The Dependency of the Value of SHM on System Characteristics

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CERDA
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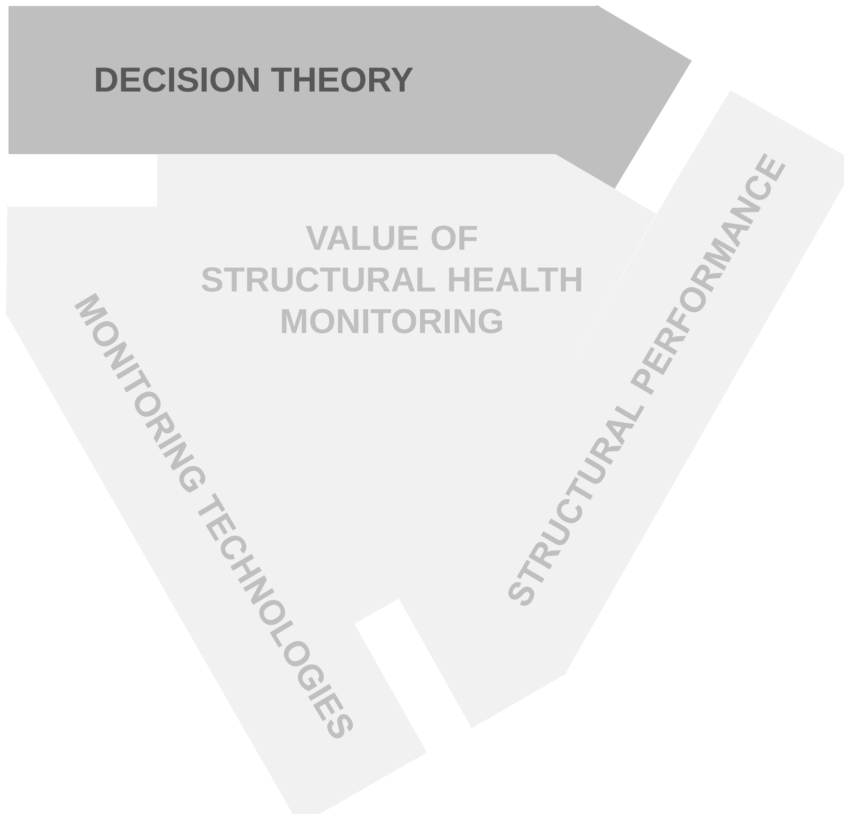
Risk and Decision Analysis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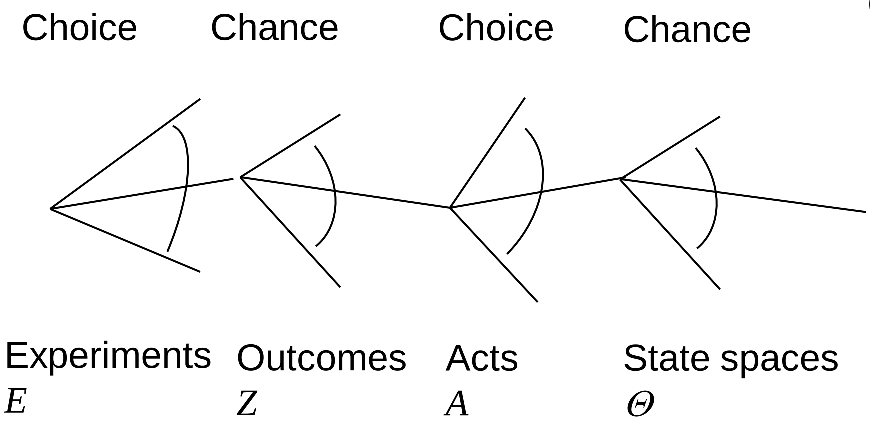
Value of Information

