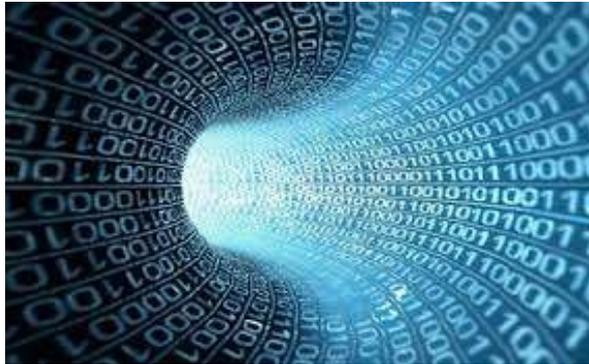
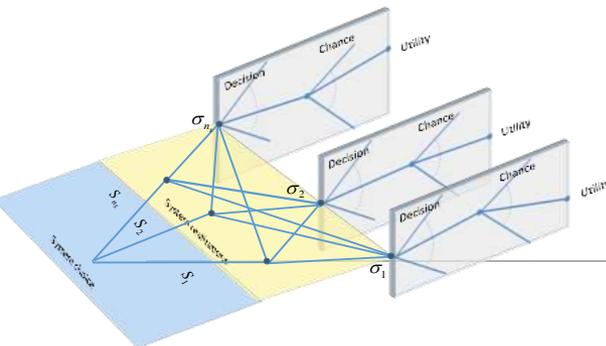
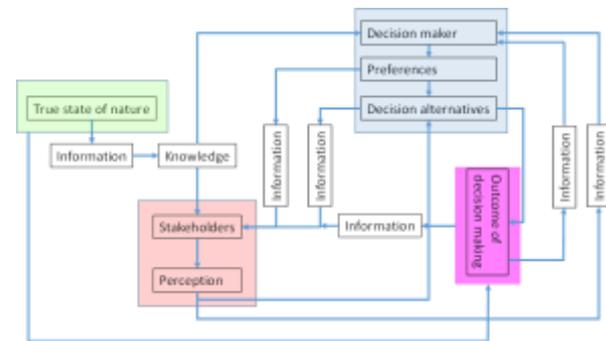


COST Action TU 1402, Final Conference,
BAM, Berlin, February, 18 2019



Theoretical Framework for Value of Information in Structural Health Monitoring

Michael Havbro Faber
Aalborg University, Denmark
&
Dimitri Val
Herriot Watt University, UK



Contents of Presentation

- Information – why and how?
- Decision analyses
- System representation and decision analysis
- Structural health monitoring
- Some conclusions



Information – why and how?

Some fundamentals

One perspective to our understanding of the universe is that this is based on and may be represented in the form of information.

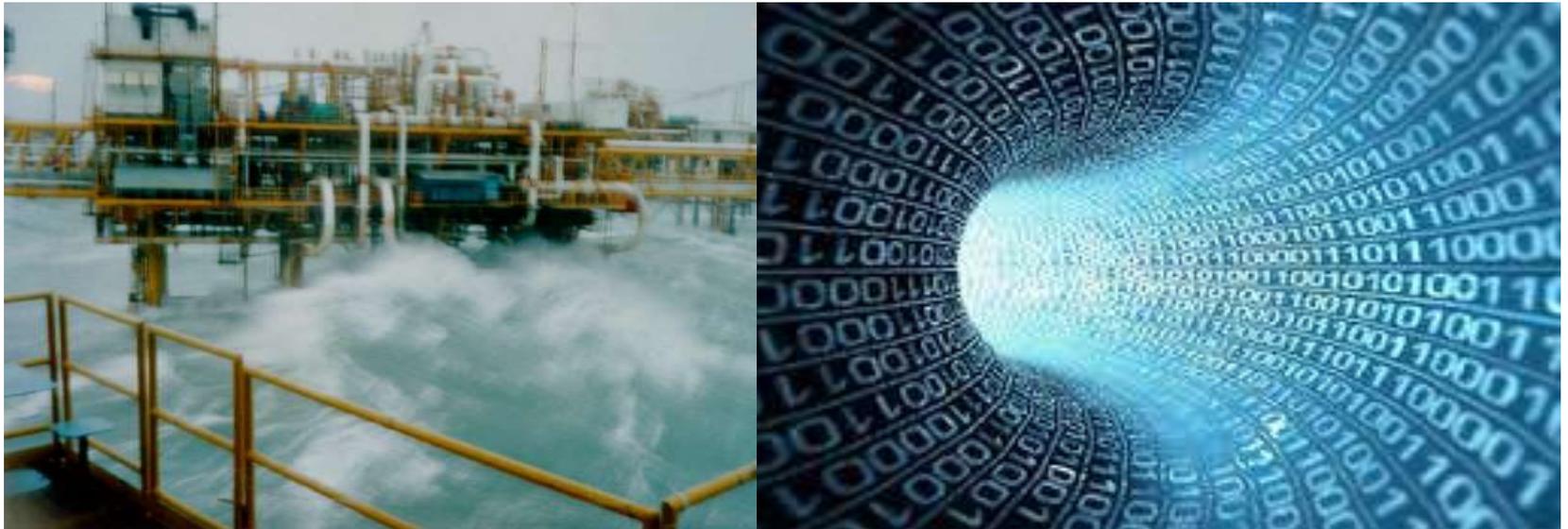
This is me !



Information – why and how?

Some fundamentals

When we are managing a system – what we are really doing is that we are manipulating information



Information – why and how?

Representing and synthesizing information

Mathematical frameworks for representing and synthesizing information include the theories of **probability, statistics and information theory**

The Three Axioms of Probability Theory

The probability theory is built up on – only – three axioms - due to Kolmogorov:

Axiom 1: $0 \leq P(E) \leq 1$

Axiom 2: $P(\Omega) = 1$

Axiom 3: $P\left(\bigcup_{i=1}^k E_i\right) = \sum_{i=1}^k P(E_i)$

When E_1, E_2, \dots are mutually exclusive



General Case

Suppose X can have one of m values... V_1, V_2, \dots, V_m

$P(X=V_1) = p_1$	$P(X=V_2) = p_2$...	$P(X=V_m) = p_m$
------------------	------------------	-----	------------------

What's the smallest possible number of bits, on average, per symbol, needed to transmit a stream of symbols drawn from X's distribution? It's

$$H(X) = -p_1 \log_2 p_1 - p_2 \log_2 p_2 - \dots - p_m \log_2 p_m$$

$$= -\sum_{j=1}^m p_j \log_2 p_j$$

$H(X)$ = The entropy of X

- "High Entropy" means X is from a uniform (boring) distribution
- "Low Entropy" means X is from varied (peaks and valleys) distribution

Copyright © 2001, 2003, Andrew W. Moore Information Girc: Slide 8



Why Information?

Representing knowledge

Bayesian probability theory facilitates that knowledge is consistently represented in terms of our prior degree of belief and any observation/information we may believe is relevant

we have

Likelihood

Prior

$$P(E_i|A) = \frac{P(A|E_i)P(E_i)}{P(A)} = \frac{P(A|E_i)P(E_i)}{\sum_{i=1}^n P(A|E_i)P(E_i)}$$

Posterior

Bayes Rule



Reverend
Thomas Bayes
(1702-1764)

29/29 M. H. Faber Aalborg University, 2019



Information – why and how?

