Evaluating the value of SHM with longitudinal performance indicators and hazard functions using Bayesian dynamic predictions

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Outline

1. Objectives of evaluating the value of SHM
2. Joint model of longitudinal data and hazard function
3. Hazard based maintenance planning and value of SHM
4. Conclusions

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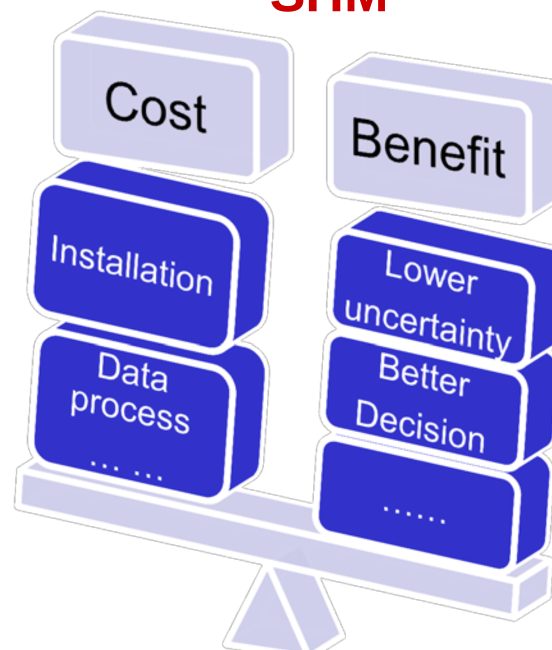
Monitoring or not?

Uncertainties

Decisions

SHM



Evaluate the value of SHM

What’s the difference?

No SHM

t_{Insp1} t_{Insp2} t_{Inspn}

– Expect Total Life-Cycle Cost (TLCC) t_{SL}

SHM

t_{Insp1} t_{Insp2} t_{Inspn}

=

Value of SHM

Evaluating the Value of SHM (VoSHM)

VoSHM

The difference between the expected TLCC.

Pre-posterior analysis in framework of decision theory.

Inspection/Repair plan.

Probability calculation, decision rules…

Basis Joint modeling of longitudinal data and **hazard function**.

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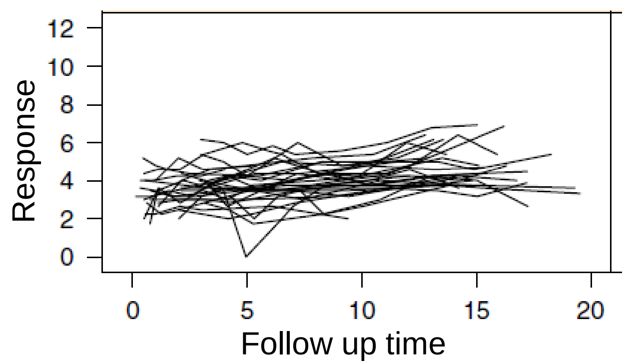
Longitudinal data and hazard function

Medical Research

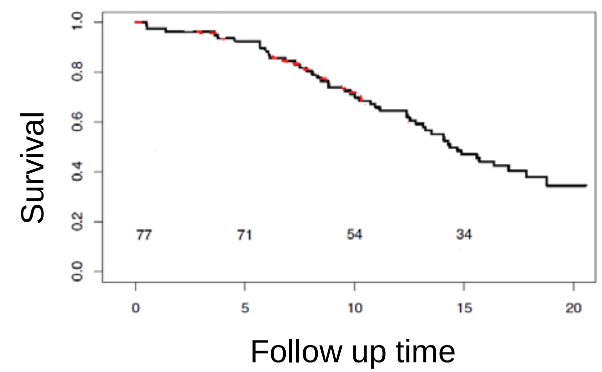
Longitudinal (time series) data:

Response measurements



Time-to-event data:

Time-to-death



similarity

Predict risk of death for a patient with same disease.

Structural Engineering

Historical PI measurements

of a group of structures/components

Or Datasets from simulation

Hazard function

Longitudinal data and hazard function

1) Structural performance time series

$$y(t) = m(t) + \epsilon(t)$$

$m(t)$~ the underlying structural state

with random effects *b* ~ variance *D.*

$\epsilon(t)$ ~ observation error, N(0, σ_1)

2) Survival process defining the hazard function

$$h(t) = \lim_{s \to 0} \frac{Pr(t < T < t + s | T > t)}{s}$$

$$= h_0(t) \exp\{\boldsymbol{\alpha}^T f(b, t)) + \alpha_n\}$$

$h_0(t)$~baseline hazard $\boldsymbol{\alpha}$ ~regression parameters.