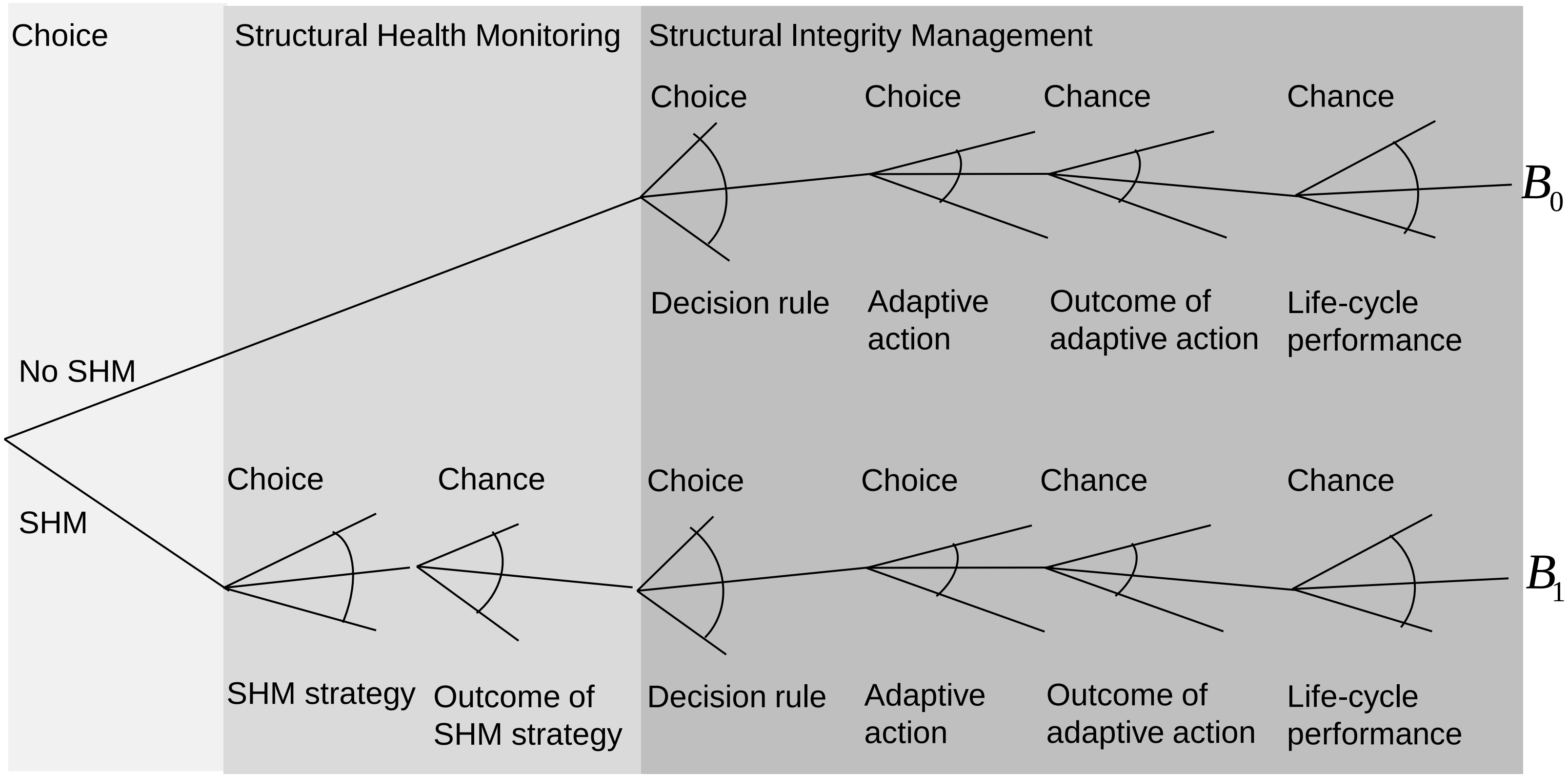


The Value of Information theory was developed by Raiffa and Schlaifer in 1961.

- Extensive and normal form analysis

Quantification of the value of structural health monitoring

- The value of structural health monitoring is calculated as the difference between life cycle benefits B_1 and B_0: $V = B_1 - B_0$



Quantification of the value of structural health monitoring

Value of SHM: $V = B_1 - B_0$

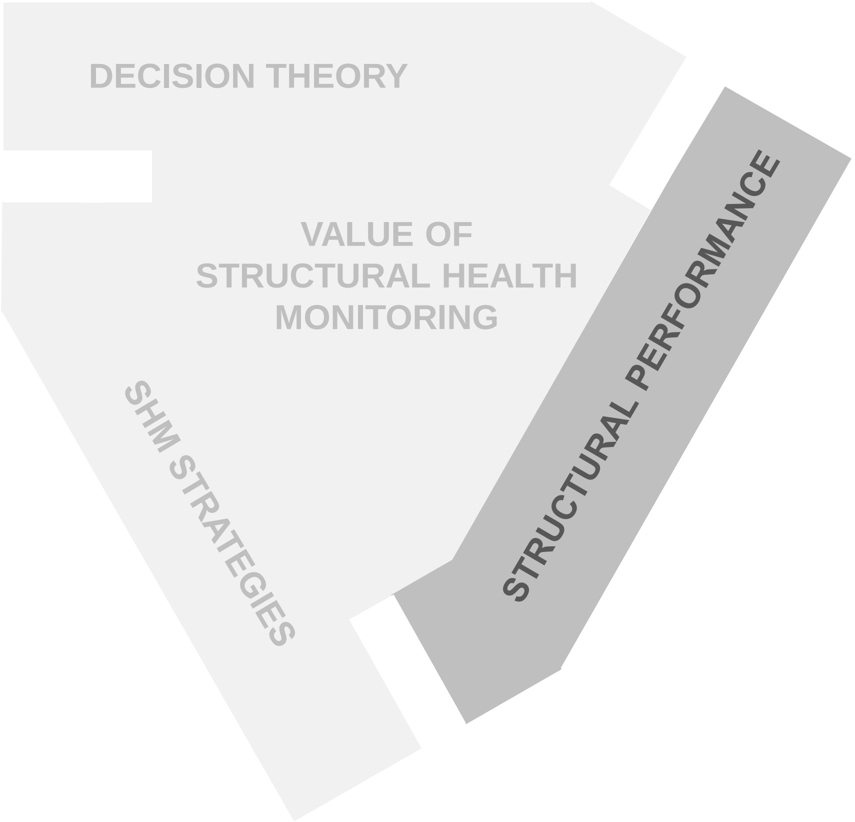
B_0: Life cycle benefit without SHM
B_1: Life cycle benefit utilizing SHM