Basis for decision ranking

When we make decisions, we take basis in available knowledge and additional information which may be collected in the future

We rank decision alternatives such as to maximize fulfilment of our preferences

The maximization is based on the expected value of the utility

Utility is a function expressing the degree to which our preferences are fulfilled

postulated by Bernoulli (1738)

shown by von Neumann and Morgenstern (1947)



Information – why and how?

Potential of pre-posterior decision analysis not exploited

Rational decision making in structural engineering was recognized as a main objective already in early works by Freudenthal (1953).

Theoretical and methodical developments on Bayesian decision analysis by e.g. Raiffa and Schlaifer (1961) were recognized and advocated by e.g. Benjamin and Cornell as a strong framework for providing rational decision support.

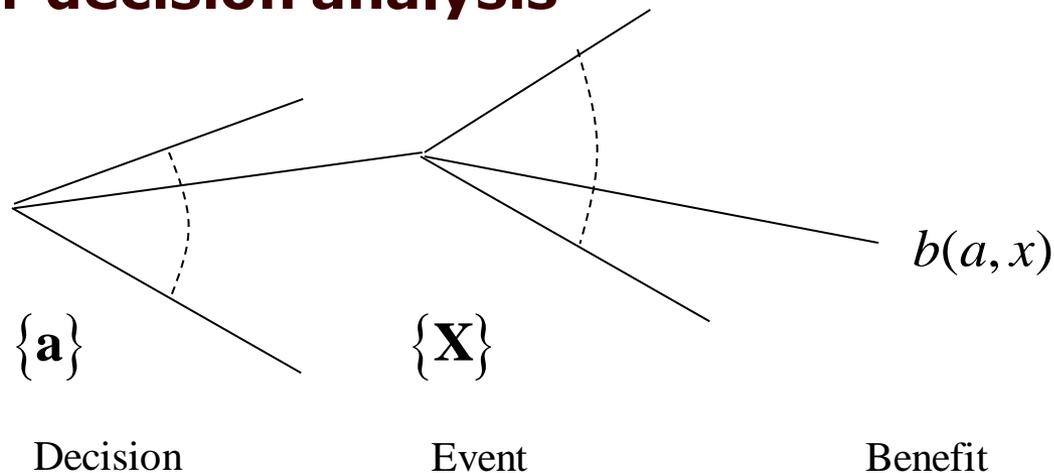
Despite these insights and efforts, applied decision analysis has not gained the impact it could have.

Especially the potential of the pre-posterior and Value of Information (VoI) analysis has not been realized/exploited.



Decision Analysis

Prior decision analysis



Information is bought by choice of prior density

Optimal decision maximizes the expected value of utility (benefit)
(von Neumann & Morgenstern)

$$B_0^* = \max_a E' [b(a, X)] = \max_a \int b(a, x) f'_X(x, a) dx$$



Decision Analysis

Posterior decision analysis

By sampling information $\hat{\mathbf{z}}$ from the sample space $\{\mathbf{X}\}$ using an experiment e we may update the probabilistic description of X

$$f_X''(x, a | \hat{\mathbf{z}}) = \frac{L(x | \hat{\mathbf{z}}) f_X'(x, a)}{\int L(x | \hat{\mathbf{z}}) f_X'(x, a)}$$

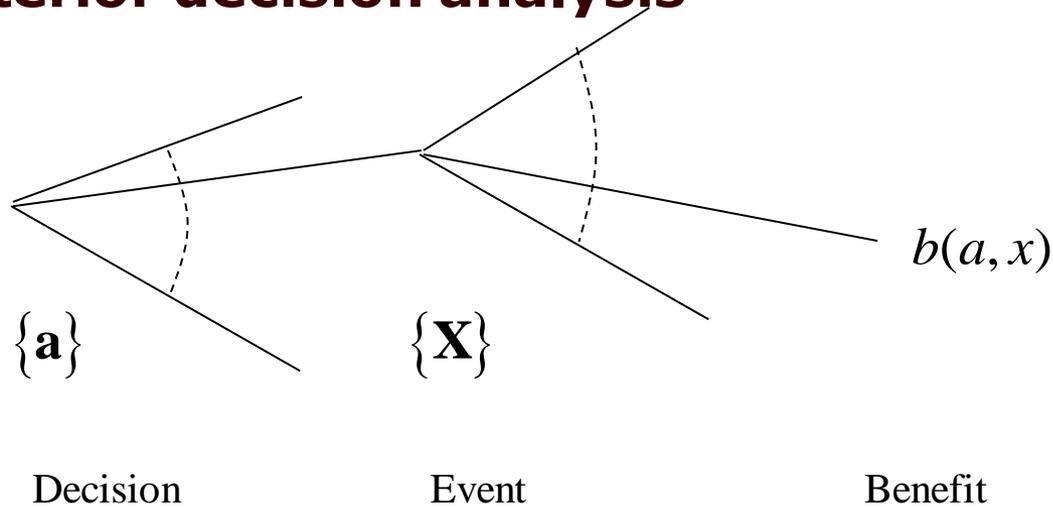
Of course the likelihood of the sample $\hat{\mathbf{z}}$ depends on the experiment e why we write

$$L(x | \hat{\mathbf{z}}) = L(x | \hat{\mathbf{z}}, e)$$



Decision Analysis

Posterior decision analysis

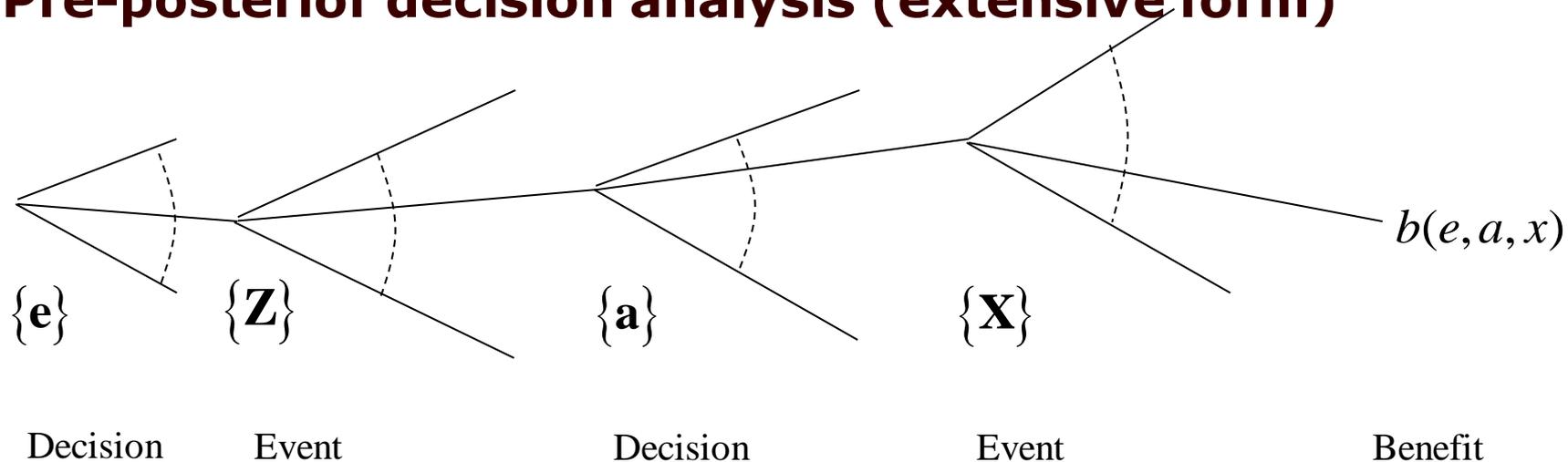


$$\max_a E''[b(a, X)] = \max_a \int b(a, x) f_X''(x, a | \hat{z}) dx$$



Decision Analysis

Pre-posterior decision analysis (extensive form)



