PhD Training School on the Value of Structural Health Monitoring Information
Lake Como, Italy, November 6-9, 2017.

COST Action TU1402: Quantifying the Value of Structural Health Monitoring


# Introduction and fundamentals of Bayesian decision analysis


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# Contents of Presentation

- **Preamble**
- **Literature**
- **Decision analysis in engineering**
- **Example**

# Preamble

"The important thing is **not to stop** questioning.
Curiosity has its own reason for existing."

A. Einstein

# Literature


- **Raiffa, H. & Schlaifer (1961). R. Applied Statistical Decision Theory. Harward University Press, Cambridge University Press, Cambridge, Mass.**
- **Faber, M. H. (2009) Risk and Safety in Engineering, Lecture Notes, ETH, 2009.**
- **Faber, M. H. Statistics and Probability Theory – In pursuit of Engineering Decision Support, Springer, 2012.**
- **Faber, M. H. and S. Thöns (2013). On the Value of Structural Health Monitoring. ESREL 2013. Amsterdam, The Netherlands.**

# Context of Decision Analysis in Engineering

## Risk informed decision support

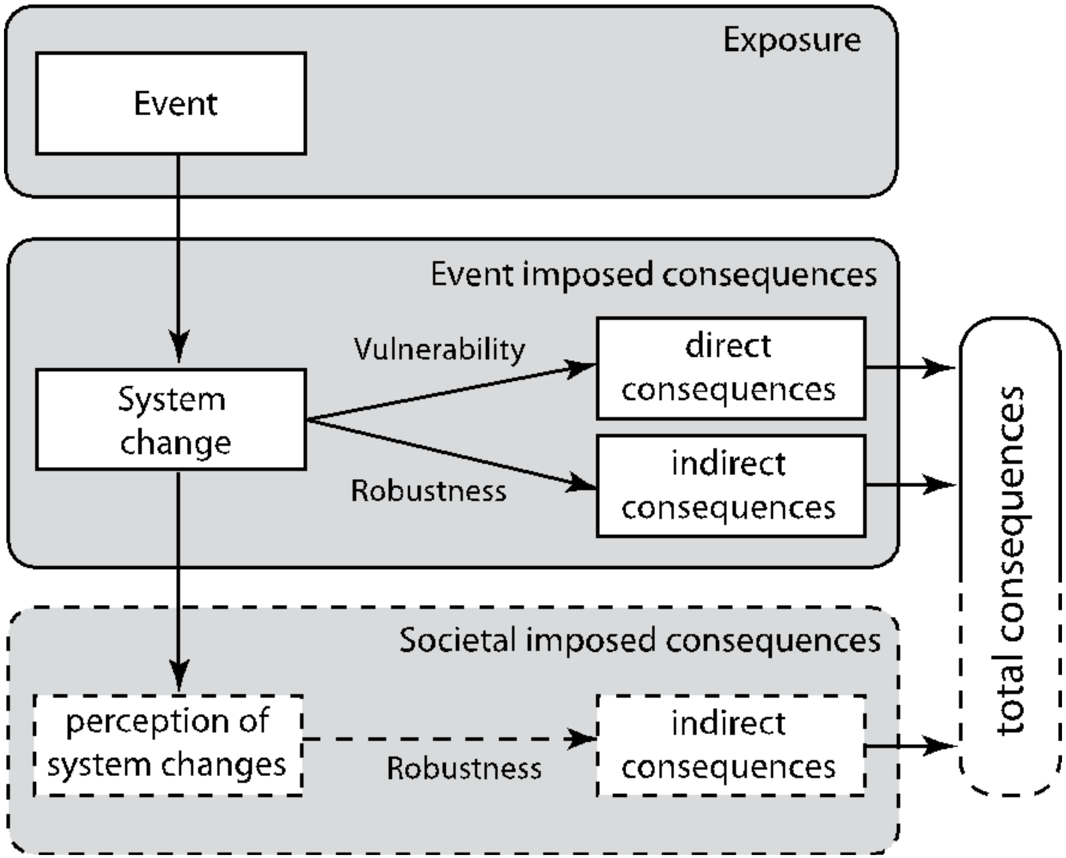


Bayesian probabilistic modeling and decision analysis provides a theoretical and methodical framework for:

- Quantifying knowledge related to structural performances
- Updating knowledge related to structural performances
- Quantifying the value of additional information
- Optimizing and ranking decision alternatives
- Documenting risks and rationale for actions

# Systems Risk Modeling

## How are consequences generated?



# Systems Risk Modeling

## Structural safety and information management

**Models of real world**

**Decisions**

**Real world**

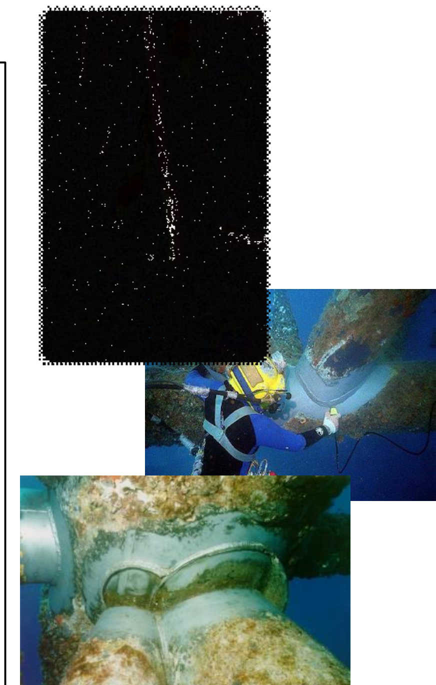
Exposures/loads

Vulnerability / direct con.

Robustness / indirect con.

Site investigations
Load control
Environment control
….
Materials
Component design
….
System concept
Maintenance/monitoring
Evacuation strategy
…

Observations/measurements



Safety, risk and life-cycle costs

# Systems Risk Modeling

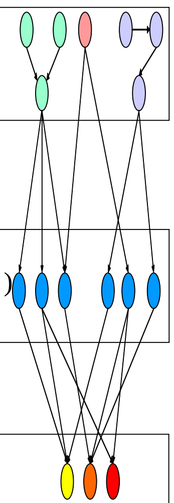
## Structural safety and information management

**Models of real world**

$p(\mathbf{EX})$

$$R_D = \sum_{k=1}^{n_{EXP}} \sum_{l=1}^{n_{CSTA}} p(\mathbf{C}_l | EX_k) c_D(\mathbf{C}_l) p(EX_k)$$

$$R_{ID} = \sum_{k=1}^{n_{EXP}} \sum_{l=1}^{n_{CSTA}} \sum_{m=1}^{n_{SSTA}} c_{ID}(S_m, c_D(\mathbf{C}_l)) \times p(S_m | \mathbf{C}_l, EX_k) p(\mathbf{C}_l | EX_k) p(EX_k)$$



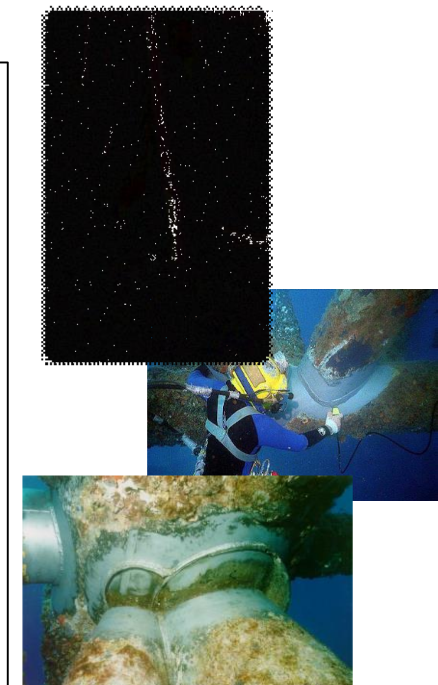
JCSS, Probabilistic Model Code

**Decisions**

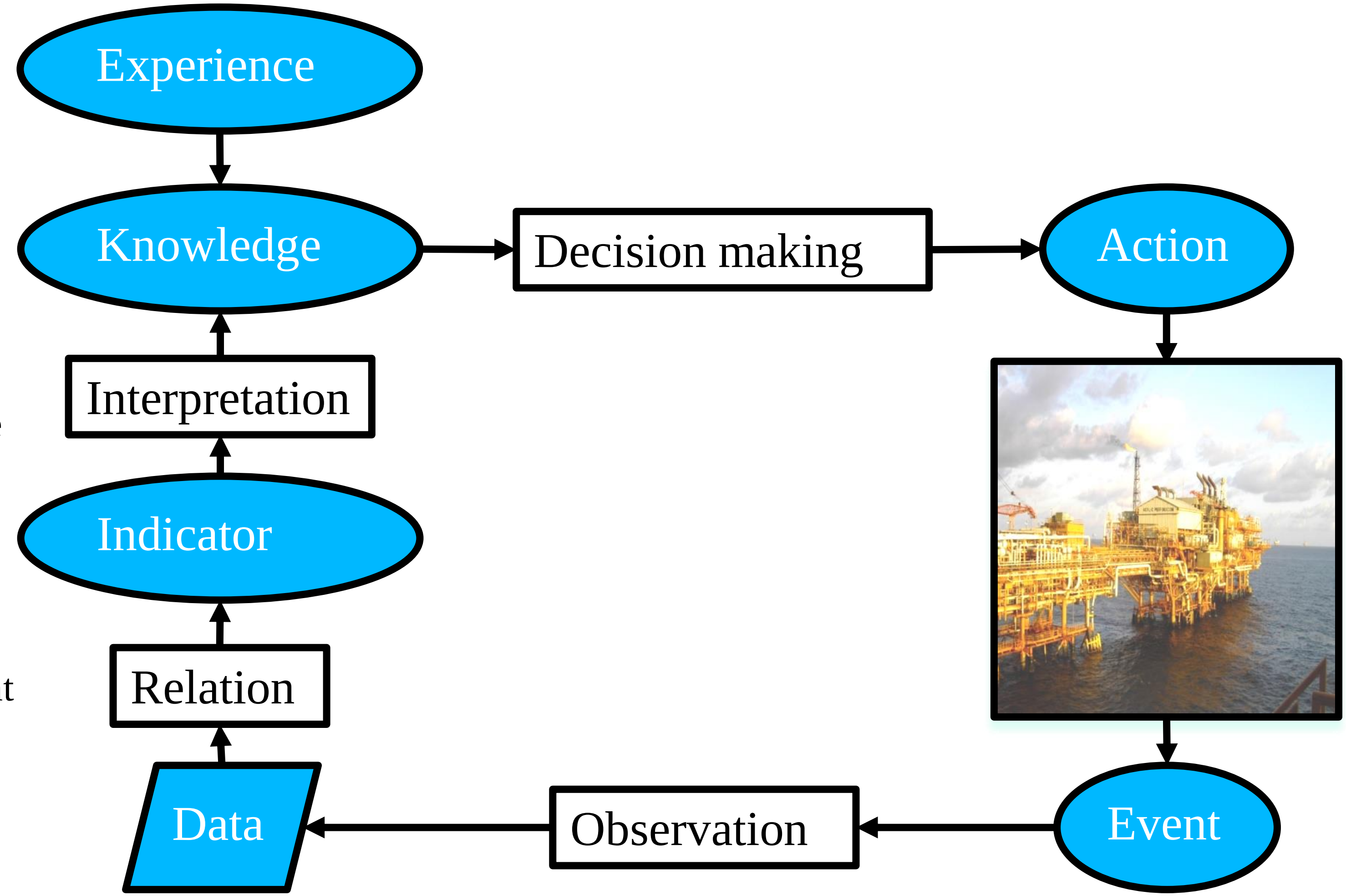
Site investigations
Load control
Environment control
….
Materials
Component design
….
System concept
Maintenance/monitoring
Evacuation strategy
…