Life cycle benefits: $B_0 = \max_{\mathbf{a},\mathbf{d}} E_{\mathbf{Z}_E}\left[E_{\mathbf{Z}_A}\left[\mathbf{d}\left(\mathbf{a},\mathbf{Z}_E,\mathbf{Z}_A\right),\mathbf{Z}_E,\mathbf{Z}_A\right]\right]$

$$B_1 = \max_{\mathbf{s}} E_{\overline{\mathbf{Z}}_E}\left[E_{\overline{\mathbf{Z}}_A}\left[\max_{\mathbf{a},\mathbf{d}} E_{\mathbf{X}|\overline{\mathbf{Z}}_E,\overline{\mathbf{Z}}_A}\left[\mathbf{X},\overline{\mathbf{Z}}_E,\overline{\mathbf{Z}}_A,\mathbf{s},\overline{\mathbf{d}}\left(\overline{\mathbf{a}},\mathbf{X},\overline{\mathbf{Z}}_E,\overline{\mathbf{Z}}_A\right)\right]\right]\right]$$

$\mathbf{X}$, $\mathbf{Z}_A$, $\mathbf{Z}_E$: Random variables for uncertain monitoring results, aleatory and epistemic uncertainties

s, d, a: SHM strategies, decision rules and adaptive actions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Structural deteriorating system performance

$$P\left(F_S\right)=\int_{\Omega_{F_S}} f_{\mathbf{Z}}\left(\mathbf{z}\right)d\mathbf{z}$$

$$g_{FS}\left(\mathbf{Z},M_S\right)=M_R R_S\left(t\right)-M_S S_S$$

With a structural system model the performance throughout the life cycle is calculated taking into account the fatigue deterioration.

Structural deteriorating system performance

SN limit state function:

$$g\left(\mathbf{Z},M\right)=\Delta-\nu\cdot t\frac{E\left[\Delta\sigma^{m}\right]}{K}$$

Expected stress ranges:

$$E\left[\Delta\sigma^{m}\right]=\left(Mk\right)^{m}\Gamma\left(1+\frac{m}{\lambda};\left(\frac{s_0}{k}\right)^{\lambda}\right)$$

FM limit state function:

$$g_i^{FM}=a_{i,c}-a_i\left(t\right)$$

The Fatigue deterioration is modelled with an SN and an FM approach.

Symbols:

Δ:	Fatigue resistance
K, m:	Parameters of SN-model
t:	Time
ν:	Annual number of stress cycles
M:	Model uncertainty
k, λ:	Weibull location and shape parameter
s_0:	Cut off stress range
$a_{i,c}$:	Critical crack depth
$a_i(t)$:	Crack depth distribution

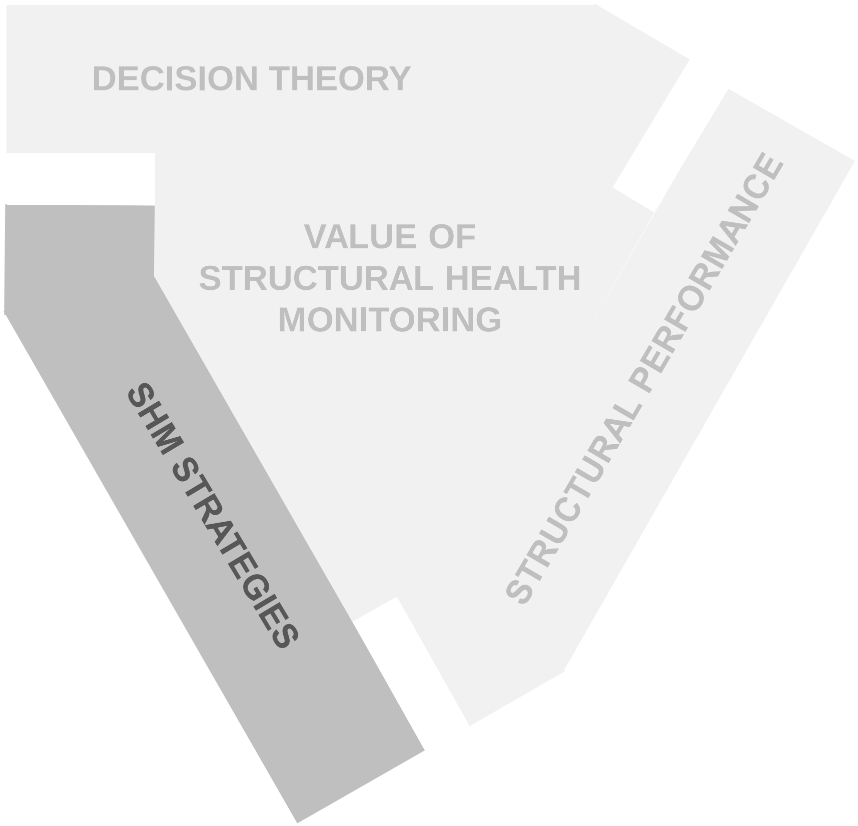
Structural deteriorating system performance

$$R_i\left(t\right) = R_{i,0}\left(1 - D_i\left(t\right)\right)$$

$$D_i\left(t\right) = r_R \frac{a_i\left(t\right)}{d_i}$$

The component resistances are reduced by the growth of fatigue cracks over time.