The optimal experiment e may be found from

$$B_1^* = \max_e E_Z \left[\max_a \int b(e, a, x) f_X''(x, a | Z) dx \right]$$



Decision Analysis

Principal engineering decisions

Any design decision may be supported by the prior decision analysis

- a choice concerning structural system, materials, dimensions corresponds to a choice of the (prior) probabilistic model of \mathbf{X}

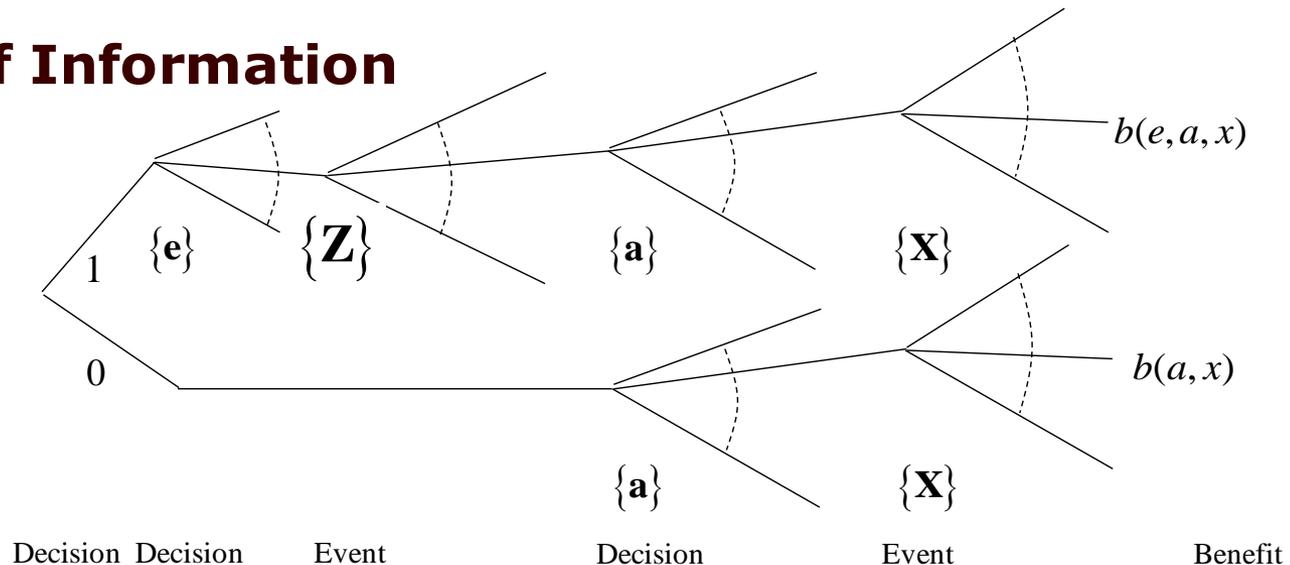
Any decision on assessments, inspections or monitoring may be supported by the pre-posterior decision analysis

- a choice concerning assessment method, inspection method and monitoring scheme will influence the (posterior) probabilistic model of \mathbf{X}



Decision Analysis

Value of Information



The value of information VoI is determined from:

$$VoI = \max_e E_Z \left[\max_a \int b(e, a, x) f_X''(x, a | \mathbf{Z}) dx \right] - \max_a \int b(a, x) f_X'(x, a) dx$$

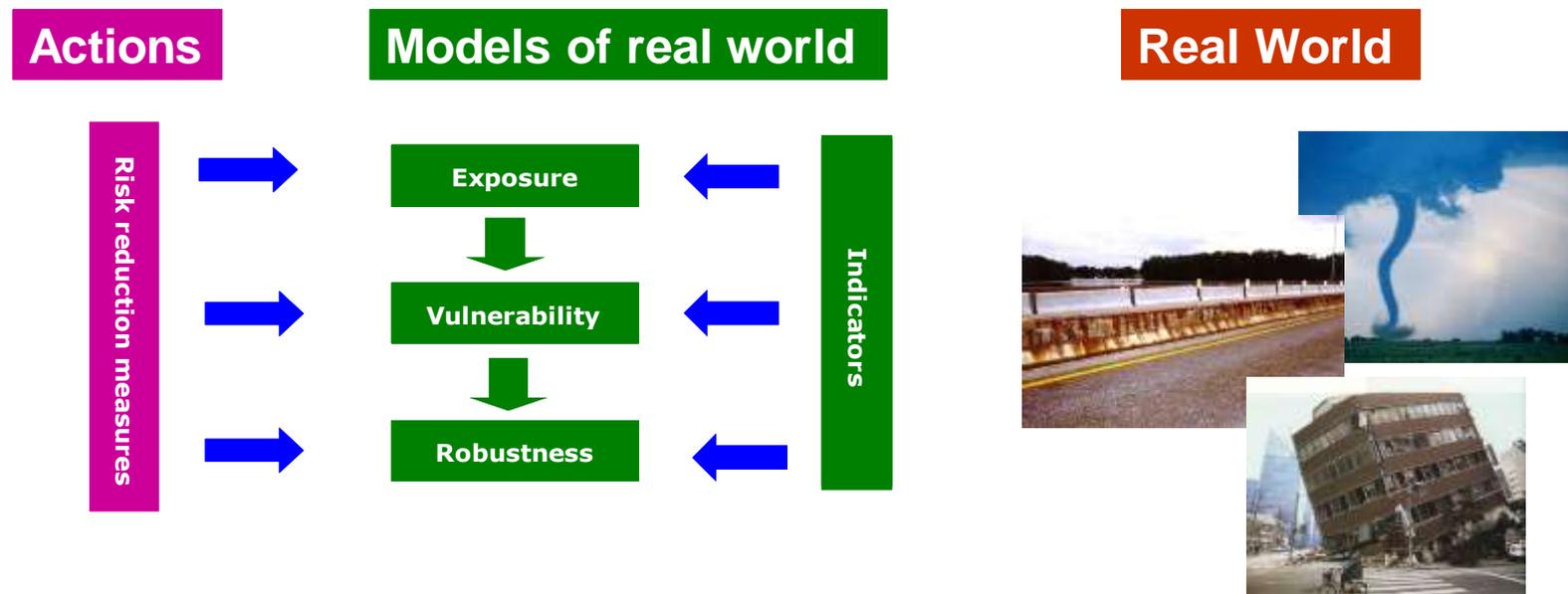
Shows the coupling between buying prior and pre-posterior information



