Factsheet:
Classes of decision analysis

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Outline

1. General overview
    - -Scope of the factsheet
    - -Application areas
    - -Critical appraisal
2. Basis / theory / methods
    - -Decision tree
    - -Decisions based on prior information: prior analysis
    - -Decisions based on additional information: posterior analysis
    - -Decisions based on “unknown information” : pre-posterior analysis
3. Value of information (VoI)

1 General overview

✓ **Scope of the factsheet:**

Three difference classes of decision analysis: Prior analysis, Posterior analysis and **Pre-posterior analysis;**

Value of Information (VoI) is defined.

✓ **Application areas:**

For optimizing the collection of information →a better decision.

- **In civil engineering**: the basis for the computation of the VoI for optimizing inspections and structural health monitoring in deteriorating structures.
- **Also in other field:** transportation infrastructure management, geotechnical engineering, natural hazards, oil exploration, environmental health risk management, etc.

1 General overview

✓ **Critical appraisal:**

Posterior and pre-posterior analysis take into account the uncertainties in the decision making process:

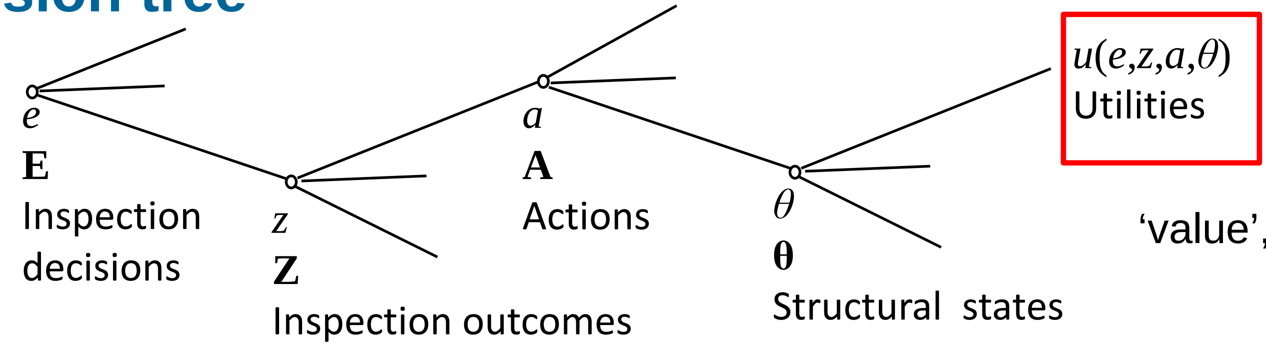
→ well-considered and structured way for making optimal decisions under uncertainty.

It sometimes requires significant computational efforts and statistical modelling:

→can be cumbersome in case one wants to apply this methodology to practical engineering applications.

2 Basis / theory / methods

-Decision tree



'value', 'use' or 'utility'

Notion from Bayesian decision theory (Raiffa and Schlaifer,1961)

E: the set of possible inspection actions (e.g., inspection date, type of inspection, location, etc.).

Z: the set inspection outcomes, which provide information on the actual structural state;

A: the set of possible maintenance actions (e.g., do nothing, repair, replace, etc.);

θ: the set of structural states, representing different levels of structural damage, which are usually time-dependent;