Likelihoods

**Real world**



# Systems Risk Modeling

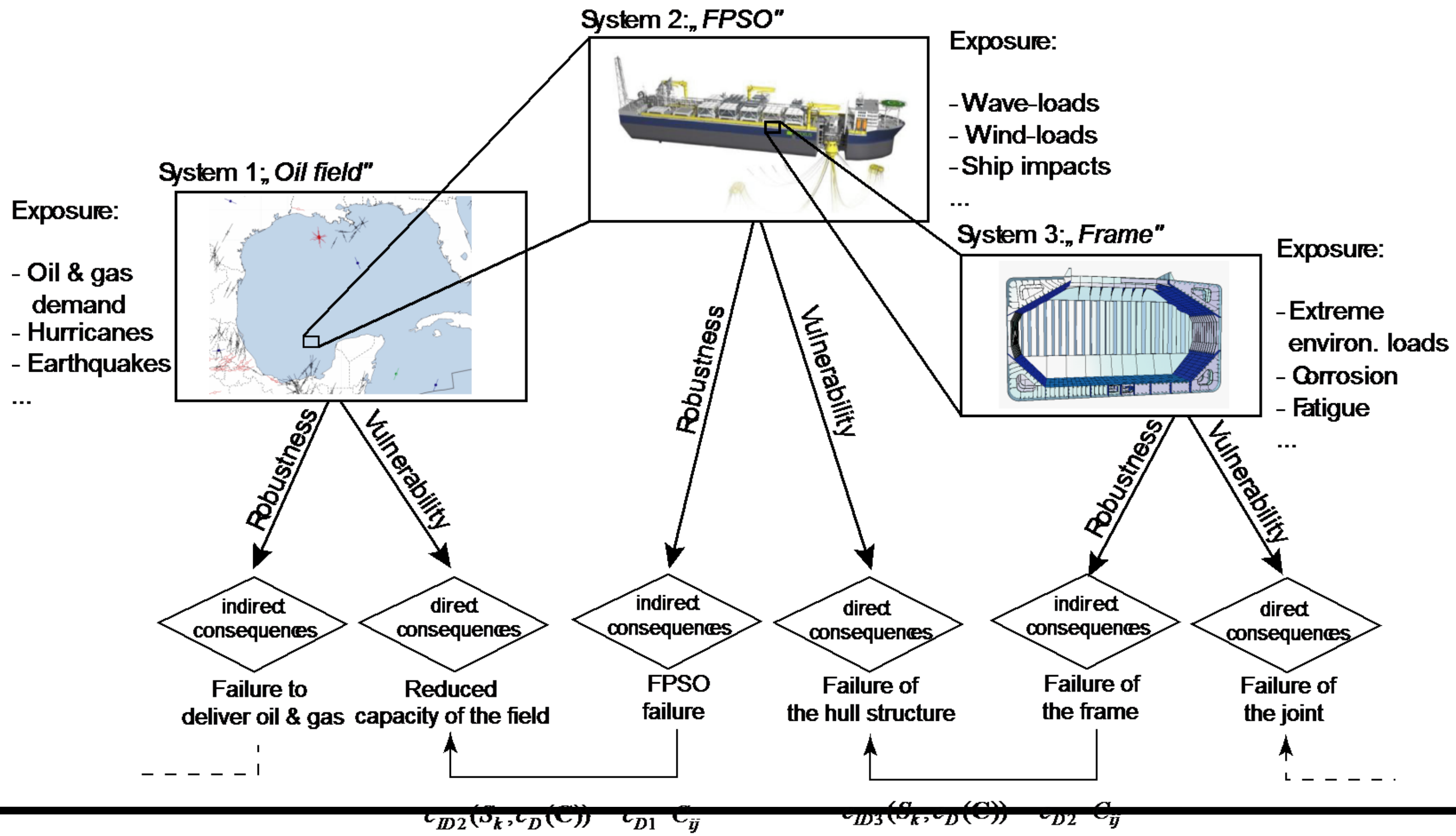



From indicator to performance (likelihood)

From measurement to indicator

# Systems Risk Modeling

## Scale – dependent on considered decision alternatives



# Decision Analysis in Engineering

## Potential of pre-posterior decision analysis not exploited

Rational decision making in structural engineering was recognized as a main objective already in early works by Freudenthal.

Theoretical and methodical developments on Bayesian decision analysis by e.g. Raiffa and Schlaifer were recognized and advocated by e.g. Benjamin and Cornell as a strong framework for providing rational decision support.

Despite these insights and efforts, applied decision analysis has not gained the impact it could have.

Especially the potential of the pre-posterior and Value of Information (VoI) analysis has not been realized/exploited.

# Decision Analysis in Engineering

## Renewed view on decision analysis in structural safety

The aim of this presentation is to take a renewed view on structural safety as a decision making problem and:

- Point at different classes of engineering decision problems which may be addressed by decision analysis.

- Highlight that management of structural safety may be seen as an information management problem.

- Show that the prior, posterior and pre-posterior decision analysis may be applied to quantify the benefit associated with information.

- Illustrate the use of Value of Information analysis

# Decision Analysis in Engineering

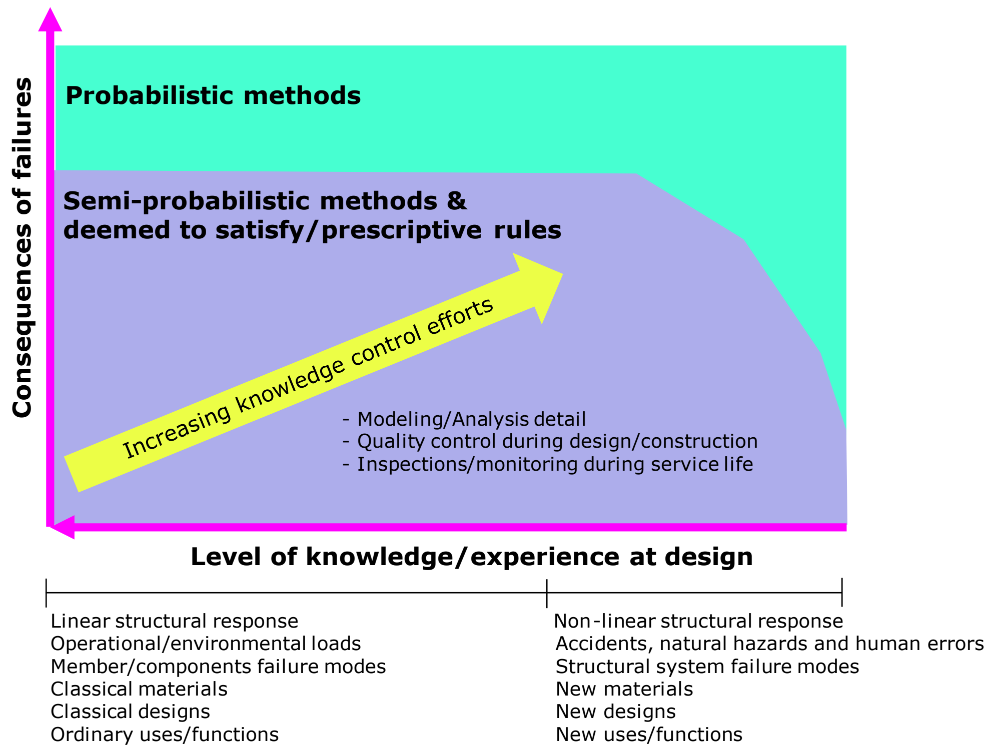
**Choices during the service life of structures:**

-Site investigations (characteristics, amount/extent)
-Laboratory experiments (characteristics, amount/extent,..)
-Design methods (analysis, codes,..)
-Construction concept (process, phases, interim structures,..)
-Structural concept (static system, dimensions, materials,..)
-Quality control (design, manufacturing, construction,..)
-Assessments (characteristics, techniques, amount/extent,..)
-Maintenance strategy (inspection, repair, quality,..)
-Monitoring strategy (characteristics, techniques, quality,..)
-Decommissioning concept (process, assessments,..)

The choices define the prior knowledge concerning structural performances, i.e. risk, safety and service life costs, but also the options to influence these over time.

# Decision Analysis in Engineering

## Safety Management in Structural Design Codes



# Decision Analysis in Engineering

## Prototype development

Health monitoring of new structural concepts intended for larger productions, facilitates concept optimization with respect to life-cycle benefit, before the initiation of a series production.

By instrumentation and subsequent monitoring and analysis of monitoring results it is possible to gather knowledge on important (model) uncertainties associated with the response and performance of the prototype.

Such information may be utilized for the purpose of optimizing design decisions which in turn can be related to the service life benefit.

# Decision Analysis in Engineering

## Code making and code calibration for the design and assessment of structures

Systematic and strategically undertaken monitoring of structures may facilitate that design basis for the considered category/type of structure is modified or adapted in accordance with the information collected.
The monitoring could e.g. focus on information concerning the model uncertainties associated with codified design equations, reflecting uncertainty in the relevant load-response transfer functions.
The value of monitoring in this application would be realized through the improved design rationale facilitating that material and costs are minimized and risk, safety and reliability are controlled at adequate acceptable and affordable levels.