System Representation and Decision Analysis

Problem framing

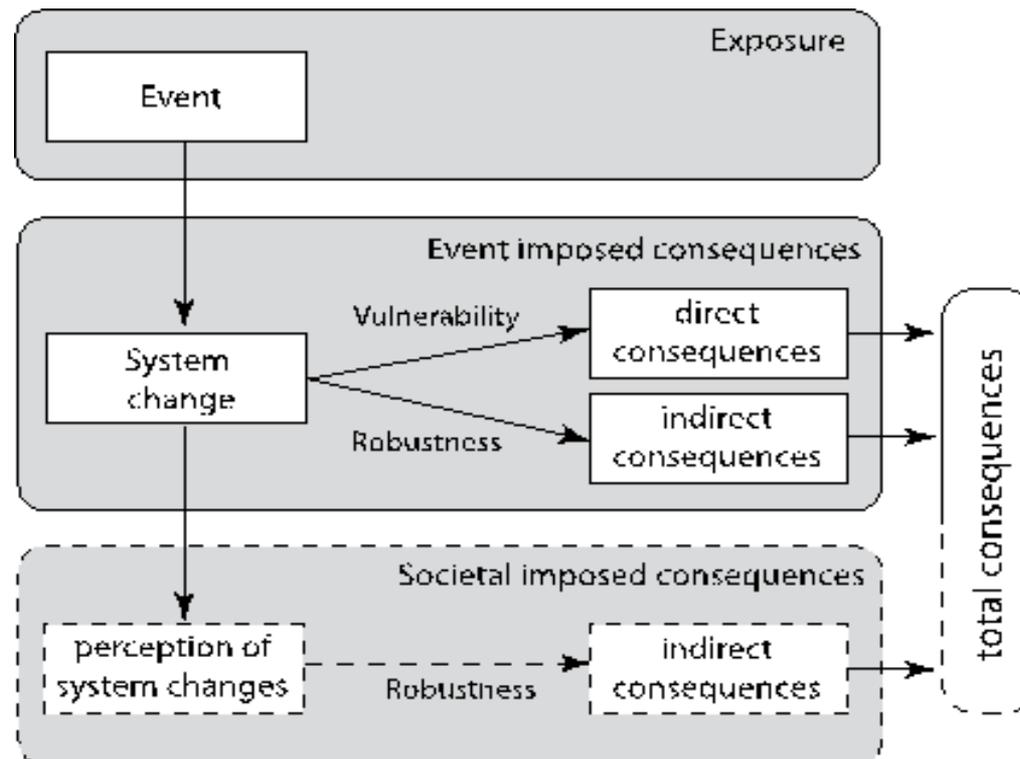
Information and knowledge forms the basis for decision ranking



System Representation and Decision Analysis

Problem framing

Information and knowledge forms the basis for decision ranking



System Representation and Decision Analysis

Problem framing

- Fundamentally we do not know what the truth (real world) is.
- We do not fully appreciate how knowledge and information relates to truth.
- Debatable which knowledge and information is relevant in a given context.
 - In society any knowledge and information is on the “free market”.
 - In science and engineering:
 - knowledge and information might be influenced by what is fundable, expected or desired
 - tendency to mix “truth” with information and assumptions

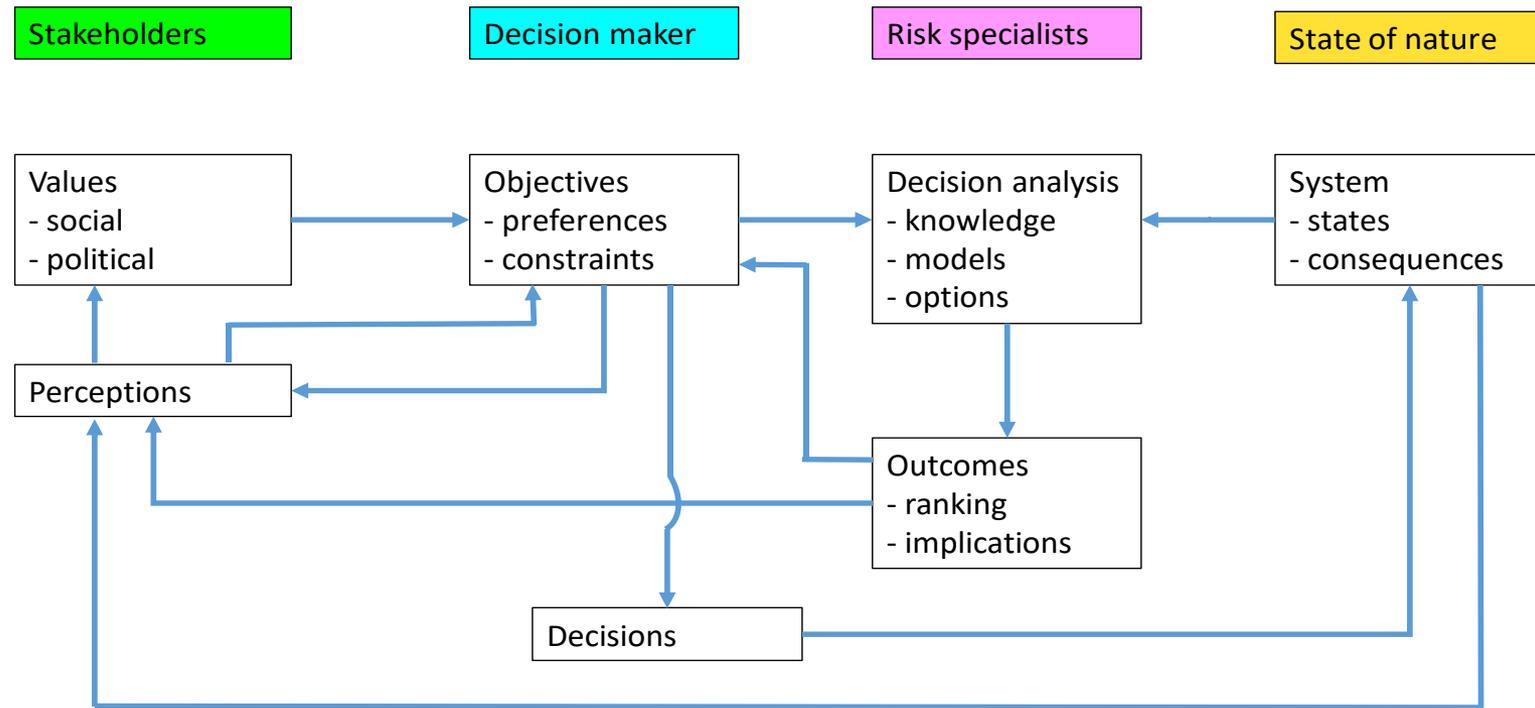


System Representation and Decision Analysis

Nielsen, Tølbøll, Qin and Faber, 2019. Faith and Fakes –
Dealing with Critical Information in Decision Analysis
Special issue of Civil Engineering and Environmental Systems

Problem framing

Information and knowledge
influence all aspects of decision problems



System Representation and Decision Analysis

Problem framing

Information may be associated with different problems

Information may be :