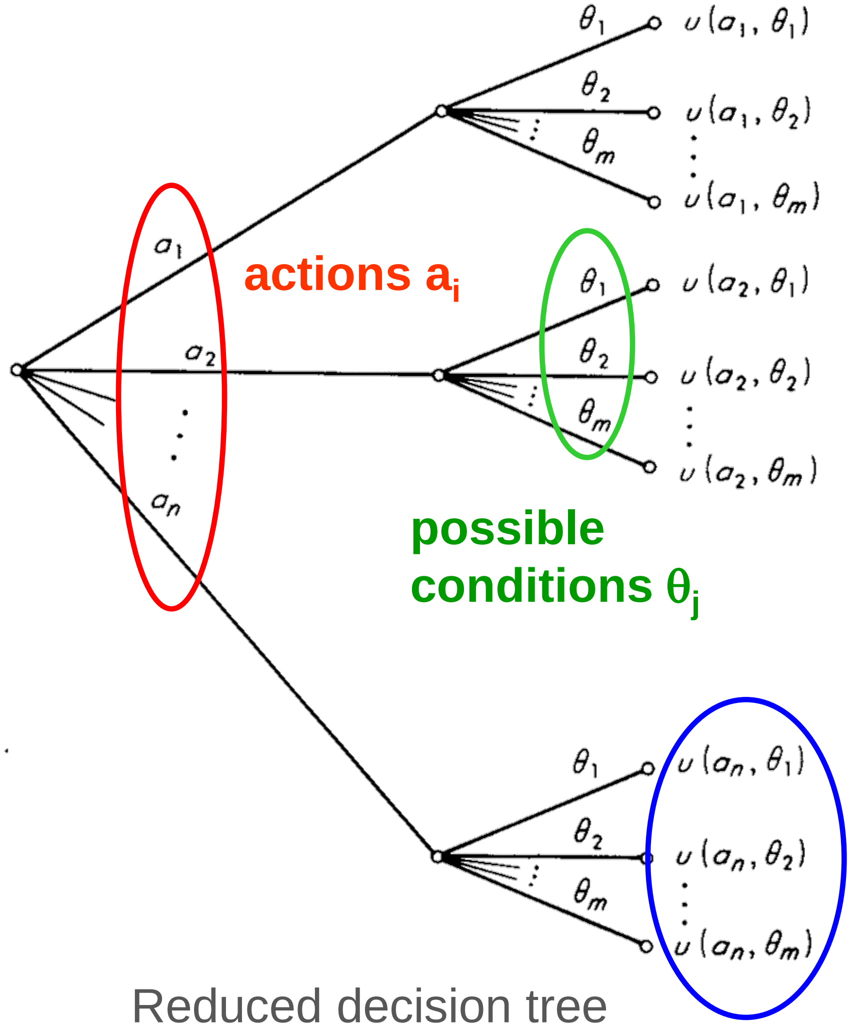
- Decisions based on prior information: prior analysis

Prior analysis: a situation when decision is to be made based on previously available, often generic, information.

Probabilities are assigned to different conditions θ_j

→ Prior probabilities $P'[\theta_j]$

After setting utilities of possible action-state combinations, $u(a_i, \theta_j)$, the expected utilities corresponding to the different actions can be calculated.



actions a_i

possible conditions θ_j

associated utilities $u(a_i,\theta_j)$

Reduced decision tree

The expected utility of action a_i is given by:

$$E'_{\theta}\left[u(a_i,\theta)\right] = \sum_{j=1}^{m} u(a_i,\theta_j)P'\left[\theta_j\right]$$

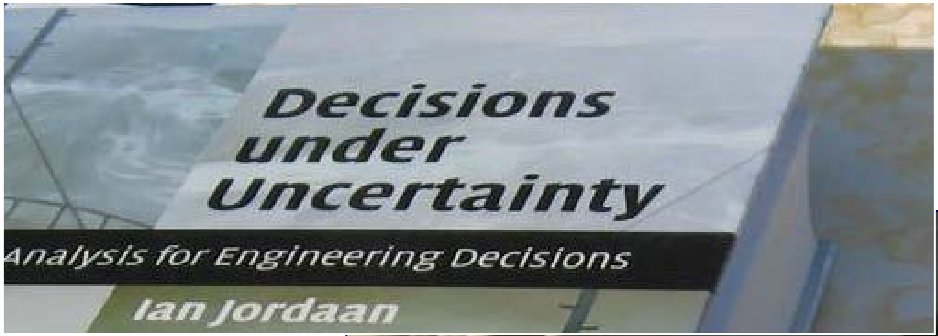
E'_{θ} denotes the expectation operation with respect to prior probabilities P’[θ]

Consequently, the decision analysis consists of choosing the action, a*, which results in the largest expected utility, u*, i.e.,