# Decision Analysis in Engineering

**In devising warning measures to allow for loss reduction in situations where structures, or systems involving structures, due to accumulated damage or extreme load events perform unreliably**

Monitoring may adequately facilitate that indications of possible adverse performances or damages of structures in operation can be observed, and utilized as trigger for remediate actions.
The information collected from monitoring could relate to changes in stiffness properties monitored e.g. in terms of dynamic and static responses.
The value of monitoring would relate to the possibility of loss reduction by shutting down the function or reducing the loading of the structure, before human lives, environment and structure are lost and/or damaged further.

# Decision Analysis in Engineering

## For the optimization of maintenance strategies

Collection of information concerning the performance of a structure may facilitate improved decision basis for optimizing inspection and maintenance activities.

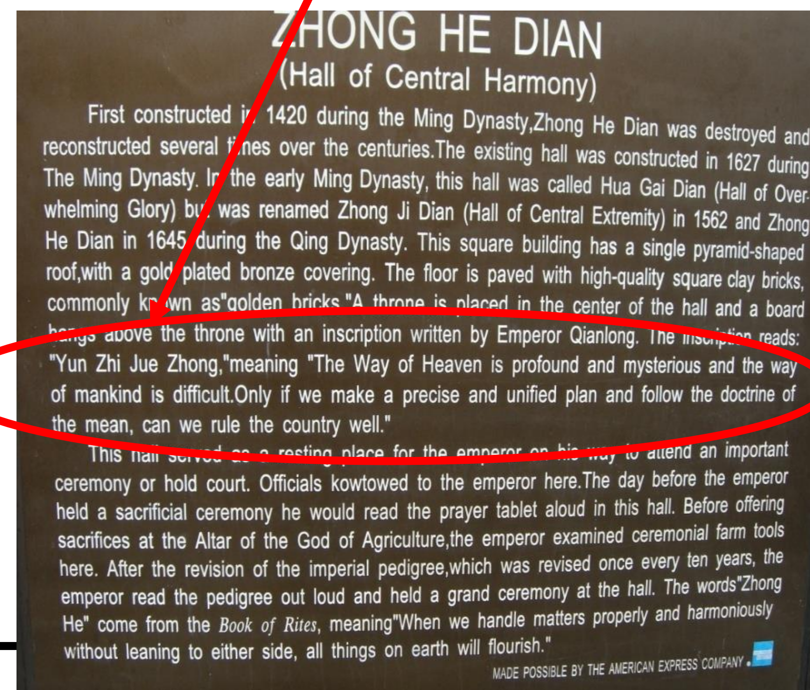
The monitoring may provide information of relevance for improving the understanding of the performance and response of the structure and this improved understanding may in turn be utilized during the life of the structure to adapt inspection and maintenance activities accordingly.

# Decision Analysis in Engineering

Emperor Qianlong
Qing dynasty
Reign :1735 – 1796

Daniel Bernoulli 1738
Expected utility hypothesis

von Neumann and Morgenstern 1947
4 Axioms of utility theory:
Ranking based on expected value
of utility (VNM rational)



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# Decision Analysis in Engineering

## Decisions and decision maker

Society – and decision making for society

For the purpose of decision making it is important to establish

- preferences/values of decision maker
- available resources, e.g. budget limitations
- exogenously given boundary conditions
- rights and responsibilities

# Decision Analysis in Engineering

## Decisions and decision maker

Society – and decision making for society

It is useful to define decision making levels at different scales

- Supranational authority
- National authority and/or regulatory agencies
- Local authority
- Private owner
- Private operator
- Specific stakeholders

# Decision Analysis in Engineering

## Attributes of decision outcomes

Decisions aim to achieve an objective

The degree of achievement is measured by attributes

- natural attributes (measurable, e.g. costs and loss of lives)
- constructed attributes (a function of natural attributes e.g. GDP)
- proxy attributes (indicators which measure the perceived degree of fulfilment of an objective)

# Decision Analysis in Engineering

## Preferences among attributes - utility

The attributes associated with a decision outcome may be translated into a degree of achievement of the objective by means of a utility function

different attributes are brought together on one or several scales

multi attribute decision making implies a weighing of different attributes

# Decision Analysis in Engineering

## Constraints on decision making

In principle – any society may define what they consider to be acceptable decisions

Typically decisions are constrained – e.g. in terms of maximum acceptable risks to

- persons
- qualities of the environment

# Decision Analysis in Engineering

## Principal engineering decisions

Any design decision may be supported by the prior decision analysis

- a choice concerning structural system, materials, dimensions corresponds to a choice of the (prior) probabilistic model of $\mathbf{X}$

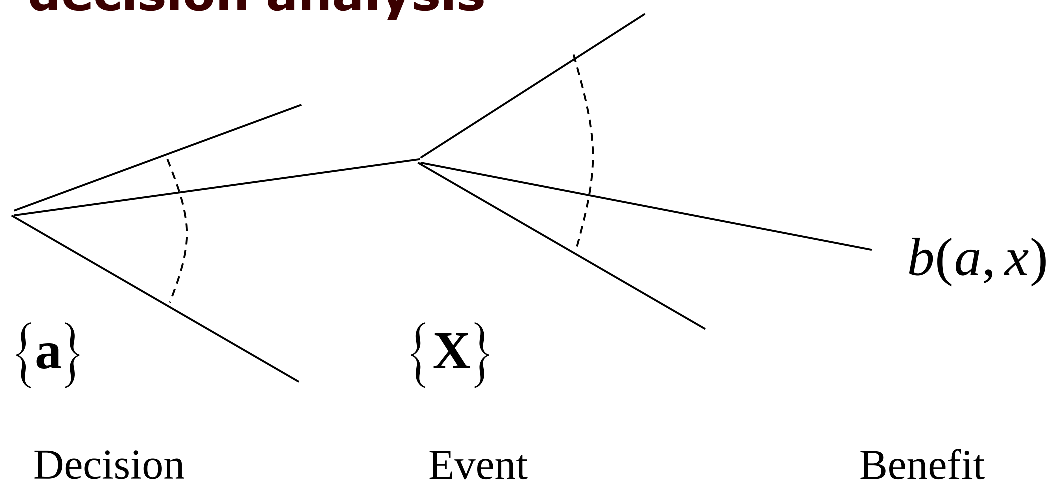
Any decision on assessments, inspections or monitoring may be supported by the pre-posterior decision analysis

- a choice concerning assessment method, inspection method and monitoring scheme will influence the (posterior) probabilistic model of $\mathbf{X}$

# Decision Analysis in Engineering

## Prior decision analysis

Information is bought by choice of prior density

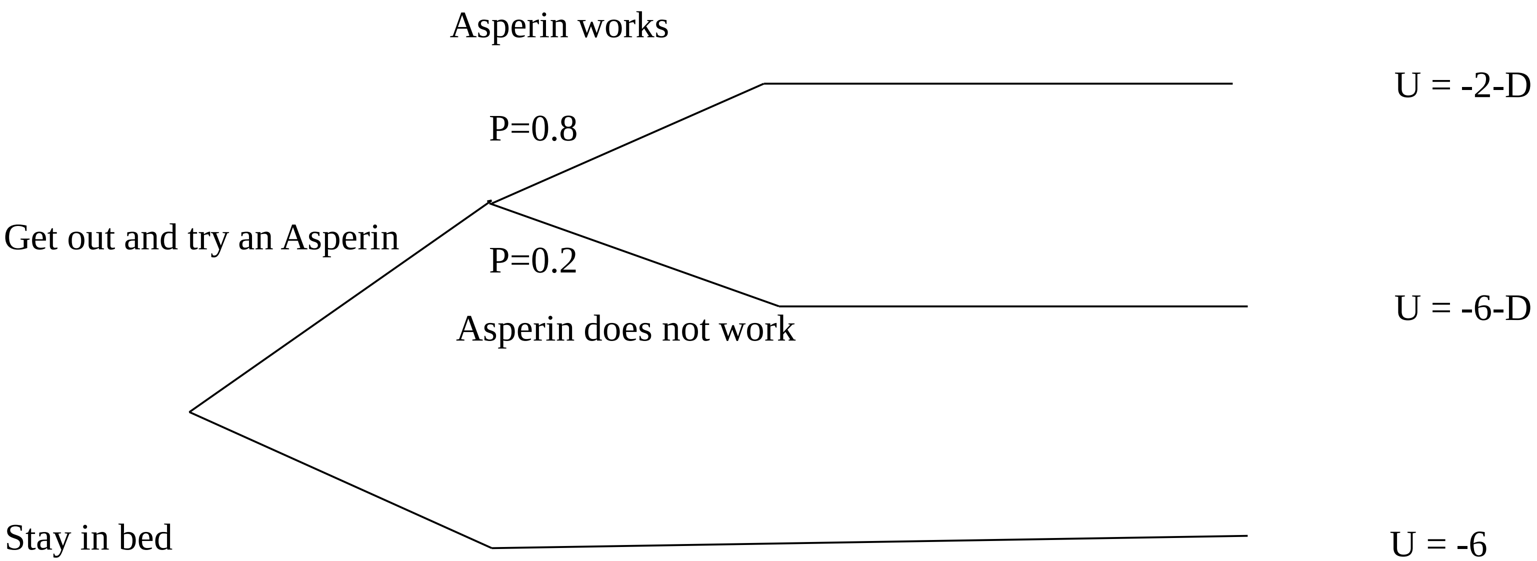



Optimal decision maximizes the expected value of utility (benefit)
(von Neumann & Morgenstern)

$$B_0^* = \max_a E'\left[b(a,X)\right] = \max_a \int b(a,x) f_X'(x,a)dx$$

# Decision Analysis in Engineering

## Hangover dilemma




$$E\left[U(go)\right] = 0.8(-2-D) + 0.2(-6-D) = -2.8 - D$$

$$E\left[U(stay)\right] = -6$$

$$\Rightarrow \quad D^* = 3.2$$

# Decision Analysis in Engineering

## Posterior decision analysis

By sampling information $\mathbf{z}$ using an experiment $e$ we may update the probabilistic description of $X$