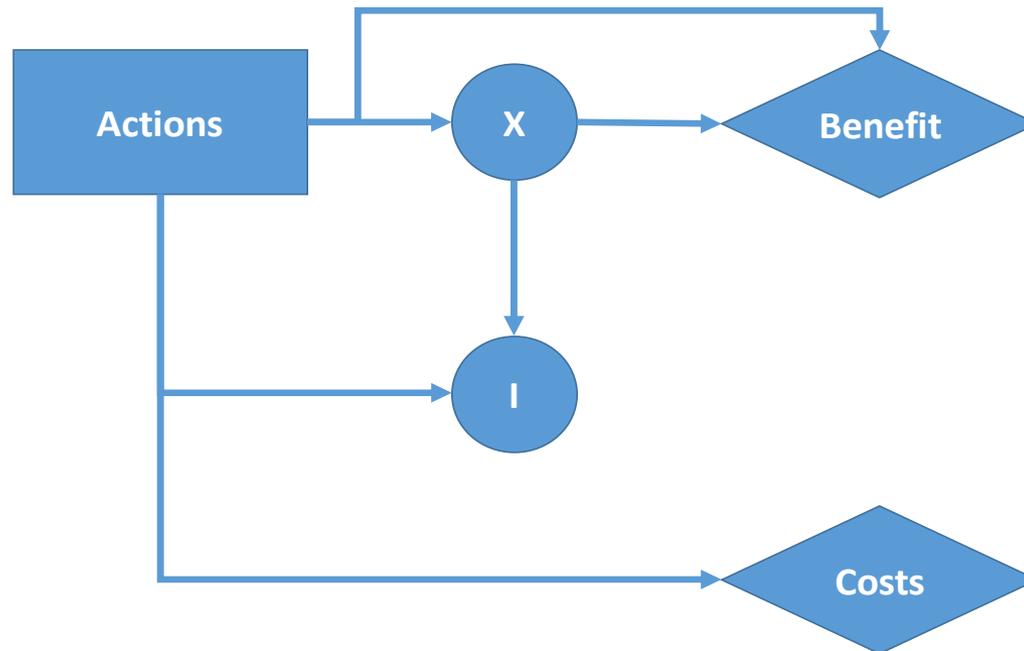
- Relevant and precise.
- Relevant but imprecise.
- Irrelevant.
- Relevant but incorrect.
- Disrupted or delayed.



System Representation and Decision Analysis

Problem framing

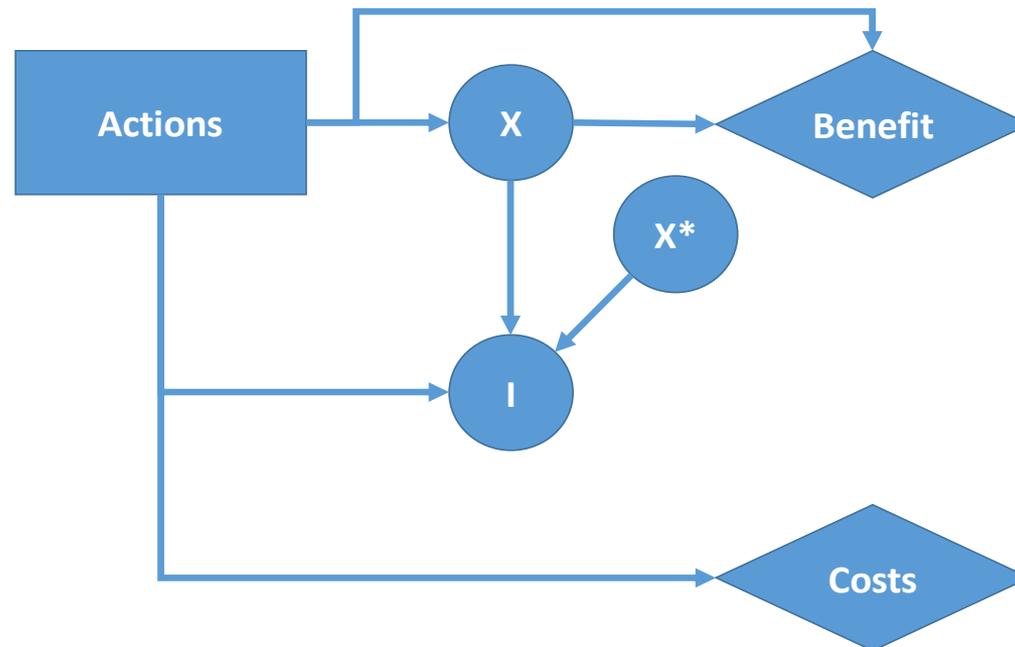
Information may be collected to support decision ranking



System Representation and Decision Analysis

Problem framing

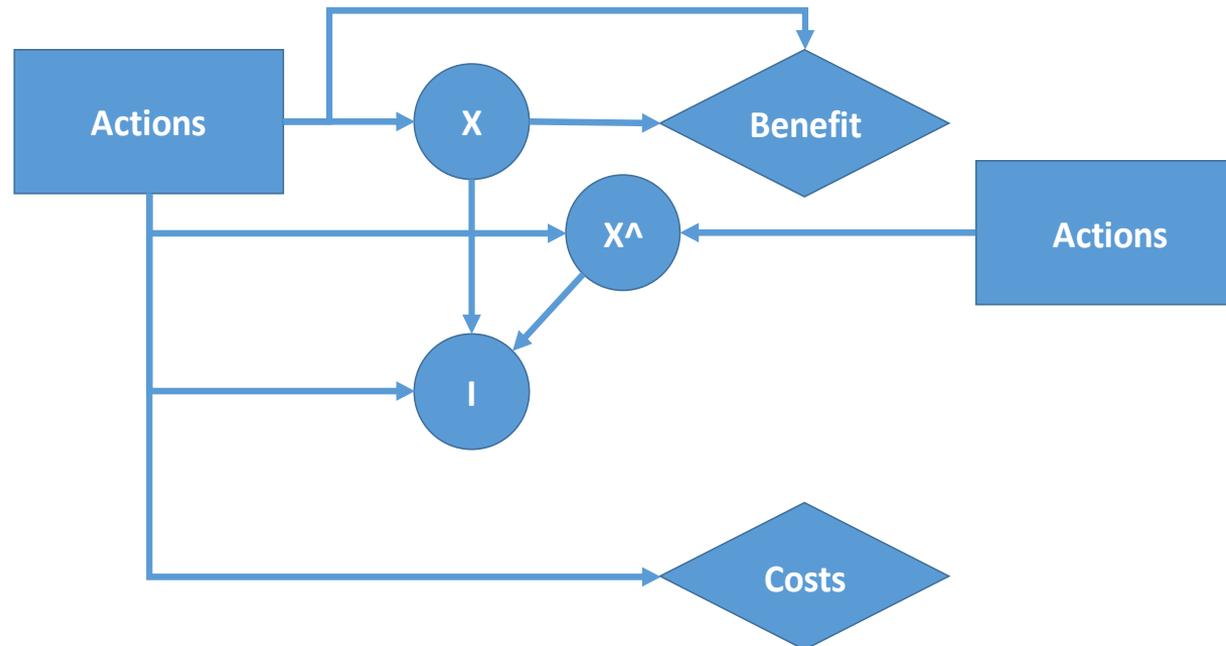
Collected information may originate from unanticipated system with no intent – irrelevant information



System Representation and Decision Analysis

Problem framing

Collected information may originate from unanticipated system with intent – fake information



System Representation and Decision Analysis

Approach – systems and information

Appreciate that the systems we are dealing with are not known.