- Continuous deterioration state $D_i\left(t\right)$ described with
  - Initial component resistance $R_{i,0}$
  - Resistance reduction factor r_R
  - Crack size to wall thickness ratio $\frac{a_i\left(t\right)}{d_i}$
- Resistance reduction factor can be determined by the crack to thickness ratio induced lost cross sectional area
- The component resistances and the deterioration states are correlated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Structural deteriorating system performance with SHM

Probability of system failure given SHM information:

$$P\left(F_S \mid M_S\right) = \int_{\Omega_{F_S}} f_{\mathbf{Z},U}\left(\mathbf{z}, u \mid \hat{M}_S\right) d\mathbf{z}du$$

$$g_{FS}\left(\mathbf{Z}, M_S\right) = M_R R_S\left(t\right) - \hat{M}_S U S_S$$

The SHM strategy constitutes load monitoring.

- Modeling of SHM information with the realizations of the model uncertainties
- SHM uncertainty is accounted for with U

Structural deteriorating system performance with SHM

Expected stress ranges for a monitored hot spot:

$$E\left[\Delta\sigma_i \mid \hat{M}_L\right] =$$

$$\left(\hat{M}_L M_\sigma M_{HS} M_Q U_L k\right)^m \Gamma\left(1+\frac{m}{\lambda};\left(\frac{s_0}{k}\right)^\lambda\right)$$

The SHM strategy constitutes load monitoring.

- Modeling of SHM information with the realizations of the model uncertainties
- SHM uncertainty is accounted for with U

Service life integrity management and risk model

The service life benefits are calculated with:

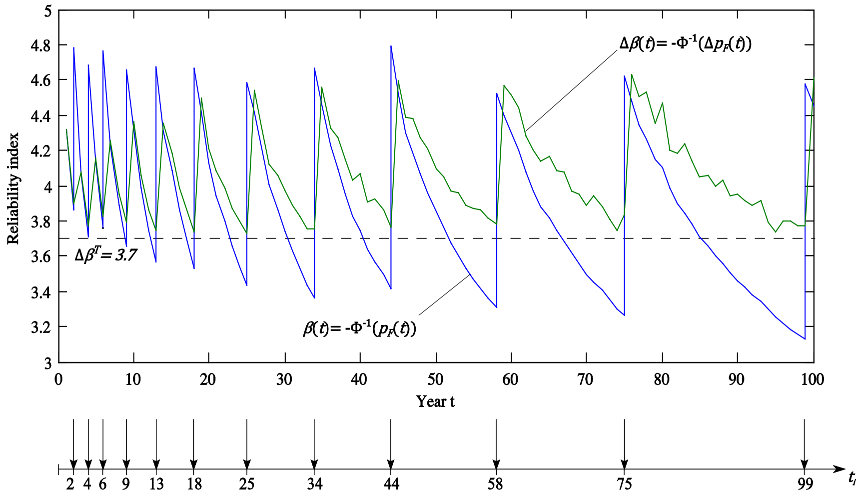
- The expected inspection costs $E\left[C_{i,Insp}\right]$
- The expected repair costs $E\left[C_{i,R}\right]$
- The component deterioration risks $R_{i,D}$
- The structural system failure risks R_{F_S}

$$B_0\left(d\left(\mathbf{a},\mathbf{Z}\right),\mathbf{Z}\right)=-\left(\sum_{i=1}^{n}\left(E\left[C_{i,Insp}\right]+E\left[C_{i,R}\right]+R_{i,D}\right)+R_{F_S}\right)$$

$$B_1\left(d\left(\mathbf{a},\mathbf{X},\mathbf{Z}\right),\mathbf{s},\mathbf{X},\mathbf{Z}\right)=-\left(\sum_{i=1}^{n}\left(E\left[C_{i,Insp}^{SHM}\right]+E\left[C_{i,R}^{SHM}\right]+E\left[C_{i,SHM}\right]+R_{i,D}^{SHM}\right)+R_{F_S}^{SHM}\right)$$

Thöns, S., R. Schneider and M. H. Faber (2015). Quantification of the Value of Structural Health Monitoring Information for Fatigue Deteriorating Structural Systems. 12th International Conference on Applications of Statistics and Probability in Civil Engineering (ICASP12), . Vancouver, Canada.

Service life integrity management and risk model



Reliability based inspection and repair planning is utilized as a decision rule.