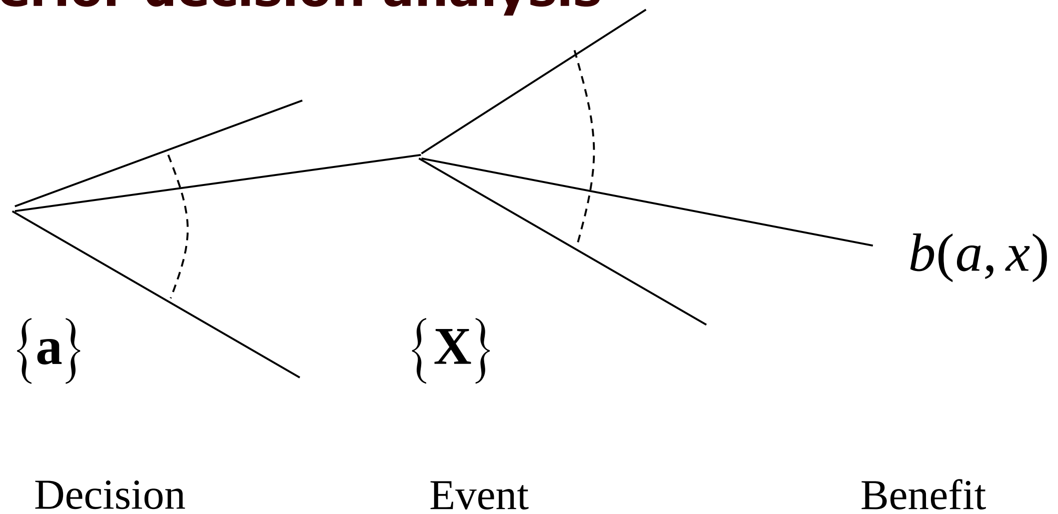
$$f_X''(x,a|\mathbf{z}) = \frac{L(x|\mathbf{z})\, f_X'(x,a)}{\int L(x|\mathbf{z})\, f_X'(x,a)}$$

Of course the likelihood of the sample $\mathbf{z}$ depends on the experiment $e$ why we write

$$L(x|\mathbf{z}) = L(x|\mathbf{z}, e)$$

# Decision Analysis in Engineering

## Posterior decision analysis

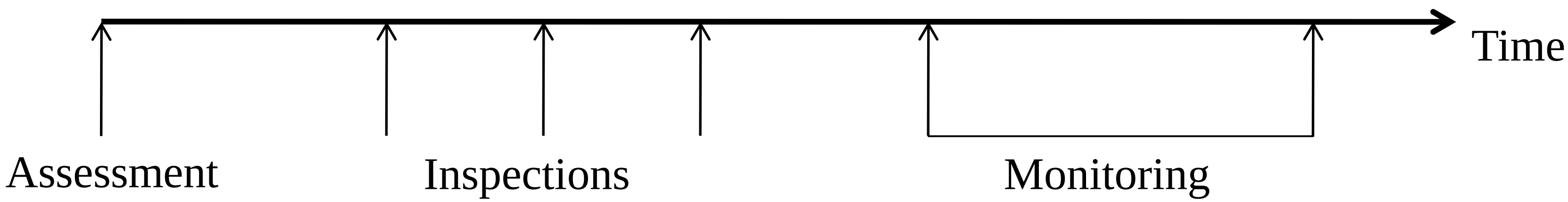



$$\max_a E''\left[b(a,X)\right] = \max_a \int b(a,x)\, f''_X(x,a|\hat{\mathbf{z}})dx$$

# Decision Analysis in Engineering

## Principal engineering decisions

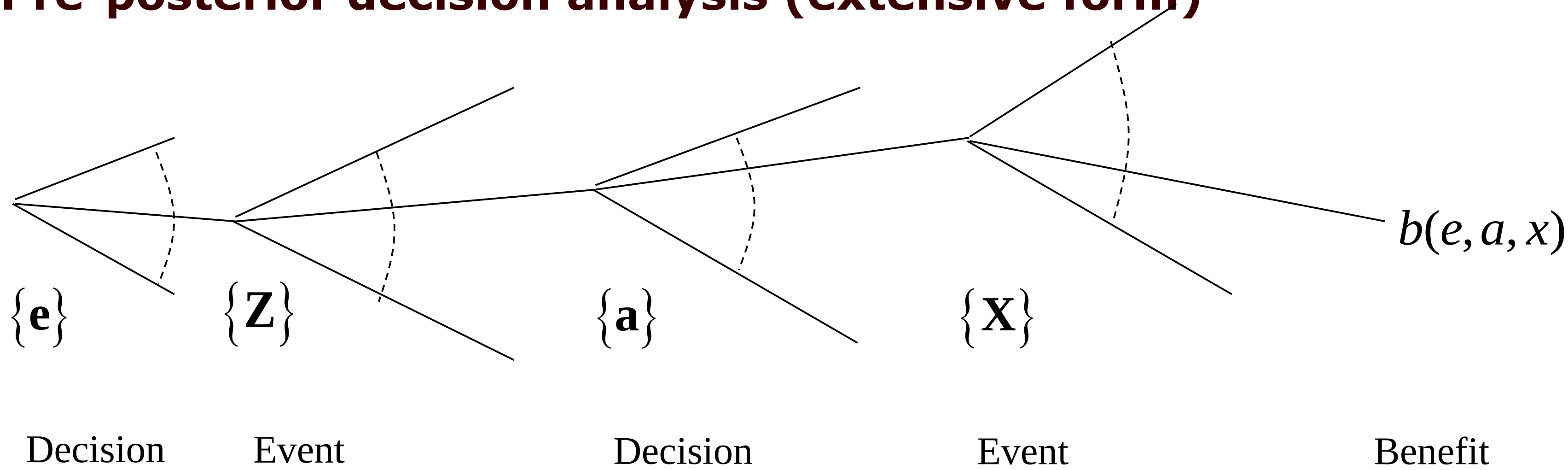
In the decision analysis structure there is no principal difference between assessment, inspection and monitoring activities




The only difference concerns the number of times at which information is collected and utilized for updating the prior probabilistic model

# Decision Analysis in Engineering

## Pre-posterior decision analysis (extensive form)

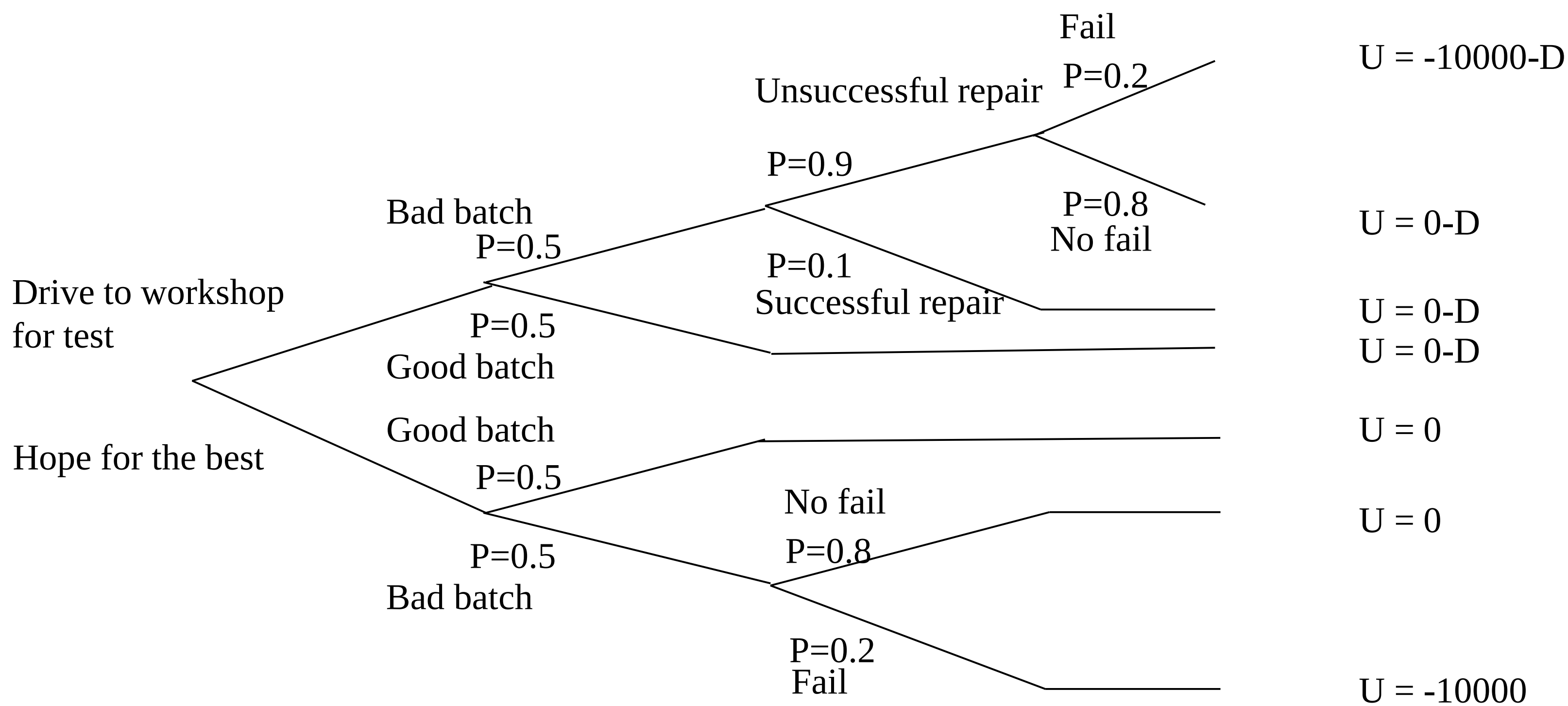



The optimal experiment $e$ may be found from

$$B_1^* = \max_e E_{\mathbf{Z}}\left[\max_a \int b(e,a,x)\, f_X''(x,a\,|\,\mathbf{Z})dx\right]$$