There may be and in general are competing possible systems.

All relevant possible systems must be accounted for in search for optimal decisions

Information flow and effects of information must be explicitly accounted for as a cause of adverse consequences – but also as means for management



System Representation and Decision Analysis

Approach – systems and information

Appreciating possible competing systems.

Accounting for all relevant scenarios.

Including possible adverse consequences originating from information.

Focus on how management of information might contribute to achieving objectives – options for buying information facilitating for adaptation.



System Representation and Decision Analysis

Approach – systems and information

There is no fundamental difference between information which is intentionally wrong and information which is unintentionally wrong

It is the context – and thereby the relevant systems to be accounted for in the decision analysis which are different

The decision analysis should therefore always account for all relevant possible competing systems



System Representation and Decision Analysis

(Multiple) System representation

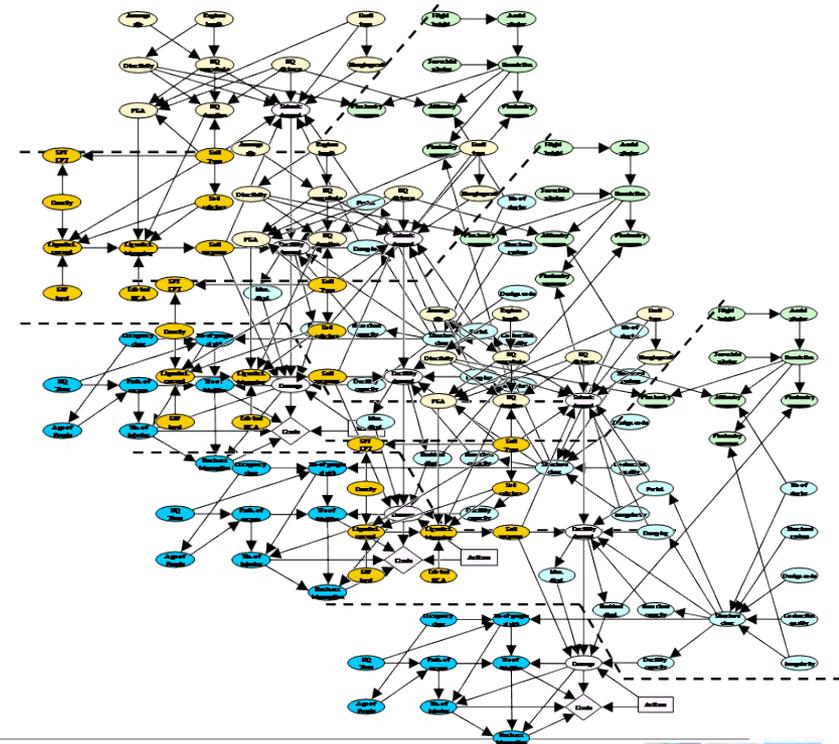
System model $\mathbf{M}(\mathbf{a}) = (\Sigma(\mathbf{a}), \mathbf{C}(\mathbf{a}), \mathbf{X}(\mathbf{a}))^T$

Graph model $\Sigma(\mathbf{a})$

Constituents model $\mathbf{C}(\mathbf{a})$

Probabilistic model $\mathbf{X}(\mathbf{a})$

Decision alternatives \mathbf{a}

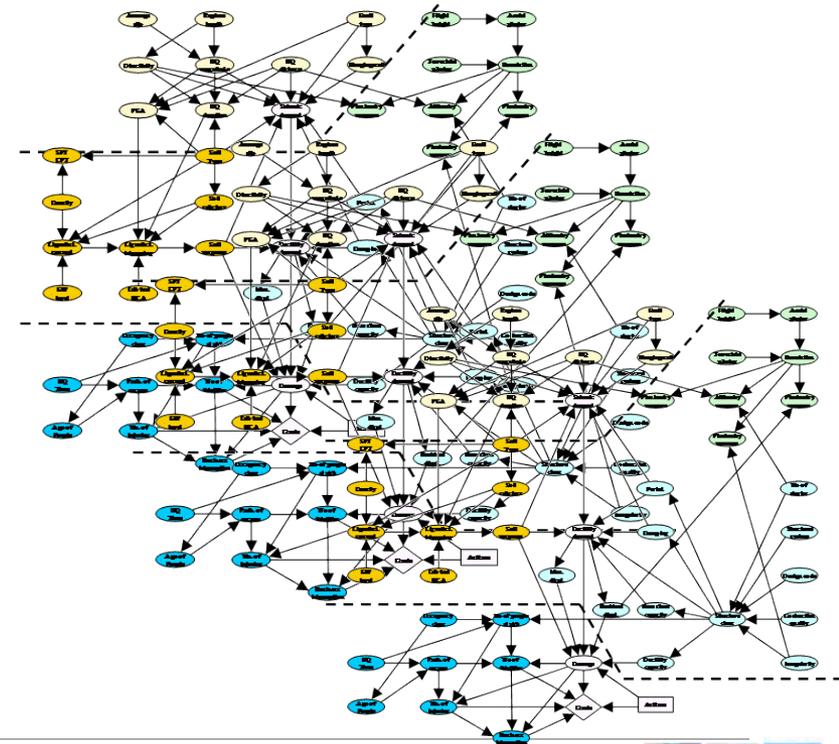


System Representation and Decision Analysis

(Multiple) System representation

$$\mathbf{M}(\mathbf{a}) = (\Sigma(\mathbf{a}), \mathbf{C}(\mathbf{a}), \mathbf{X}(\mathbf{a}))^T$$

- System models may be established using “bottom-up” approaches as in structural engineering or by “top-down” approaches as in data-mining
- Potentially a combination of the two approaches would be adequate
- Bayesian Networks lend themselves for system modelling in either case



System Representation and Decision Analysis

Systems representation

Top-down models – or data driven modelling approaches are usually assumed to be better than bottom-up models – “data cannot lie”.

It is overseen that data-driven models depend entirely on the data-bases, “experiment” plans and algorithms they take basis in – all of which are choices – and thus subjective – in the same manner as bottom-up models

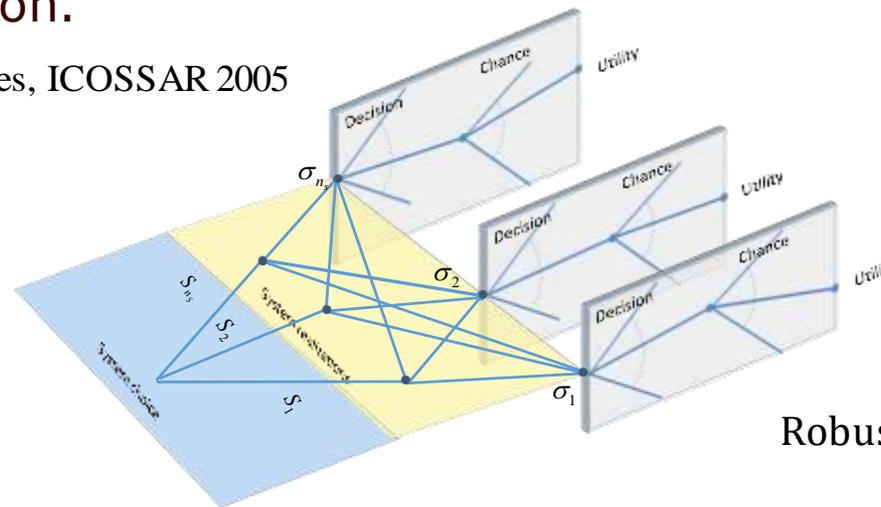


System Representation and Decision Analysis

Decision optimization subject to competing systems

Bayesian decision analysis as framework for managing information.