Adaptive actions

- Inspection and repair

Normalized cost model

- Component inspection, repair, failure
- System failure
- SHM system investment, installation and operation
- Discounted

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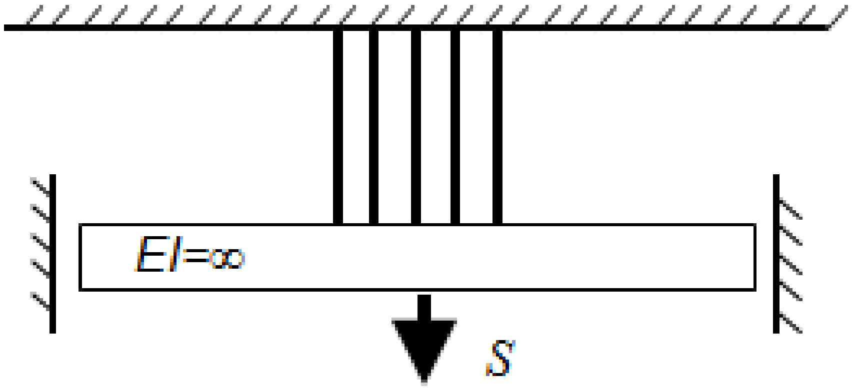
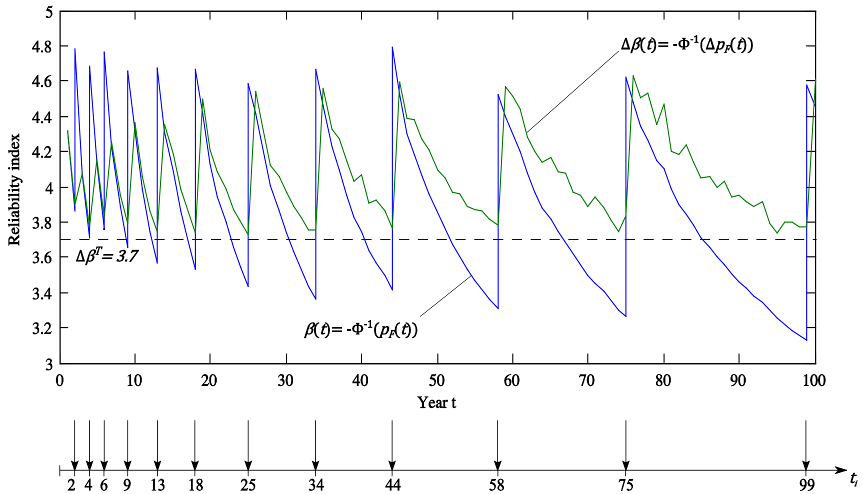
Example



A ductile and brittle Daniels system with 5 components is studied.

- Value of Information analysis
- Structural performance subjected to fatigue degradation
- SHM strategy: Load monitoring

Service life integrity management and risk model



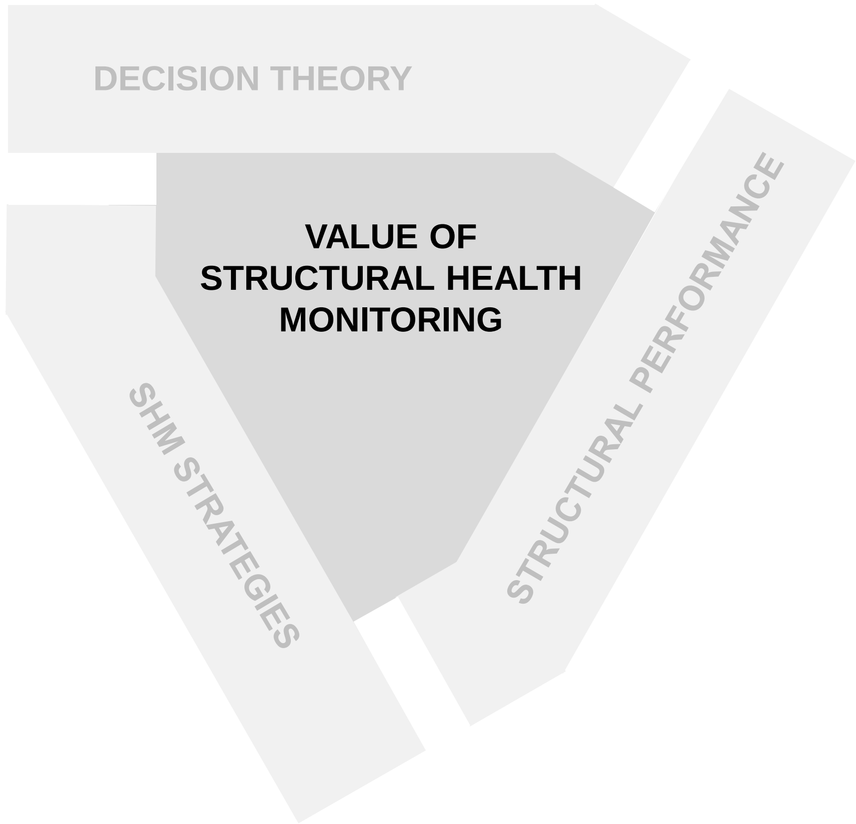
Reliability based inspection and repair planning is utilized as a decision rule.