Baseline hazard and association structure

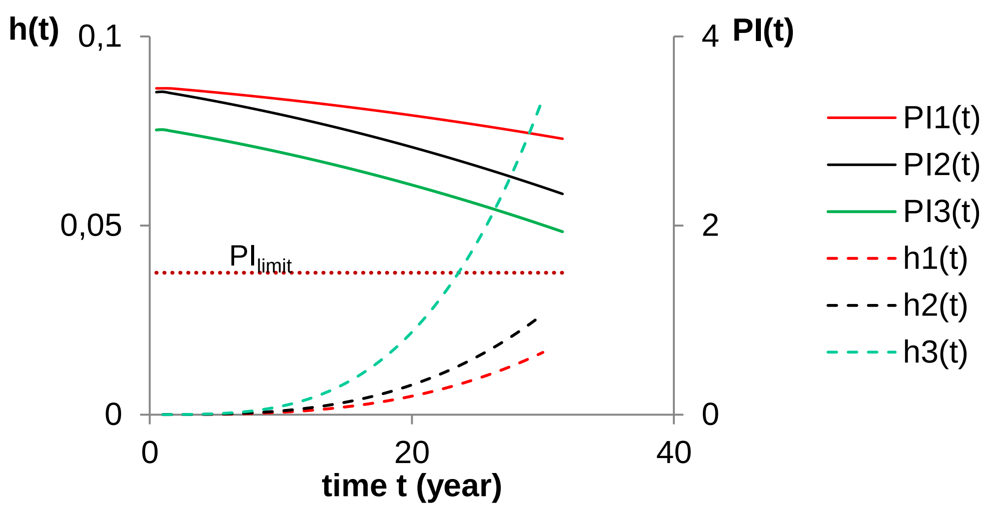
- **Baseline hazard** $h_0(t)$:

  Weibull baseline →accelerated failure time model

- **Association structure**:

  $m(t)$ and it's time dependent changing rate $m'(t)$

$$h(t) = \boxed{\sigma_2 t^{\sigma_2 - 1}} \boxed{\exp\{\alpha_1 m(t) + \alpha_2 m'(t) + \alpha_3\}}$$



Parameter estimation

$\theta = [\sigma_1, \alpha_1, \alpha_2, \alpha_3, \sigma_2, D)$

- Sample dataset $\boldsymbol{b}\ (u_0, D) \rightarrow \{\boldsymbol{y}_i, T_i\},\ \ \boldsymbol{y}_i = [y_{it}],\ \begin{cases} t = 1\sim ni \\ i = 1\sim N \end{cases}$
- $p(y_i, T_i, \delta_i \,|b_i, \theta)$

$$= \prod_{l=1}^{n_i} p\left(y_{il} \middle| b_i; \theta_y\right)\ p(T_i, \delta_i\,|b_i\,; \theta_t)\ p(b_i; \theta_b)$$

$$\propto \left[\begin{array}{c} (\sigma_1^2)^{-\frac{n_i}{2}} \exp\{-\sum_l (y_{il} - m_i(l\,))^2/2\sigma_1{}^2\} \times [\sigma_2 t^{\sigma_2 - 1} \exp\{\alpha_1 m_i(T_i) + \alpha_2 m_i{}'(T_i) + \alpha_3\}] \\ \times \exp\left[-\int_0^{T_i} \sigma_2 t^{\sigma_2 - 1} \exp\{\alpha_1 m_i(s) + \alpha_2 m_i{}'(s) + \alpha_3\} ds\right] \\ \times p(b_i; \theta_b) \end{array} \right]$$

Parameter estimation

Likelyhood : $L(\boldsymbol{D}|\theta) = \prod_{i=1}^{N} [p(y_i, T_i, \delta_i | b_i, \theta)]$