# Decision Analysis in Engineering

## Possibly defect car dilemma



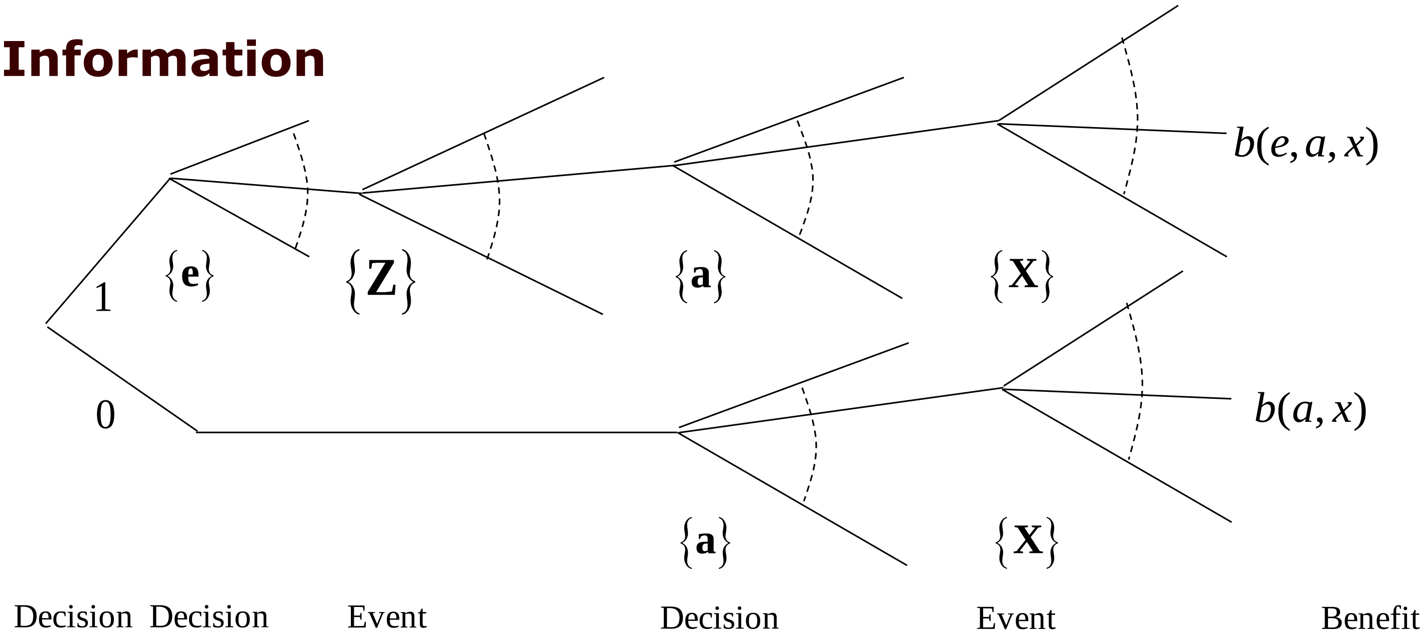

$$E\left[U(test)\right] = -0.5\times D - 0.5\times 0.1\times D - 0.5\times 0.9\times 0.8\times D - 0.5\times 0.9\times 0.2\times (10000 + D) = -D - 900$$

$$E\left[U(hope)\right] = -0.5\times 0.2\times 10000 = -1000$$

$$\Rightarrow \quad D^* = 100$$

# Decision Analysis in Engineering

## Value of Information




The value of information *VoI* is determined from:

$$VoI = \max_{e} E_{\mathbf{Z}}\left[\max_{a}\int b(e,a,x)\,f_X''(x,a\,|\,\mathbf{Z})dx\right] - \max_{a}\int b(a,x)\,f_X'(x,a)dx$$

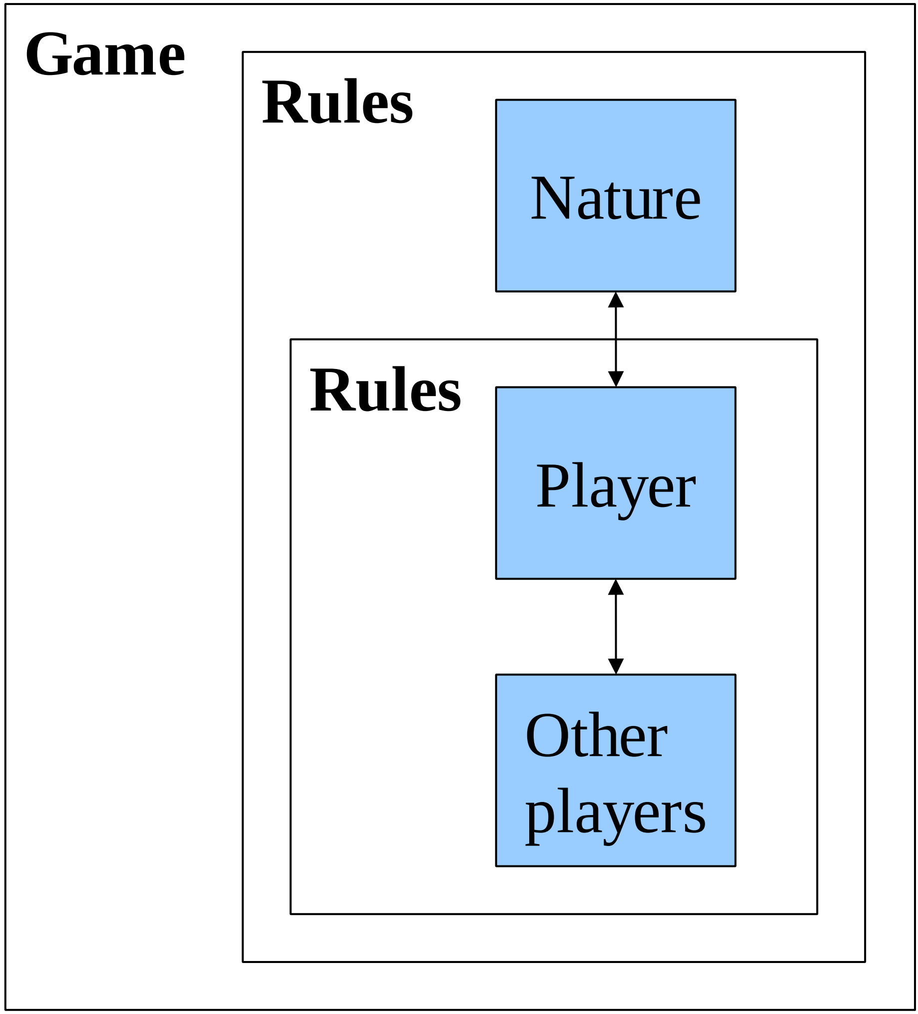
# Decision Analysis in Engineering

**Games and Risk**

**Rules (exogenous)**
- Nature

**Rules (endogenous)**
- Knowledge
- Best practices
- Regulations and standards
- Culture
- Ethics

**Drivers/Challenges**
- Preferences
- Psychology
- Asymmetric information

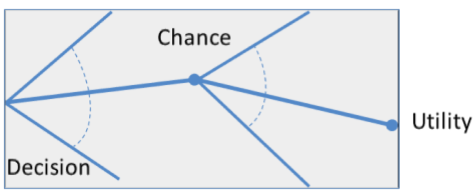


# Decision Analysis in Engineering

## Bayesian decision analysis

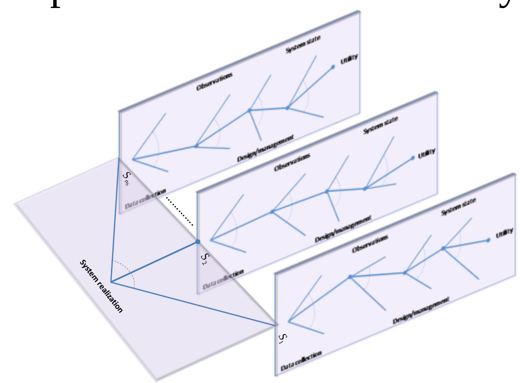
Consistent "book keeping" of the expected value of the utility associated with different decision alternatives –(Raiffa and Schlaifer (1961), von Neumann and Morgenstern (1947))

Prior decision analysis




$$\mathbf{a}^* = \underset{\mathbf{a}}{\arg\max}(E'_{\mathbf{X}}\left[U_{Total}(\mathbf{a},\mathbf{X})\right])$$

Pre-posteror/VoI decision analysis




$$\mathbf{a}^{**} = \arg\max E'_{\mathbf{X}|\hat{\mathbf{Z}}=\hat{\mathbf{z}},\mathbf{M}_S=\mathbf{m}_{S_i}}\left[U_{Total}(\mathbf{a}_i,\mathbf{X}_i)\right]$$



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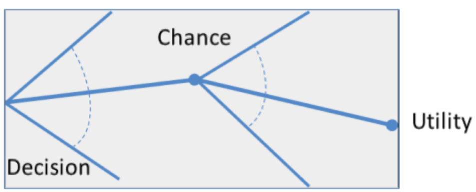
# Decision Analysis in Engineering

## Bayesian decision analysis

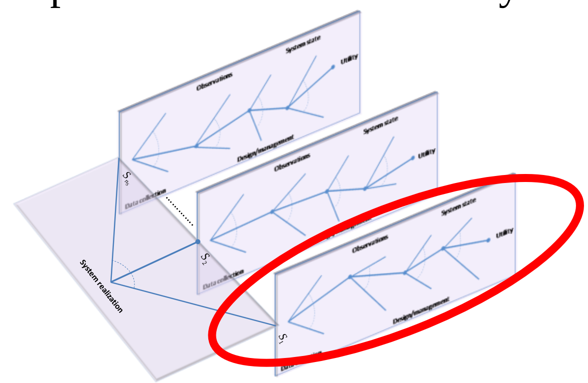
Consistent "book keeping" of the expected value of the utility associated with different decision alternatives –(Raiffa and Schlaifer (1961), von Neumann and Morgenstern (1947))

Prior decision analysis




$$\mathbf{a}^* = \underset{\mathbf{a}}{\arg\max}(E'_{\mathbf{X}}\left[U_{Total}(\mathbf{a},\mathbf{X})\right])$$

Pre-posteror/VoI decision analysis



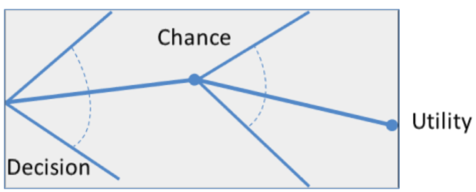

$$\mathbf{a}^{**} = \arg\max E'_{\mathbf{X}|\hat{\mathbf{Z}}=\hat{\mathbf{z}},\mathbf{M}_S=\mathbf{m}_{S_i}}\left[U_{Total}(\mathbf{a}_i,\mathbf{X}_i)\right]$$

# Decision Analysis in Engineering

**Bayesian decision analysis**

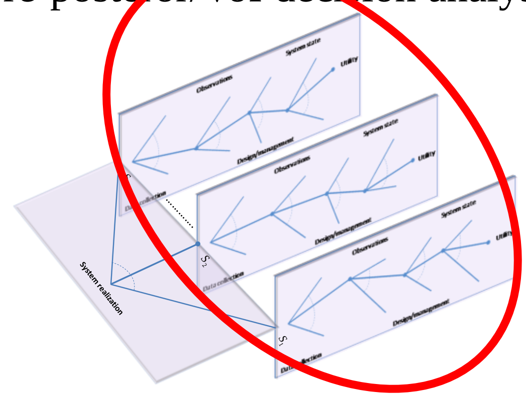
Consistent "book keeping" of the expected value of the utility associated with different decision alternatives –(Raiffa and Schlaifer (1961), von Neumann and Morgenstern (1947))