Adaptive actions

- Inspection and repair

Normalized cost model

- Component inspection: $1.0x10^{-3}$
- Component repair: $1.0x10^{-2}$
- Component failure: 1.0
- Systemfailure 100.0
- SHM system investment ($6.7x10^{-4}$), installation ($6.7x10^{-4}$) and operation ($2.0x10^{-4}$)
- Discount rate 5.0%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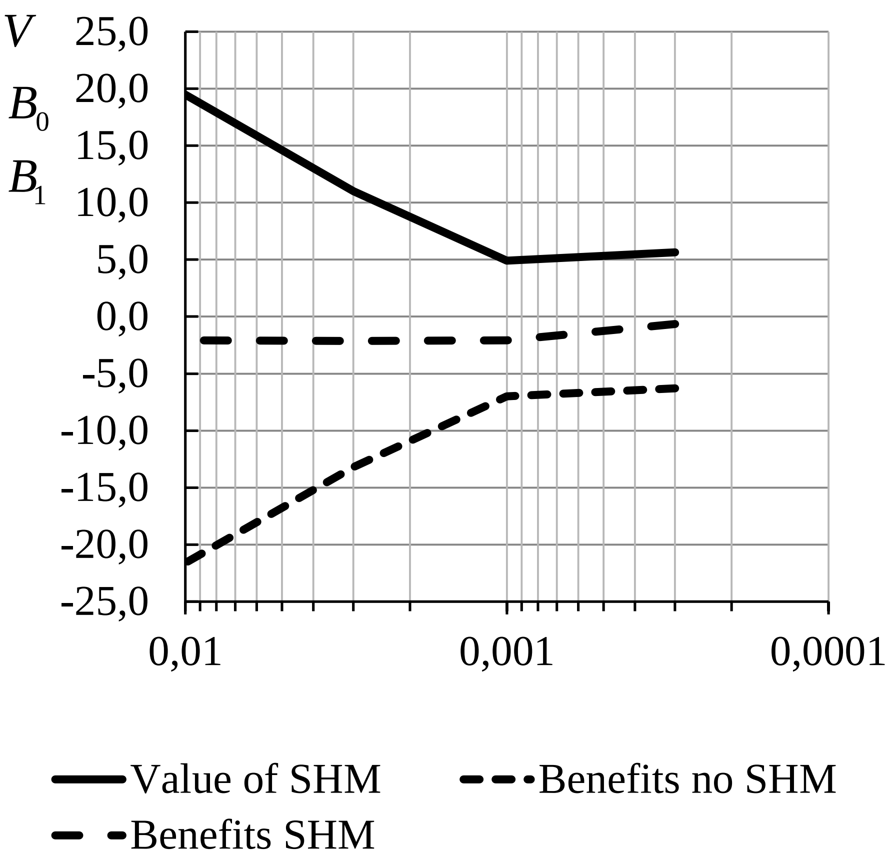
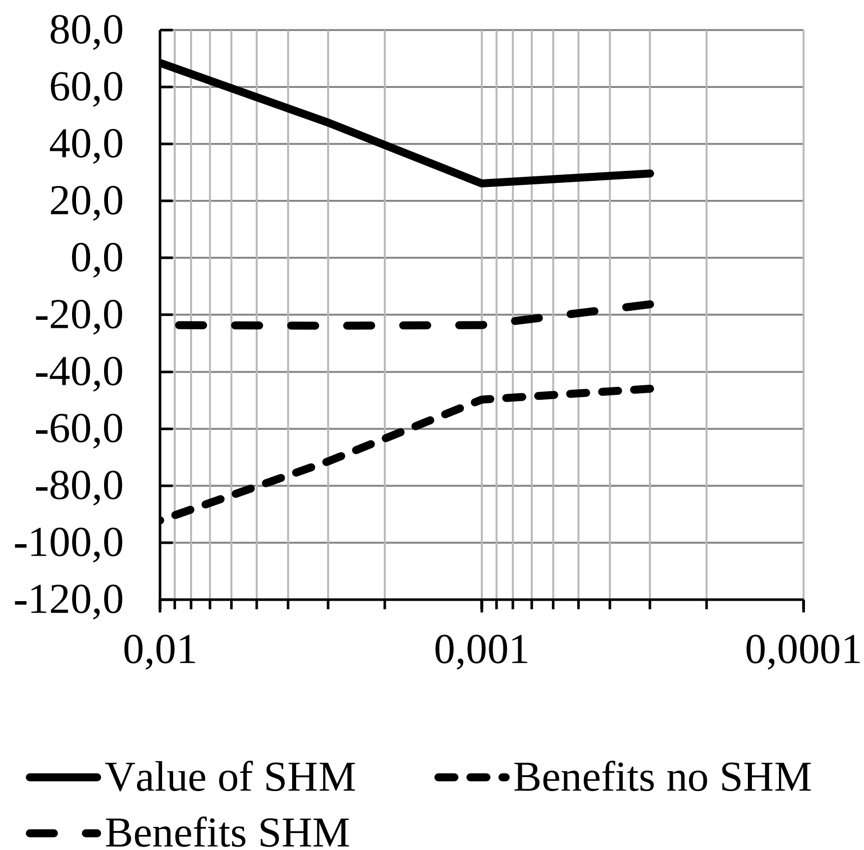
Value of SHM