Prior : $\pi(\theta)$

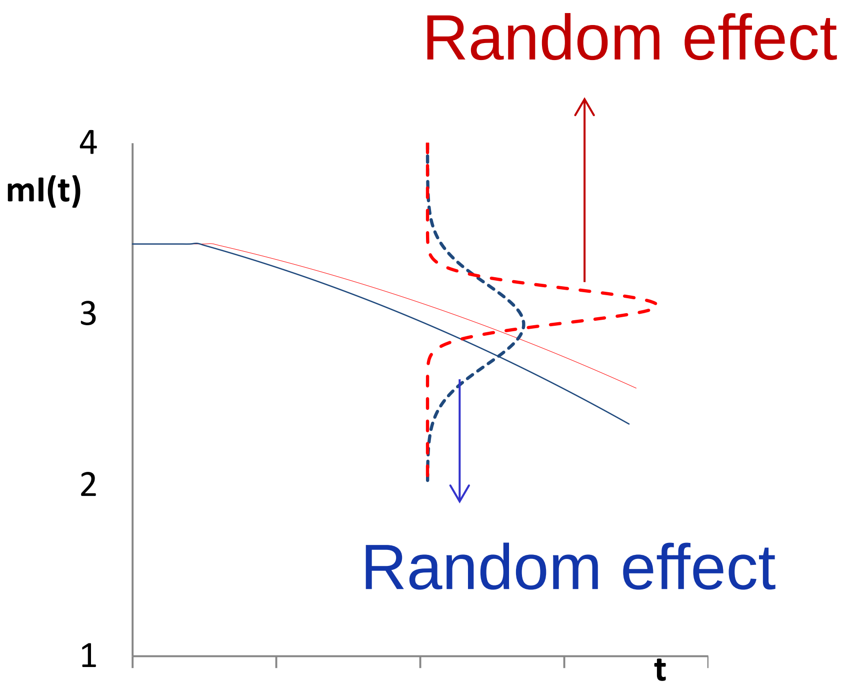
Posterior : $\pi''(\theta) \propto L(\boldsymbol{D}|\theta)\pi(\theta)$

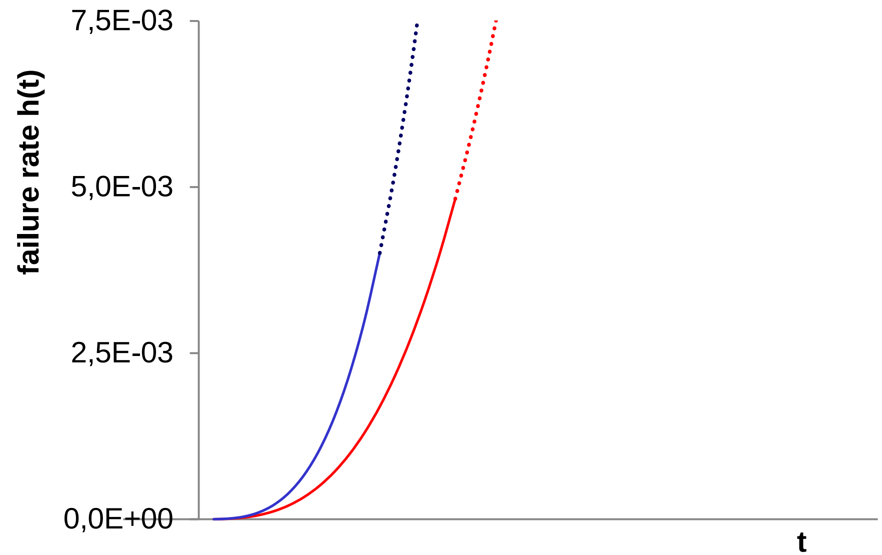
$$= \prod_{i=1}^{N} [p(y_i, T_i, \delta_i | b_i, \theta)] \, \pi(\theta)$$