Prior decision analysis




$$\mathbf{a}^* = \underset{\mathbf{a}}{\arg\max}(E'_{\mathbf{X}}\left[U_{Total}(\mathbf{a},\mathbf{X})\right])$$

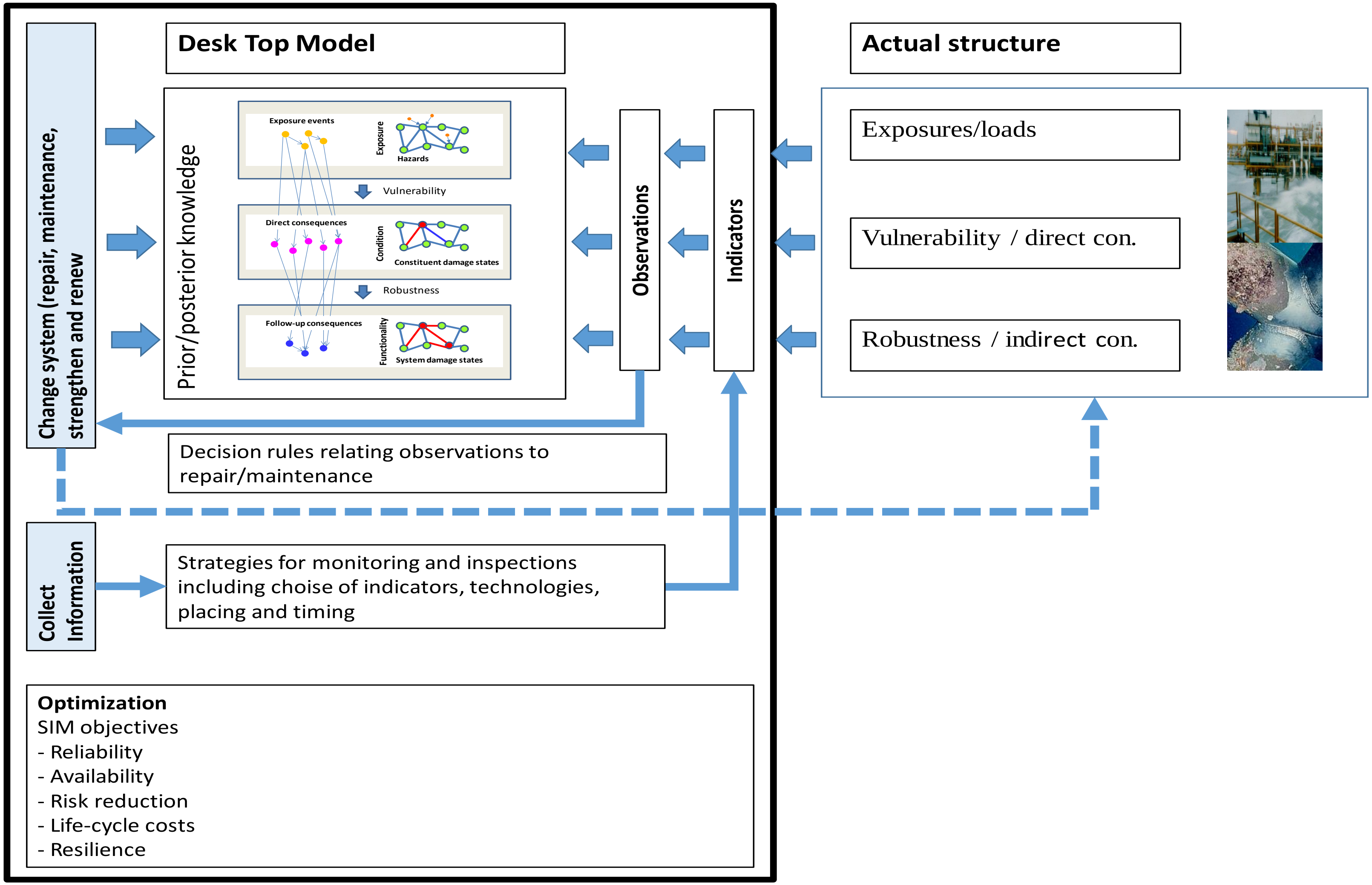
Pre-posterior/VoI decision analysis




Big world

$$\mathbf{a}^{**} = \underset{\mathbf{a}}{\arg\max}\, E'_{\mathbf{X}|\hat{\mathbf{Z}}=\hat{\mathbf{z}},\mathbf{M}_S=\mathbf{m}_{S_i}}\left[U_{Total}(\mathbf{a}_i,\mathbf{X}_i)\right]$$

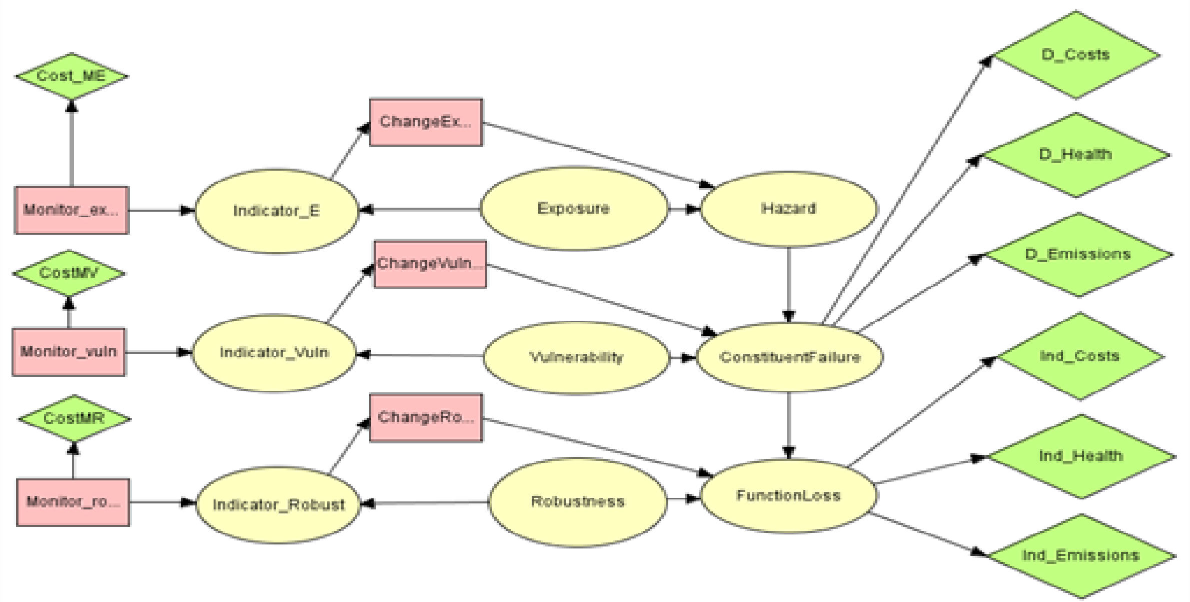
# Decision Analysis for SIM



# Decision Analysis for SIM

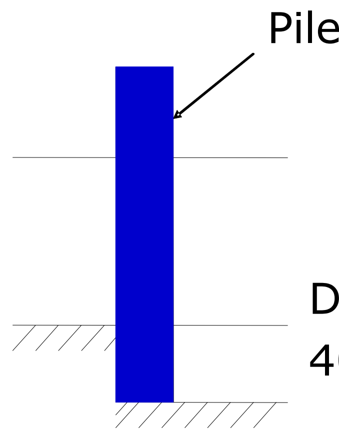
## Generic decision analysis model for VoI in SHM

COST Action 1402: "Quantifying the Value of Structural Health Monitoring"



# Example - Decision Analysis in Engineering

The decision tree




Depth of rock bed
40ft or 50ft ?

| Action alternatives | Outcome | Consequence | Utility(consequence) |
|---|---|---|---|
| 40 ft Pile | depth = 40 ft | none | 0 |
| | depth = 50 ft | splice | 400 |
| 50 ft Pile | depth = 40 ft | cutting | 100 |
| | depth = 50 ft | none | 0 |

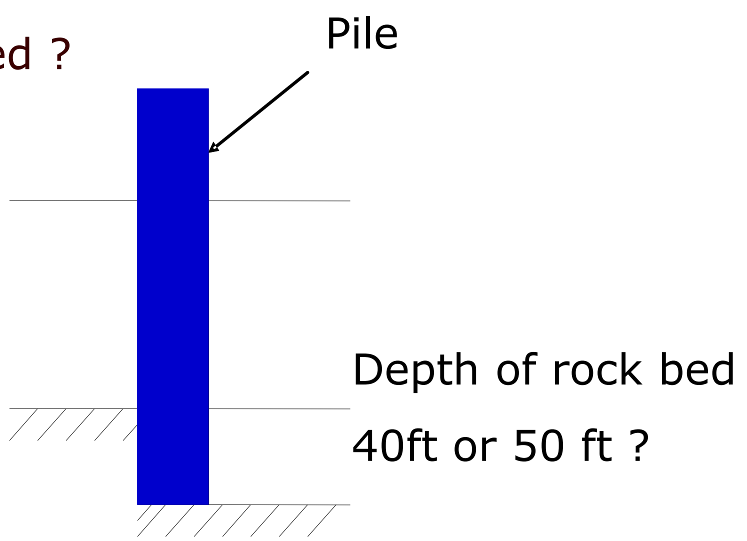
# Example - Decision Analysis in Engineering

The different types of decision analysis

- Prior
- Posterior
- Pre-posterior

Illustrated on an example :

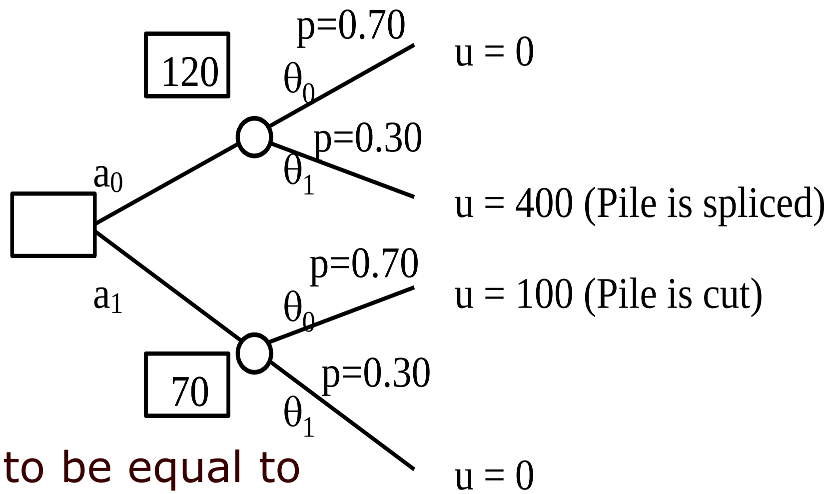
Question : What pile length should be applied ?