Faber and Maes, ICOSSAR 2005



$$\text{Robustness} = \frac{E'_{\mathbf{X}|s}(U(a^*, \mathbf{X}))}{E'_{\Sigma \setminus s} \left(E'_{\mathbf{X}|\{\Sigma \setminus s\}}(U(a^*, \mathbf{X})) \right)}$$

$$(s^*, a^*) = \max_s \left(P'(\Sigma = s) \max_a \left(E'_{\mathbf{X}|a} [U(a, \mathbf{X})] \right) + E'_{\Sigma \setminus s} \left[E'_{\mathbf{X}|\{\Sigma \setminus s\}} [U(a^*, \mathbf{X})] \right] \right)$$

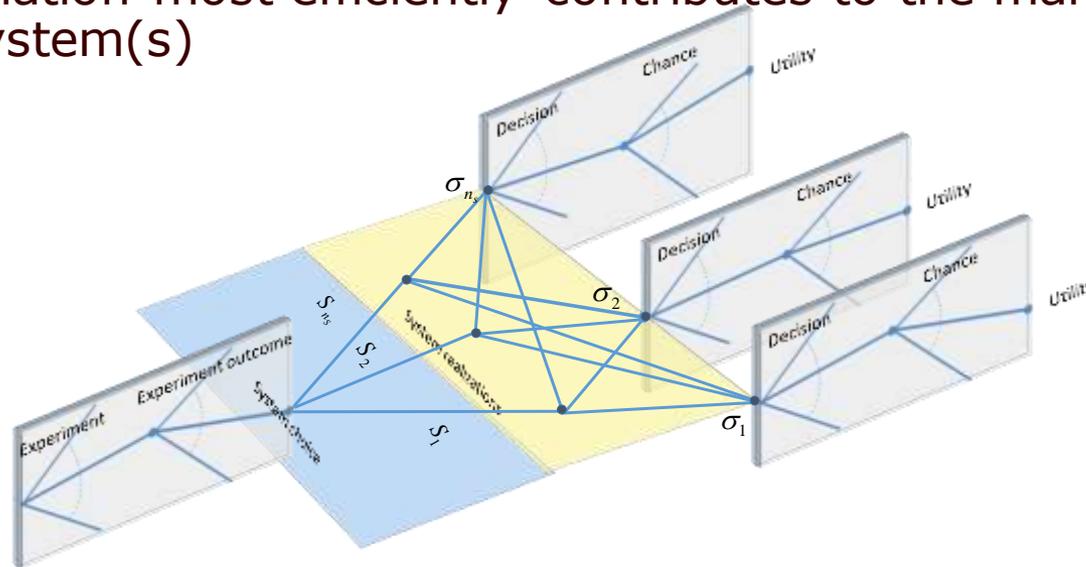
When new information is available prior probability assignments may be updated and the importance of the different possible systems will change – as well as the probability assignments within the different possible systems



System Representation and Decision Analysis

Decision optimization subject to competing systems

Pre-posterior decision analyses to identify how additional information most efficiently contributes to the management of the system(s)



$$(e^*, s^*, a^*) = \max_e E'_Z \left[\max_s \left(P''(\Sigma = s | \mathbf{z}) \max_a \left(E''_{\mathbf{X}|a} [U(a, \mathbf{X})] \right) + E''_{\Sigma \setminus s} \left[E''_{\mathbf{X}|\{\Sigma \setminus s\}} [U(a^*, \mathbf{X})] \right] \right) \right]$$



Structural Health Monitoring

The fundamental logic of SHM is:

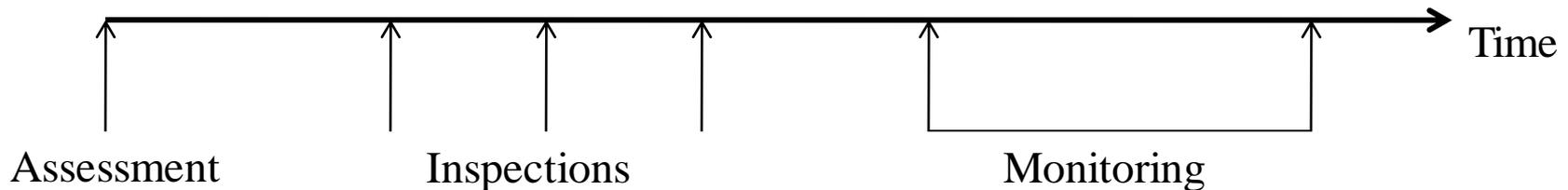
- Monitoring may provide information concerning variables which have a significant influence on the service life performance of a structure
- The information can be collected at a cost and with a given precision which depends on the technique and thereby also depends on the costs
- The information collected through monitoring facilitates that adaptive actions are taken to reduce service life costs or increase service life benefits



Structural Health Monitoring

Inspections vs monitoring?

In the decision analysis structure there is no principal difference between assessment, inspection and monitoring activities



The only difference concerns the number of times at which information is collected and utilized for updating the prior probabilistic model