MCMC methods: Gibbs sampler, MH algorithm

R package: JMbayes

Joint modeling





Joint modeling

Without SHM→ $b_i\,(D)$ → $h(t)$

With SHM→ $\hat{b}_i(\hat{D})$ → $\hat{h}(t)$

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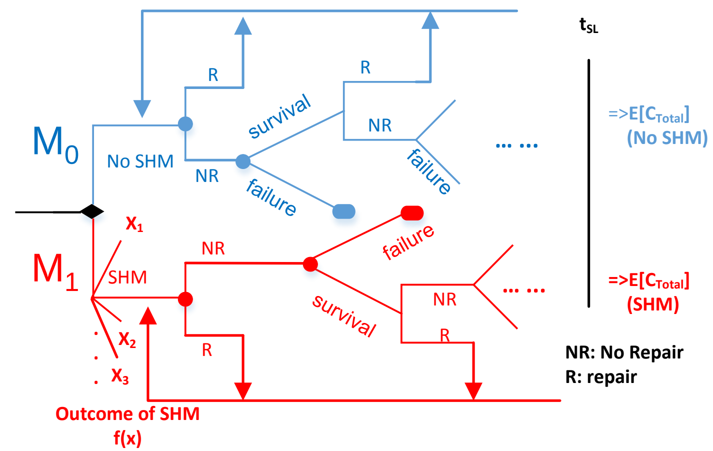
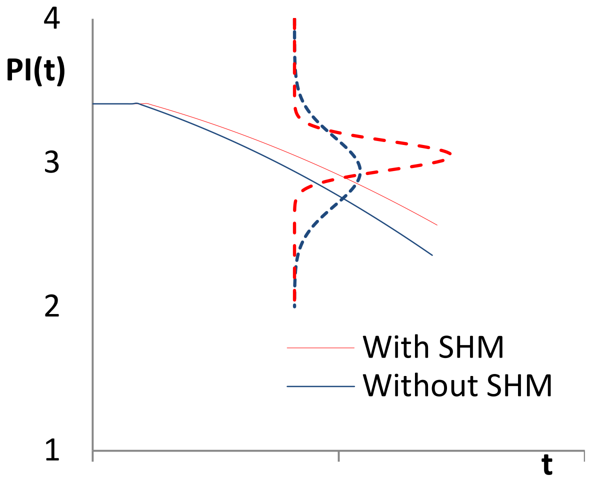
Value of SHM

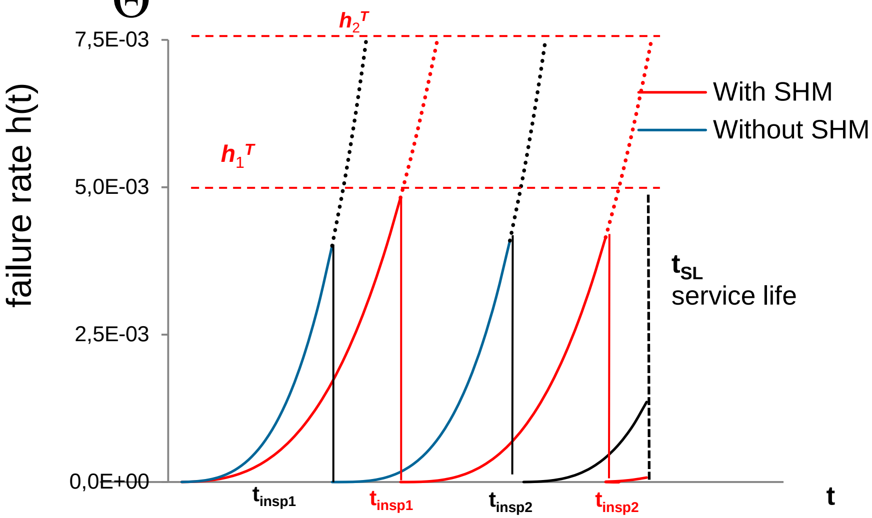
Z - Inspection outcome;
X - Monitoring outcome;
Θ - Structural state ← $f'_{\Theta}(\theta)$;
a - Maintenance action, d(e, z);
e - Inspection decision ← $h(t)\&h(t)^T$



For M_0: $$E'_{\Theta}[C_T(e,d,\theta)] = \int_{\Theta} C_T(e,d,\theta) f'_{\Theta}(\theta) d\theta$$

For M_1: $$E''_{\Theta}[C_T(x,e,d,\theta)] = \int_{\Theta} C_T(x,e,d,\theta) f''_{\Theta}(\theta|x) d\theta$$





Value of SHM

For M_0: $E'_{\Theta}[C_T(e,d,\theta)] = \int_{\Theta} C_T(e,d,\theta) f'_{\Theta}(\theta) d\theta$

For M_1: $E''_{\Theta}[C_T(x,e,d,\theta)] = \int_{\Theta} C_T(x,e,d,\theta) f''_{\Theta}(\theta|x) d\theta$

EVSI: (VoSHM) $E_{\mathrm{X}}[CVSI(x)] = E'_{\Theta}[C_T(e,d,\theta)] - \int_X E''_{\Theta}[C_T(x,e,d,\theta)] f_{\mathrm{X}}(x) dx$

