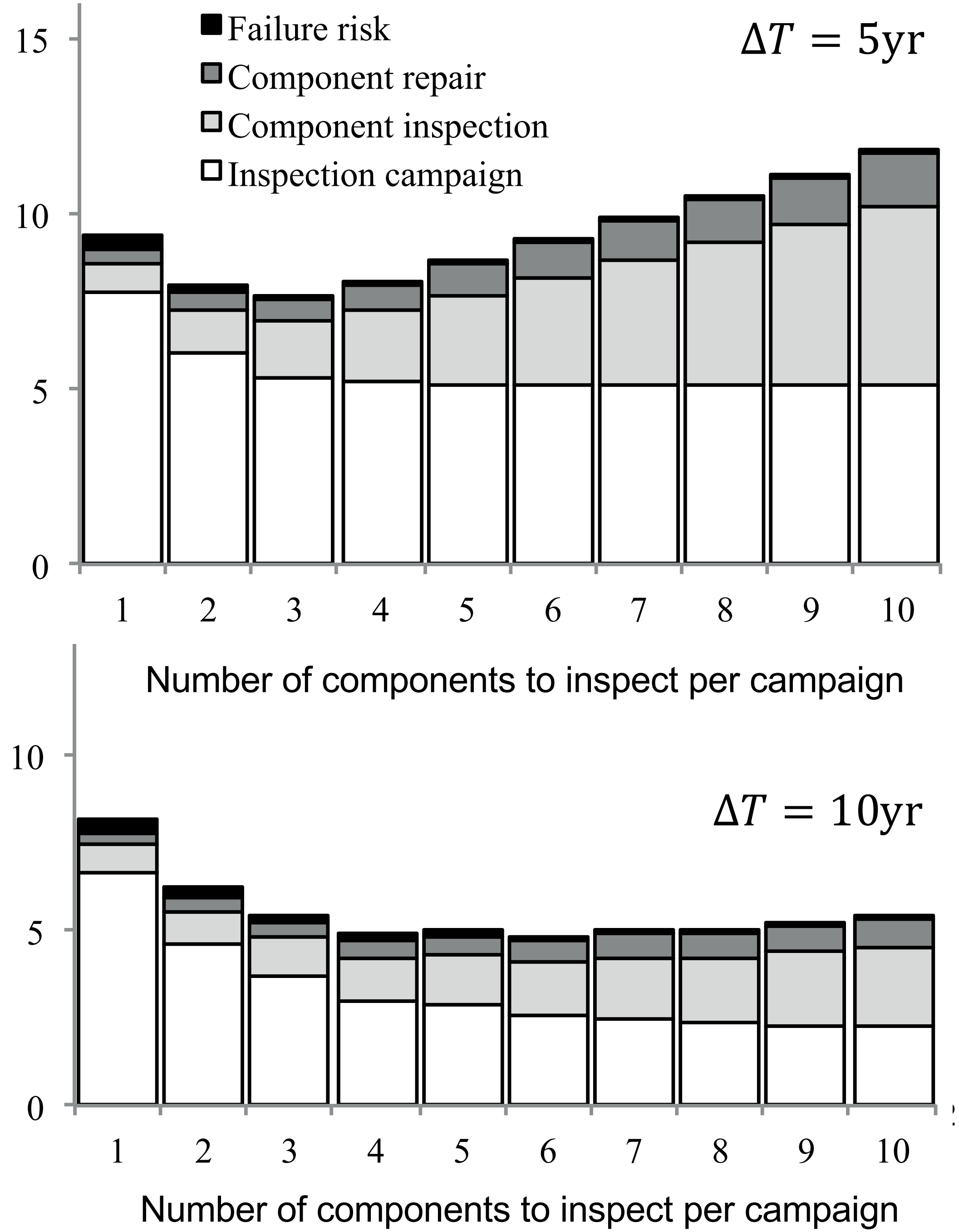
Integrating over inspection results

In the order of 10^2 to 10^3 simulated inspection histories are necessary for accuracy