Alternatives :
$a_0$ : Choose a 40 ft pile
$a_1$ : Choose a 50 ft pile

States of nature (depth to rock bed)
$\theta_0$ : Rock bed at 40 ft
$\theta_1$ : Rock bed at 50 ft

# Example - Decision Analysis in Engineering



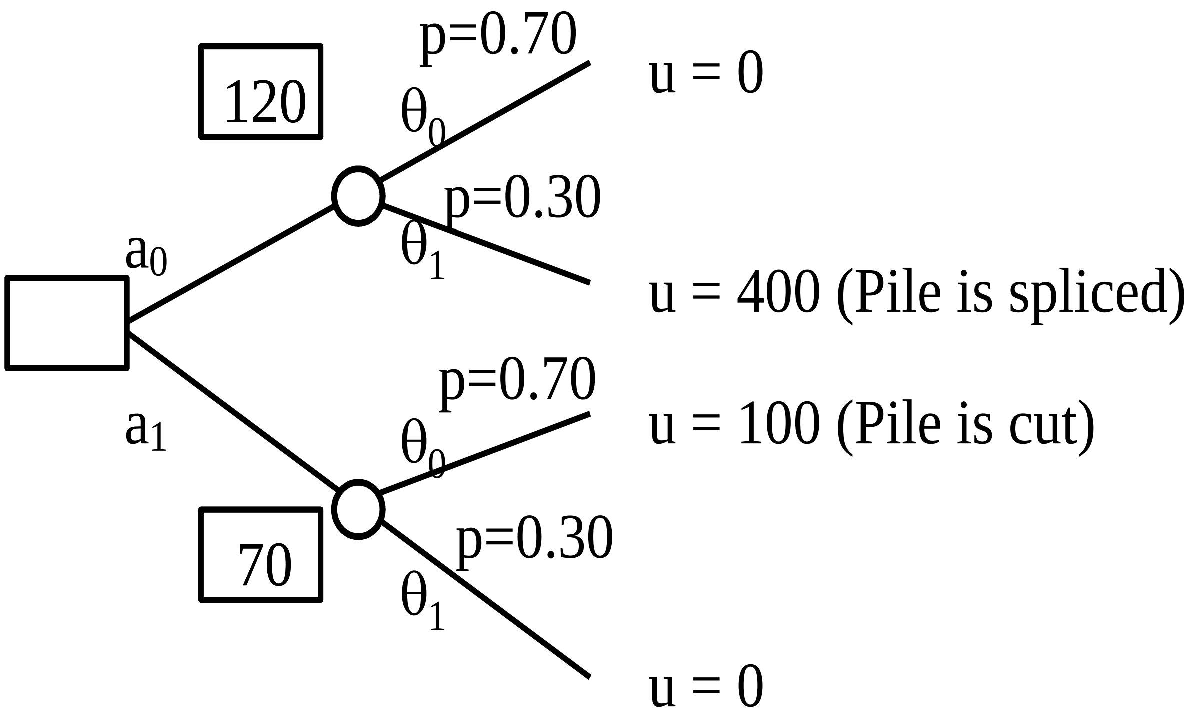

Prior Analysis

$P'[\theta_0] = 0.70$
$P'[\theta_1\ ] = 0.30$

The expected utility is calculated to be equal to

$$
\begin{aligned}
E'[u] &= \min\{u[a_0]\ , u[a_1]\} \\
&= \min\{P'[\theta_0]\times u[\theta_0|a_0] + P'[\theta_1]\times u[\theta_1|a_0]\ , \\
&\qquad P'[\theta_0]\times u[\theta_0|a_1] + P'[\theta_1]\times u[\theta_1|a_1]\} \\
&= \min\{0.7\times 0 + 0.3\times 400,\ 0.7\times 100 + 0.3\times 0\} \\
&= \min\{120, 70\} = 70 \quad \Rightarrow \text{ Decision for } \mathrm{a}_1 \text{ (50ft Pile)}
\end{aligned}
$$

# Example - Decision Analysis in Engineering




$\Rightarrow$ Choice of pile $a_1$ (50ft Pile)

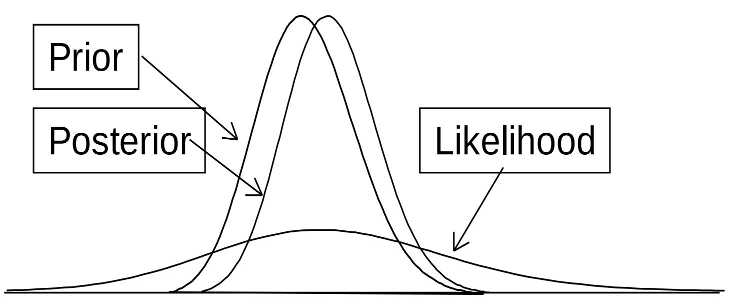
# Example - Decision Analysis in Engineering

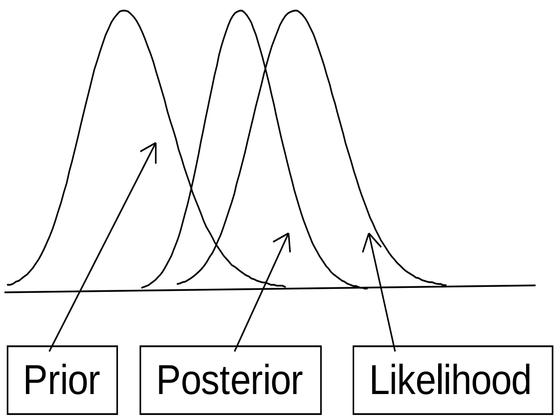
## Posterior Analysis

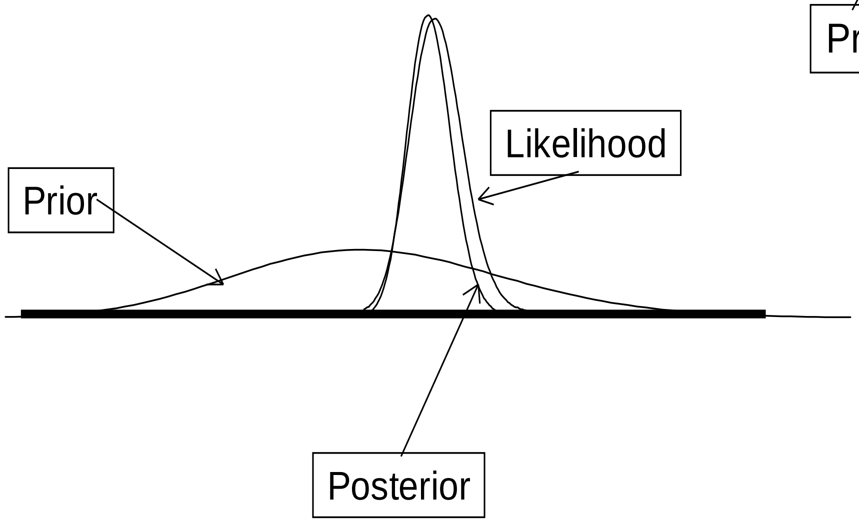
$$P''(\theta_i) = \frac{P\left[z_k \,|\theta_i\right] P'\left[\theta_i\right]}{\sum_j P\left[z_k \,|\theta_j\right] P'\left[\theta_j\right]}$$

# Example - Decision Analysis in Engineering

## Posterior Analysis



# Example - Decision Analysis in Engineering

Posterior Analysis

$$P''(\theta_i) = \frac{P\left[z_k \middle| \theta_i\right] P'\left[\theta_i\right]}{\sum_j P\left[z_k \middle| \theta_j\right] P'\left[\theta_j\right]}$$

Ultrasonic tests to determine the depth to bed rock

| Test result \ True state | $\theta_0$ 40 ft – depth | $\theta_1$ 50 ft – depth |
|---|---|---|
| $z_0$ - 40 ft indicated | 0.6 | 0.1 |
| $z_1$ - 50 ft indicated | 0.1 | 0.7 |
| $z_2$ - 45 ft indicated | 0.3 | 0.2 |

Likelihoods of the different indications/test results given the various possible states of nature – ultrasonic test methods $P\left[z_k \middle| \theta_j\right]$

# Example - Decision Analysis in Engineering

Posterior Analysis

$$P''(\theta_i) = \frac{P\left[z_k \middle| \theta_i\right] P'\left[\theta_i\right]}{\sum_j P\left[z_k \middle| \theta_j\right] P'\left[\theta_j\right]}$$

It is assumed that a test gives a 45 ft indication

$$P''\left[\theta_0\right] = P\left[\theta_0 \middle| z_2\right] \propto P\left[z_2 \middle| \theta_0\right] P\left[\theta_0\right] = 0.3\, x\, 0.7 = 0.21$$

$$P''\left[\theta_1\right] = P\left[\theta_1 \middle| z_2\right] \propto P\left[z_2 \middle| \theta_1\right] P\left[\theta_1\right] = 0.2\, x\, 0.3 = 0.06$$