For X: $f_{\mathrm{X}}(x|M) = E_{\Theta}[f_X(x|M,\theta)]$

Model uncertainty

Measurement uncertainty

Expected TLCC

$$E[C_T(e,d,t_{SL})] = E[C_F(e,d,t_{SL})] + E[C_I(e,d,t_{SL})] + E[C_R(e,d,t_{SL})] + E[C_M(e,d,t_{SL})]$$

$$E[C_F(e,d,t_{SL})] = \sum_{t=1}^{t_{SL}} \left[\left(1 - \sum_{i=1}^{t-1} p_R(e,d,i)\right) \frac{1}{(1+r)^t} \left(h(e,d,t) \left(1 - p_F(e,d,t-1)\right) C_F + p_R(e,d,t) E[C_F(e,d,t_{SL}-t)] \right) \right]$$

t_{SL} - service life, *r* - discount rate. $C_F, C_{Insp}, C_R, \boldsymbol{C_M}$

Straub, D. (2004). Generic Approaches to Risk Based Inspection Planning for Steel Structures. ETH Zürich.
Thöns, S. (2012). Monitoring based condition assessment of offshore wind turbine support structures. ETH Zürich.

Aspects to calculate expected TLCC

- Probabilities related to the decision tree:

  - $h(t) = \sigma_2 t^{\sigma_2 - 1} \exp\{\alpha_1 m(t) + \alpha_2 m'(t) + \alpha_3\}$ ⬅ Joint model

  - $p_F(t)$ $\begin{cases} 1 - \exp\left[-\int_0^t \sigma_2 t^{\sigma_2 - 1} \exp\{\alpha_1 m_i(s) + \alpha_2 m_i'(s) + \alpha_3\} ds\right] \\ \text{Weibull distribution} \end{cases}$

  - $p_{det} = \Phi\left(\frac{\delta(t) - \delta_{0.5}}{\sigma_{0.5}}\right)$ $\delta_{0.5}$, $\sigma_{0.5}$ → quality of inspection

  - $p_R = \left(\frac{\delta(t)}{\delta_{max}}\right)^{r_a}$ $(\delta(t) \leq \delta_{max})$ r_a decision parameter

Barone, G. and Frangopol, D. M. (2013). Hazard-Based Optimum Lifetime Inspection and Repair Planning for Deteriorating Structures

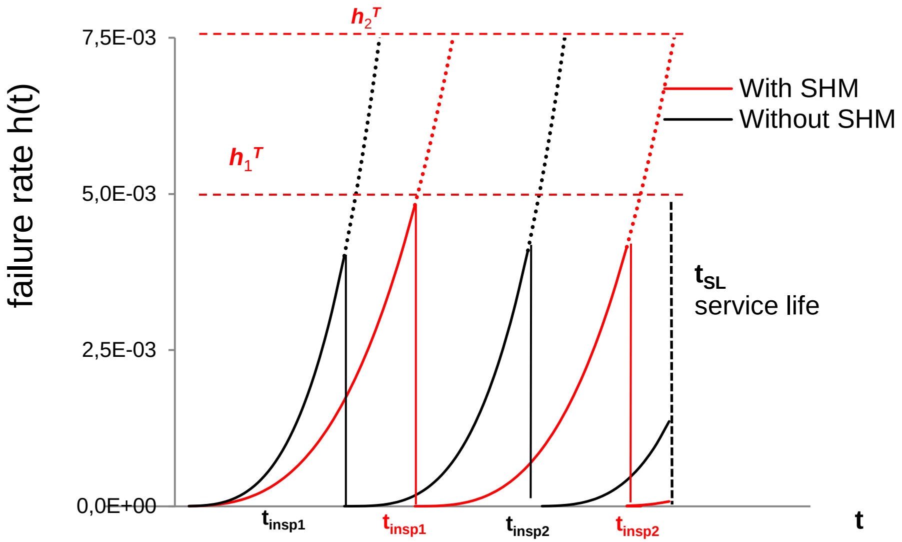
Aspects to calculate expected TLCC

- Risk acceptance criteria and decision rules:

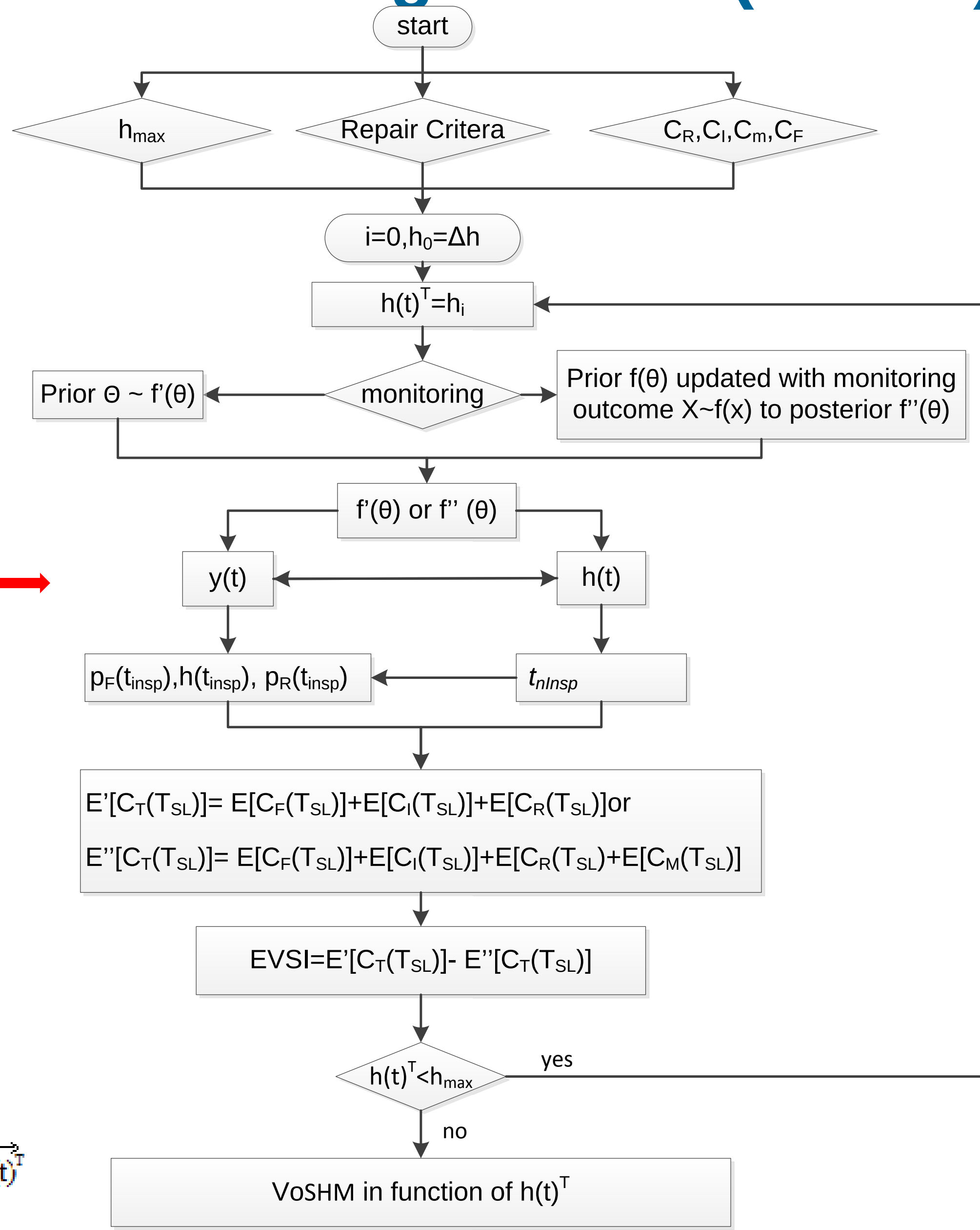
h_{max} → Maximum allowable yearly failure rate (JCSS)

Threshold approach → inspection planning

$$h(t)^T \leq h_{max}$$



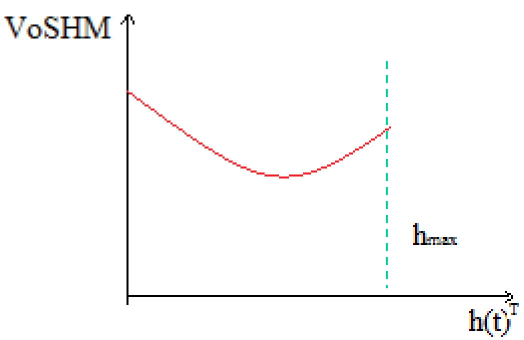
Flow chart for evaluating VoSHM (for M1)



With joint modeling

SHM outcome distribution is modeled

Change $h(t)^T$



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- Derived hazard function is used as a tool for determining inspection/repair plans;

- The uncertainties related to the SHM outcomes are considered and incorporated in the joint model, leading to an updated inspection/repair planning and expected TLCC;

- The difference between the prior and posterior expected TLCC is defined as the VoSHM.

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Thank you for your attention!

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