Ductile Daniels System



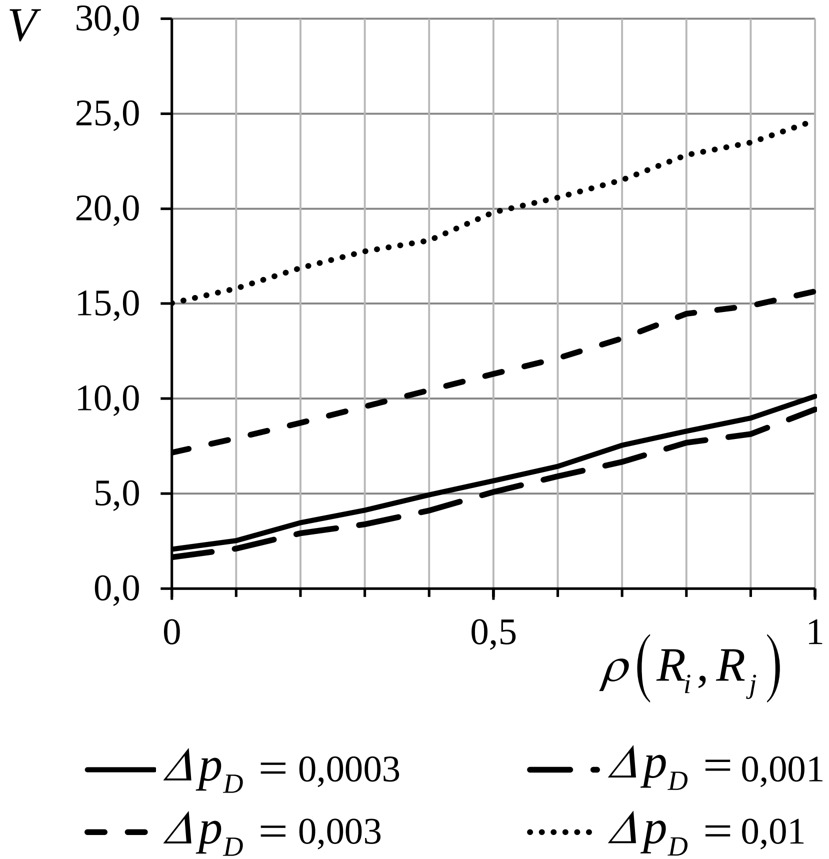
Brittle Daniels System



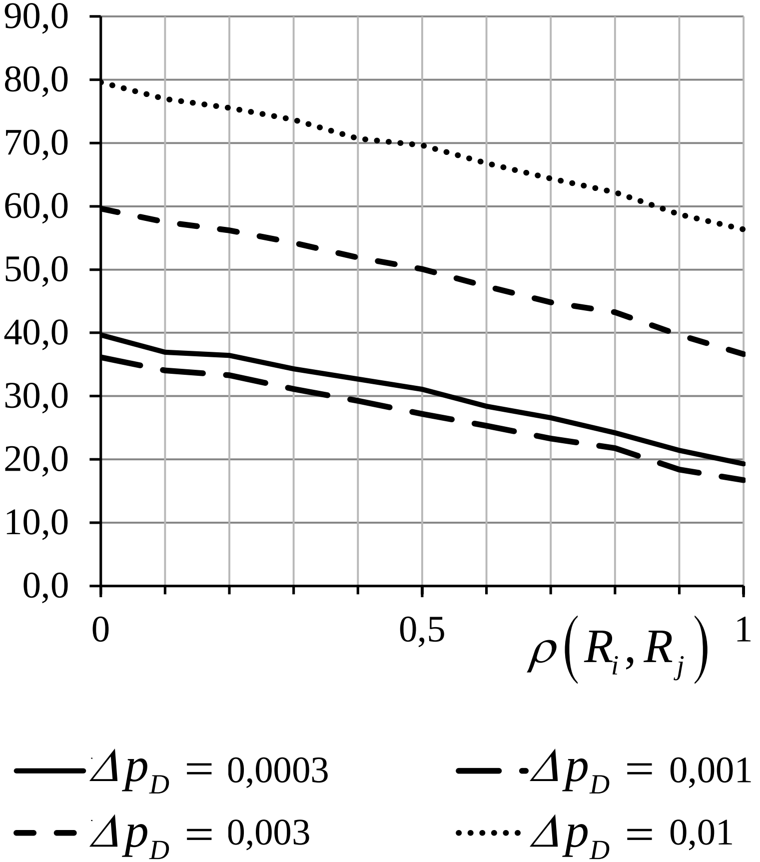
$\rho\left(R_i, R_j\right) = 0.5 \qquad \rho\left(D_i, D_j\right) = 0.6 \qquad P\left(F_{c,i}\right) = 1.0 \cdot 10^{-2}$