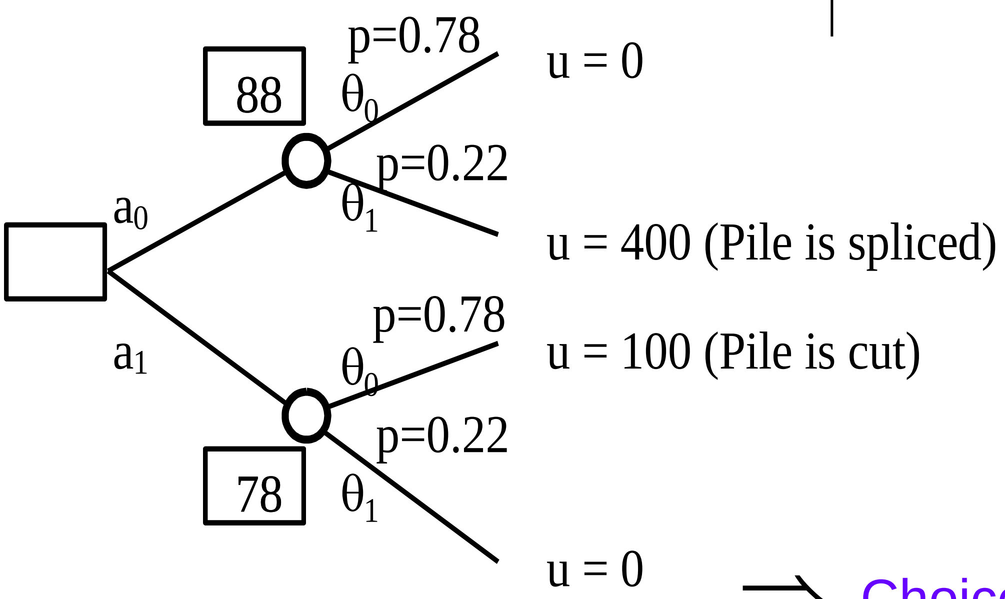
$$P''\left[\theta_0 \middle| z_2\right] = \frac{0.21}{0.21 + 0.06} = 0.78$$

$$P''\left[\theta_1 \middle| z_2\right] = \frac{0.06}{0.21 + 0.06} = 0.22$$

# Example - Decision Analysis in Engineering

## Posterior Analysis

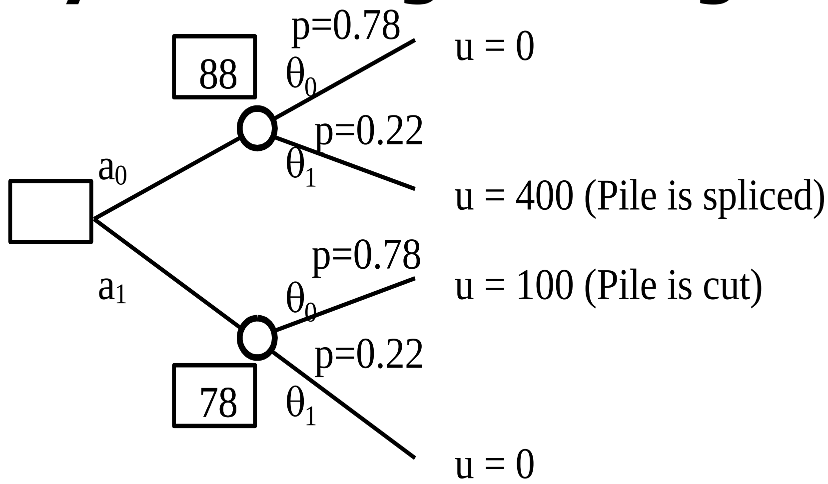
Test result indicates 45ft to rock bed




$\Rightarrow$ Choice of alternative $a_1$ (50ft Pile)

# Example - Decision Analysis in Engineering

## Posterior Analysis




$$E''\left[u\middle|z_2\right] = \min_j\{E''\left[u(a_j)\middle|z_2\right]\}$$

$$= \min\{P''\left[\theta_0\right]\times 0 + P''\left[\theta_1\right]\times 400,\ P''\left[\theta_0\right]\times 100 + P''\left[\theta_1\right]\times 0\}$$

$$= \min\{0.78\times 0 + 0.22\times 400,\ 0.78\times 100 + 0.22\times 0\}$$

$$= \min\{88\ ,\ 78\} = 78$$

$\Longrightarrow$ Choice of alternative $a_1$ (50ft Pile)

# Example - Decision Analysis in Engineering

## Pre-posterior Analysis

$$E[u] = \sum_{i=1}^{n} P'[z_i] \times E''\left[u \middle| z_i\right] = \sum_{i=1}^{n} P'[z_i] \times \min_{j=1,m}\{E''\left[u(a_j) \middle| z_i\right]\}$$

$$P'[z_i] = P\left[z_i \middle| \theta_0\right] \times P'[\theta_0] + P\left[z_i \middle| \theta_1\right] \times P'[\theta_1]$$

$$P'[z_0] = P\left[z_0 \middle| \theta_0\right] \times P'[\theta_0] + P\left[z_0 \middle| \theta_1\right] \times P'[\theta_1] = 0.6 \times 0.7 + 0.1 \times 0.3 = 0.45$$

$$P'[z_1] = P\left[z_1 \middle| \theta_0\right] \times P'[\theta_0] + P\left[z_1 \middle| \theta_1\right] \times P'[\theta_1] = 0.1 \times 0.7 + 0.7 \times 0.3 = 0.28$$

$$P'[z_2] = P\left[z_2 \middle| \theta_0\right] \times P'[\theta_0] + P\left[z_2 \middle| \theta_1\right] \times P'[\theta_1] = 0.3 \times 0.7 + 0.2 \times 0.3 = 0.27$$

# Example - Decision Analysis in Engineering

## Pre-posterior Analysis

$$E''\left[u \middle| z_0\right] = \min_j \{E''\left[u(a_j) \middle| z_0\right]\}$$

$$= \min\{\overbrace{\underbrace{P''\left[\theta_0 \middle| z_0\right]\times 0}_{\textbf{do nothing}} + \underbrace{P''\left[\theta_1 \middle| z_0\right]\times 400}_{\textbf{splicing}}}^{\mathbf{a_0}},\ \overbrace{\underbrace{P''\left[\theta_0 \middle| z_0\right]\times 100}_{\textbf{cutting}} + \underbrace{P''\left[\theta_1 \middle| z_0\right]\times 0}_{\textbf{do nothing}}}^{\mathbf{a_1}}\}$$

$$= \min\{0.93\times 0 + 0.07\times 400,\ 0.93\times 100 + 0.07\times 0\}$$

$$= 0.07\times 400 + 0.93\times 0 = 28$$

# Example - Decision Analysis in Engineering

## Pre-posterior Analysis

$$E''\left[u \middle| z_1\right] = \min_j \{E''\left[u(a_j) \middle| z_1\right]\}$$

$$\overbrace{\text{do nothing} \qquad \text{splicing}}^{\mathbf{a_0}} \qquad \overbrace{\text{cutting} \qquad \text{do nothing}}^{\mathbf{a_1}}$$

$$= \min\{P''\left[\theta_0 \middle| z_1\right] \times 0 + P''\left[\theta_1 \middle| z_1\right] \times 400,\ P''\left[\theta_0 \middle| z_1\right] \times 100 + P''\left[\theta_1 \middle| z_1\right] \times 0\}$$

$$= \min\{0.25 \times 0 + 0.75 \times 400,\ 0.25 \times 100 + 0.75 \times 0\}$$

$$= 0.25 \times 100 + 0.75 \times 0 = 25$$

# Example - Decision Analysis in Engineering

## Pre-posterior Analysis

The minimum expected costs based on pre-posterior decision analysis – not including costs of experiments

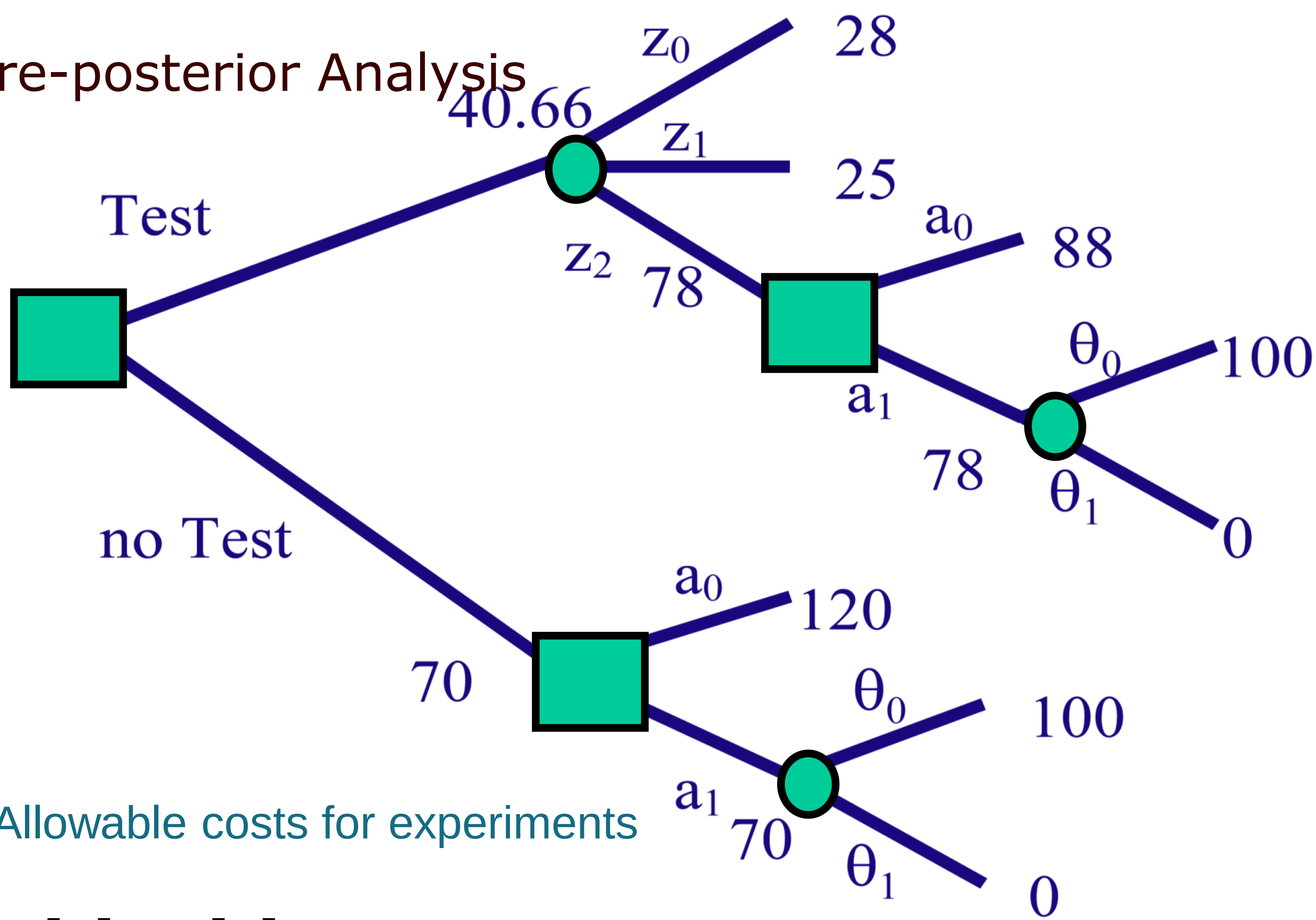
$$E[u] = \sum_{i=1}^{n} P'[z_i] \times E''[u|z_i] = 28 \times 0.45 + 25 \times 0.28 + 78 \times 0.27 = 40.00$$

Allowable costs for the experiment

$$E'[u] - E[u] = 70.00 - 40.00 = 30.00$$

# Example - Decision Analysis in Engineering

Pre-posterior Analysis




Allowable costs for experiments

$$E'[u]-E[u]=70.00-40.00=30.00$$


# Thanks for your attention ☺

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