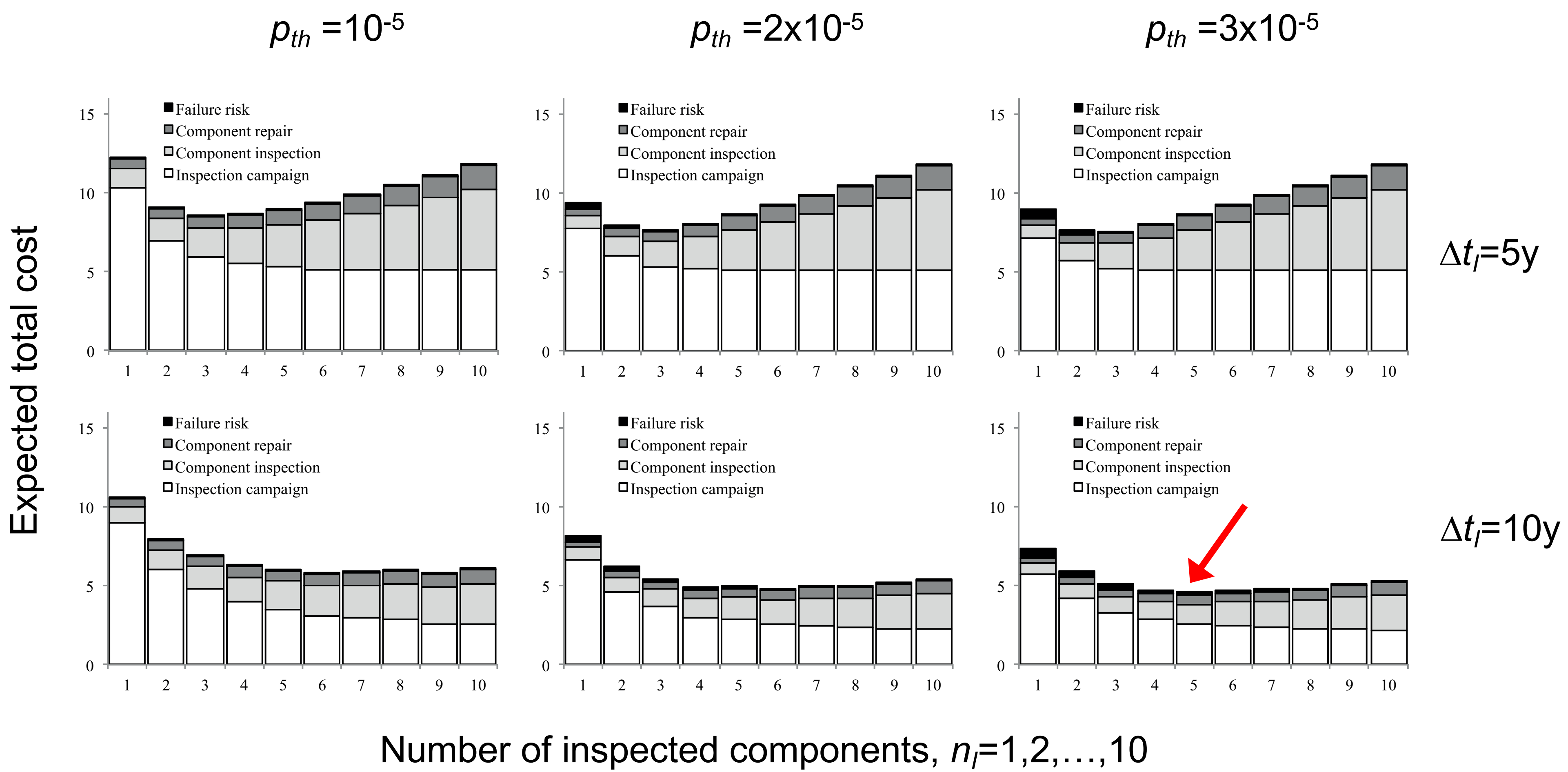


Optimal strategies

- Case 1
- Threshold $p_{th} = 2{\times}10^{-5}$

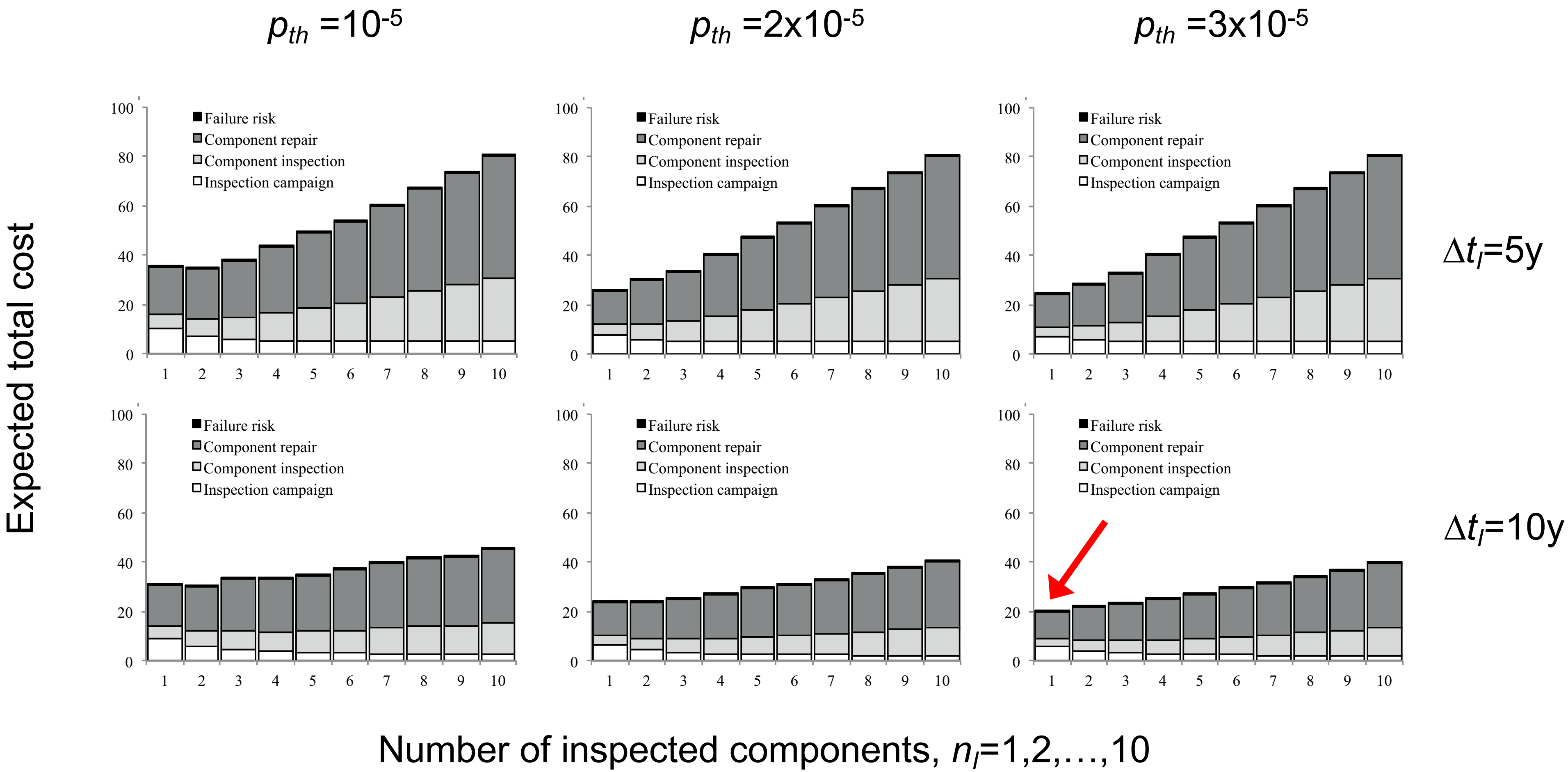


Optimal strategies (case 1)



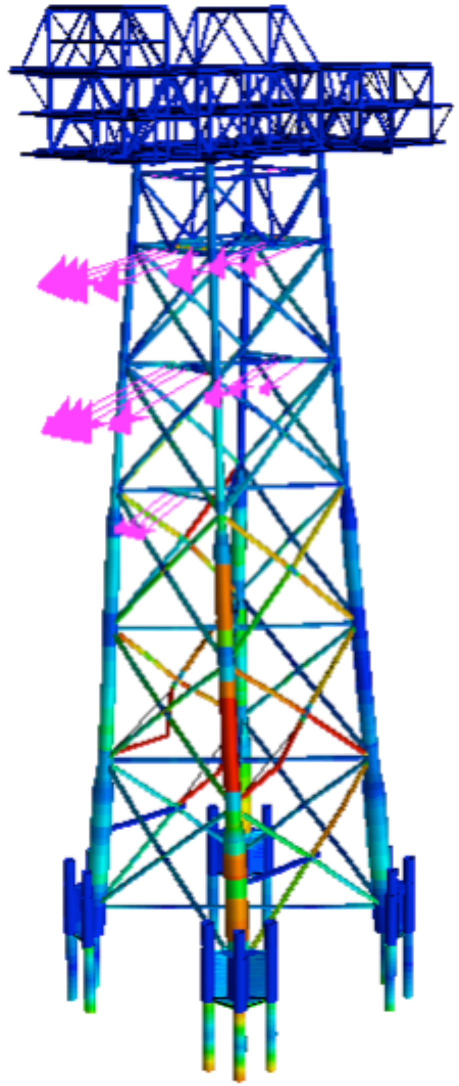
Optimal strategies (case 2)

■ Failure risk
■ Component repair
□ Component inspection
□ Inspection campaign



Current case study

- Zayas frame
- Pushover analysis to determine ultimate capacity
- 13 critical members are considered
- Two hotspots per member
- Value of information estimated based on:
  - $\Pr(F_i|\mathbf{Z})$
  - Criticality of members
  - Redundancy factor
  - …



Conclusion

- Pragmatic solution based on combining:
  - Bayesian network (for fast reliabilty updating with inspection results)
  - Monte Carlo sampling (for integrating over inspection histories)
  - Heuristics (to reduce solution space of the optimization)

- Investigations into the optimality of the proposed heuristics for general structural systems are necessary

References

Luque J., Straub D.: Reliability analysis and updating of deteriorating structural systems with dynamic Bayesian networks. *Structural Safety*, in print.

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Straub D., Der Kiureghian A. (2011). Reliability Acceptance Criteria for Deteriorating Elements of Structural Systems. *ASCE Journal of Structural Engineering*, **137**(12): 1573–1582.

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Thank you for your attention