Value of SHM for resistance dependencies

Ductile Daniels System



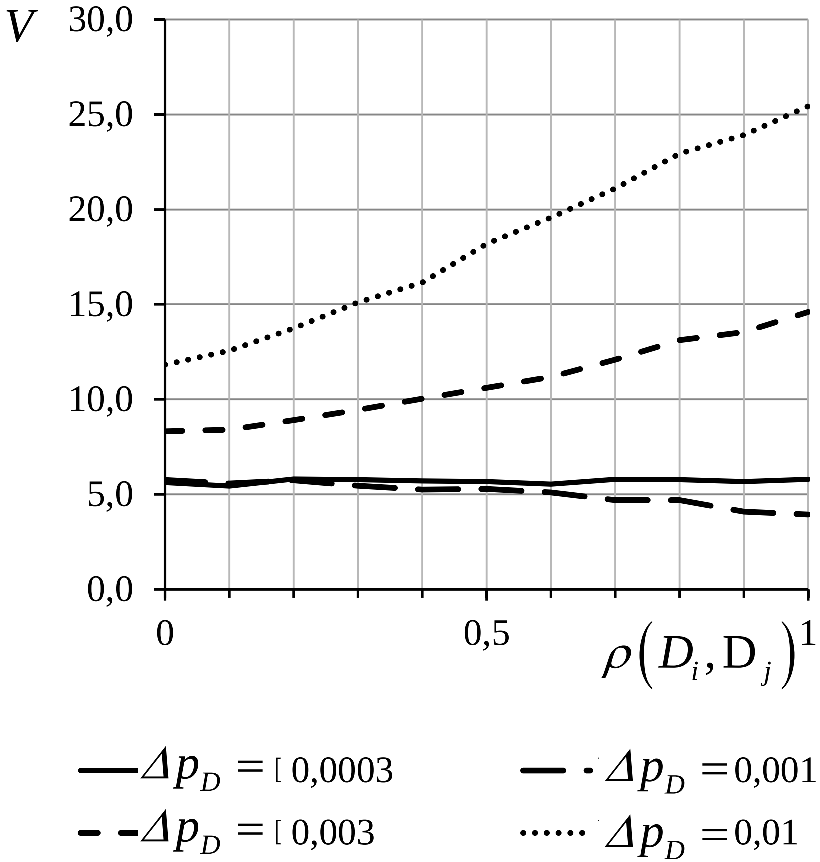
Brittle Daniels System



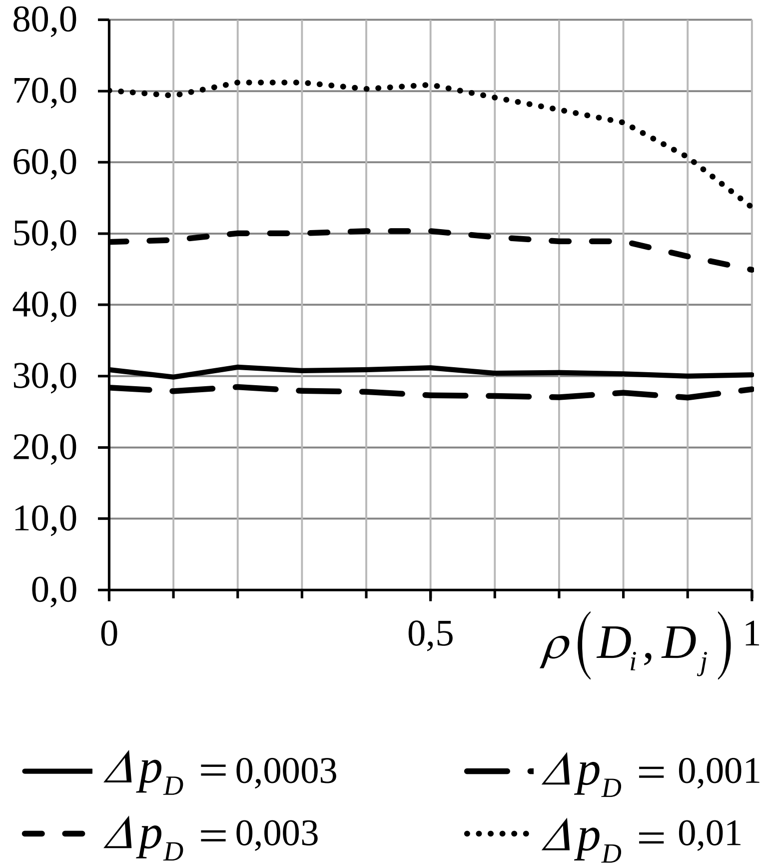
$$\rho(D_i, D_j) = 0.6 \quad P(F_{c,i}) = 1.0 \cdot 10^{-2}$$

Value of SHM for deterioration dependencies

Ductile Daniels System



Brittle Daniels System



$\rho(R_i, R_j) = 0.5 \quad P(F_{c,i}) = 1.0 \cdot 10^{-2}$

Conclusions

The Value of SHM was quantified for a ductile and a brittle Daniels system in dependency of the correlation between the resistances and the deterioration states.

- The value of SHM is dominated by the system reliability and consequences of failure.
- The value of SHM increases for ductile systems and decreases for brittle systems with increasing resistance correlation
- Similar behaviour can be observed for the deterioration state given it is relevant for the system reliability

Thank you for your attention.