Structural Health Monitoring

Points to keep in mind

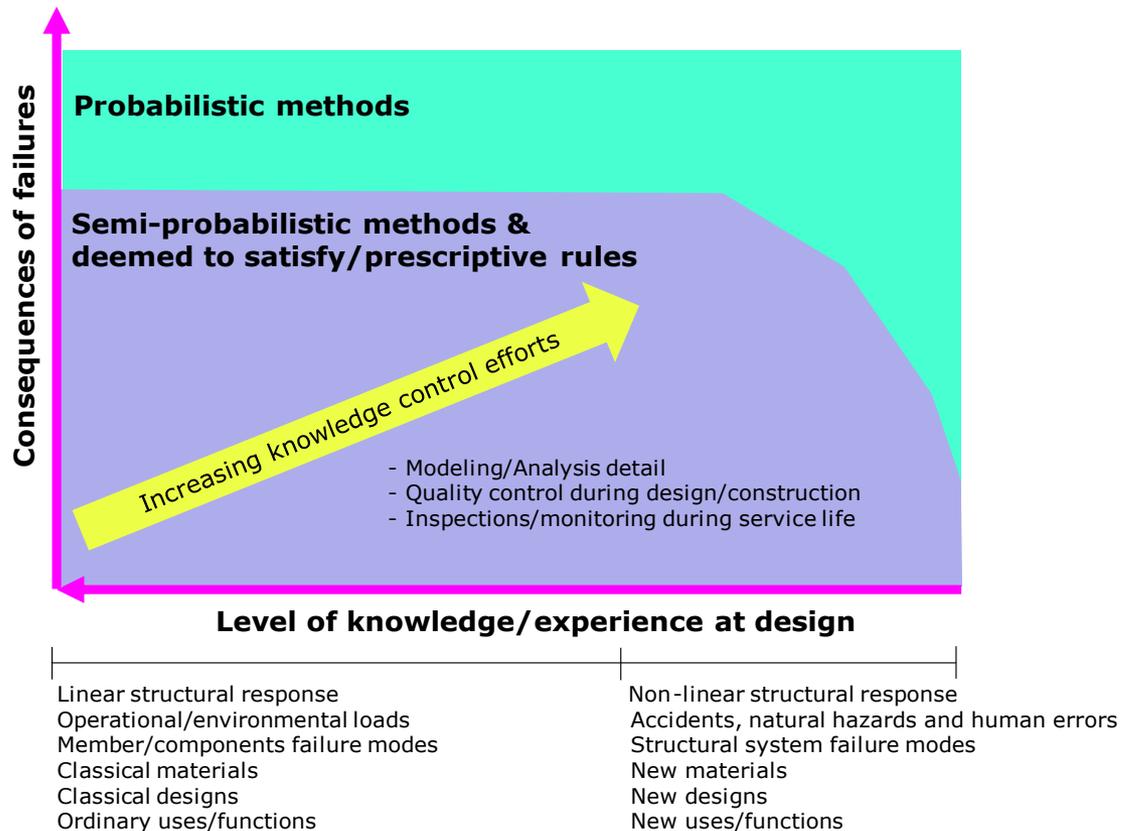
- If the collected information is not correct or biased the actions will not be optimal and may even cause basis for adaptive actions which increase the service life costs
- When assessing the benefit or value of different monitoring schemes and corresponding optimal strategies for adaptive actions the only basis for the modeling of the not yet collected information is the a-priori available data and models concerning the variables of interest.

The benefit of health monitoring cannot be assessed through one or a few anticipated monitoring results



Structural Health Monitoring

Safety Management in Structural Design Codes



Structural Health Monitoring

Basic principles to be appreciated

- ☹️ A performed information collection **does not** in itself improve the safety of a structure
 - 😊 It **improves** our estimate of the safety
 - ☹️ A planned information collection **does not** in itself improve the safety of a structure
 - ☹️ Planned information collection **does not** improve our estimate of the safety
- ! To ensure the safety of a structure planning of information collection must be performed in conjunction with planning of mitigating actions !



Structural Health Monitoring

Potential benefits of Structural Health Monitoring

SHM may:

- Save human lives
- Reducing CO₂ emissions
- Increasing competitiveness:



Structural Health Monitoring

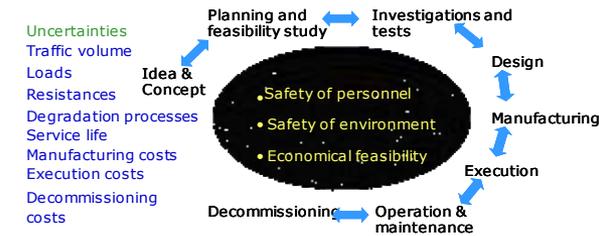
Potential applications of Structural Health Monitoring

- Service life management of structures
- Prototype development
- Code making and code calibration for the design and assessment of structures
- In devising warning measures to allow for loss reduction in situations where structures, or systems involving structures, due to accumulated damage or extreme load events perform unreliably
- For the optimization of maintenance strategies



Structural Health Monitoring

Service life management of structures



Choices during the service life of structures:

- Structural concept (static system, materials,..)
- Site investigations (characteristics, amount/extent)
- Laboratory experiments (characteristics, amount/extent,..)
- Design methods (analysis, codes,..)
- Construction concept (process, phases, interim structures,..)
- Quality control (design, manufacturing, construction,..)
- Assessments (characteristics, techniques, amount/extent,..)
- Maintenance strategy (inspection, repair, quality,..)
- Monitoring strategy (characteristics, techniques, quality,..)
- Decommissioning concept (process, assessments,..)

The choices define the prior knowledge concerning structural performances, i.e. risk, safety and service life costs, but also the options to influence these over time.



Structural Health Monitoring

In devising warning measures to facilitate loss reduction

Monitoring may adequately facilitate that indications of possible adverse performances or damages of structures in operation can be observed, and utilized as trigger for remediate actions.

The information collected from monitoring could relate to changes in stiffness properties monitored e.g. in terms of dynamic and static responses.

The value of monitoring would relate to the possibility of loss reduction by shutting down the function or reducing the loading of the structure, before human lives, environment and structure are lost and/or damaged further.



Structural Health Monitoring

For the optimization of maintenance strategies

Collection of information concerning the performance of a structure may facilitate improved decision basis for optimizing inspection and maintenance activities.

Monitoring may provide information of relevance for improving the understanding of the performance and response of the structure and this improved understanding may in turn be utilized during the life of the structure to adapt inspection and maintenance activities accordingly.



Structural Health Monitoring

Structural Health Monitoring (SHM) is applied at very broad scale

- There is no doubt that SHM provides valuable information and supports decisions
- In practice very little effort has been devoted on the formal and quantitative assessment of the value of SHM
- There is good reason to doubt whether present best practices on SHM are economically efficient or even in some cases relevant

Value of Information Analysis forms the theoretical framework for assessing and optimizing the feasibility of SHM

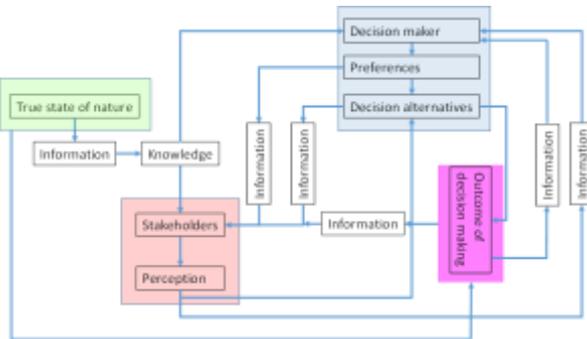
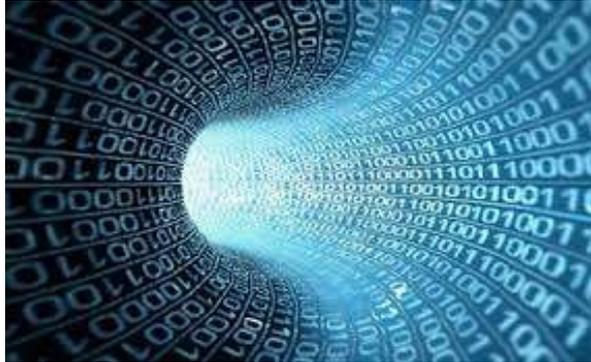


Some Conclusions

- Knowledge and information form the bases for decision making
- Bayesian probability theory is an adequate framework for representing knowledge and knowledge development through collection of new information
- Structural Health Monitoring aims to develop knowledge in support of management of structures
- Value of Information analysis from Bayesian decision analysis facilitates assessing and optimizing the benefit of Structural Health Monitoring
- Information and knowledge modeling are essential parts of Structural Health Monitoring
- Possible competing systems must be accounted for



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Thanks for your attention 😊

Michael Havbro Faber
mfn@civil.aau.dk

&

Dimitry Val
D.Val@hw.ac.uk