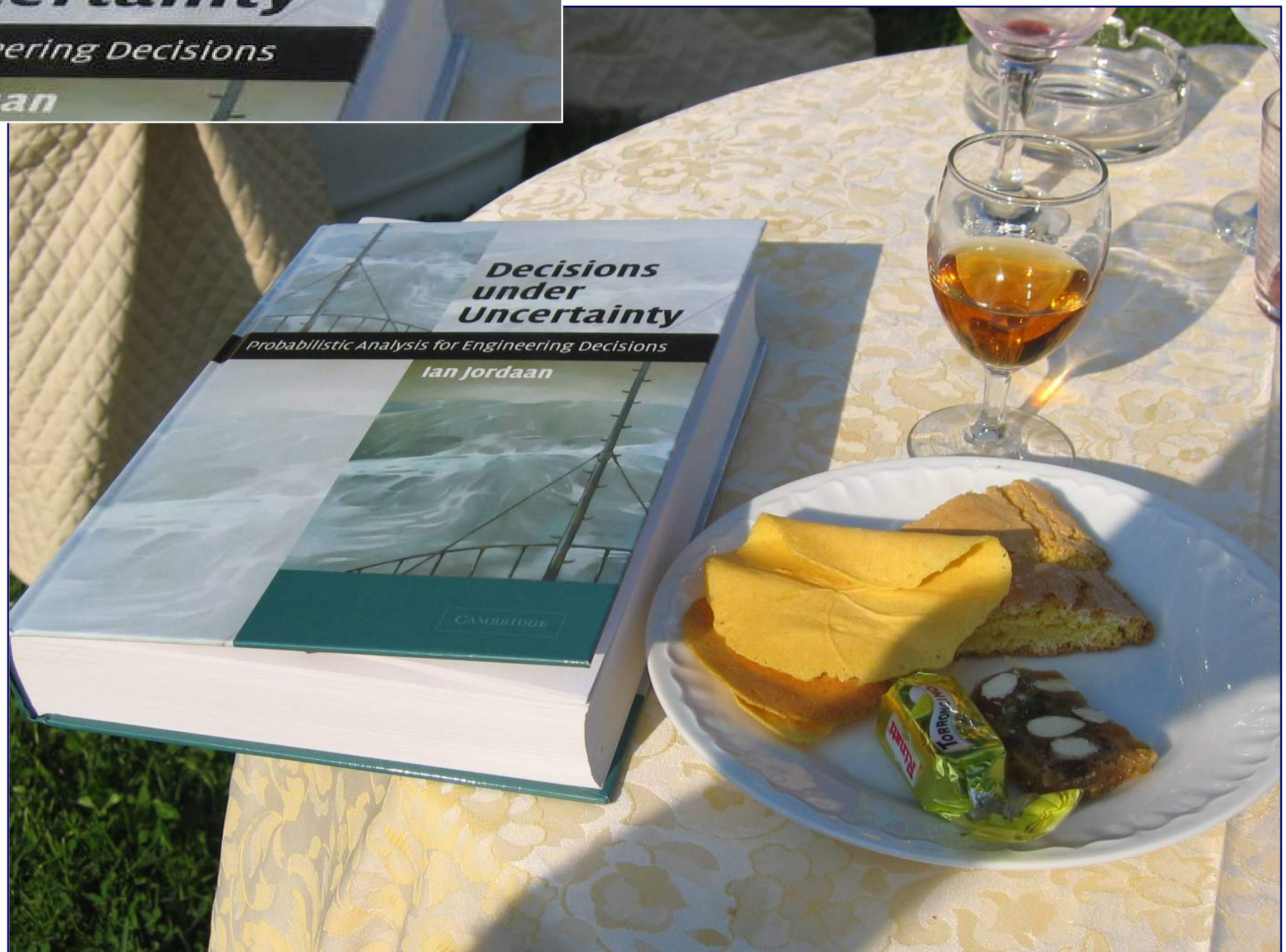
$$u^* = \max_a E'_{\theta}\left[u(a,\theta)\right]$$

Or

$$a^* = \arg\max_a E'_{\theta}\left[u(a,\theta)\right]$$



to decide = to chose between …



Decisions based on additional information: posterior analysis

Additional information → update probabilities

z_k: outcome of an experiment

Prior probabilities: $P'[\theta_i]$ → Posterior probabilities $P''[\theta_i|z_k]$

Bayes' theorem:

$$P''[\theta_i \mid z_k] = \frac{P[z_k|\theta_i]\,P'[\theta_i]}{\sum_{j=1}^{m} P[z_k|\theta_j]\,P'[\theta_j]}$$

$$\begin{pmatrix}\text{posterior}\;\;\text{probability}\\ \text{of } \theta_i \text{ based on}\\ \text{given outcome of}\\ \text{experiment}\end{pmatrix} = \begin{pmatrix}\text{normalizing}\\ \text{constant}\end{pmatrix}\begin{pmatrix}\text{likelihood of}\\ \text{experment}\;\;\text{outcome}\\ \text{based on } \theta_i\end{pmatrix}\begin{pmatrix}\text{prior}\;\;\text{probability}\\ \text{of } \theta_i\end{pmatrix}$$

Incorporation of new and old information through $P[z_k|\theta_i]$ and $P'[\theta_i]$ resp.

After determining posterior probabilities → similar decision making as previous

Decisions based on additional information: posterior analysis

Maximize expected utility

$$u^* = \max_a E''_{\theta|z}\left[u(a,\theta)\right]$$

Find the corresponding optimal terminal action

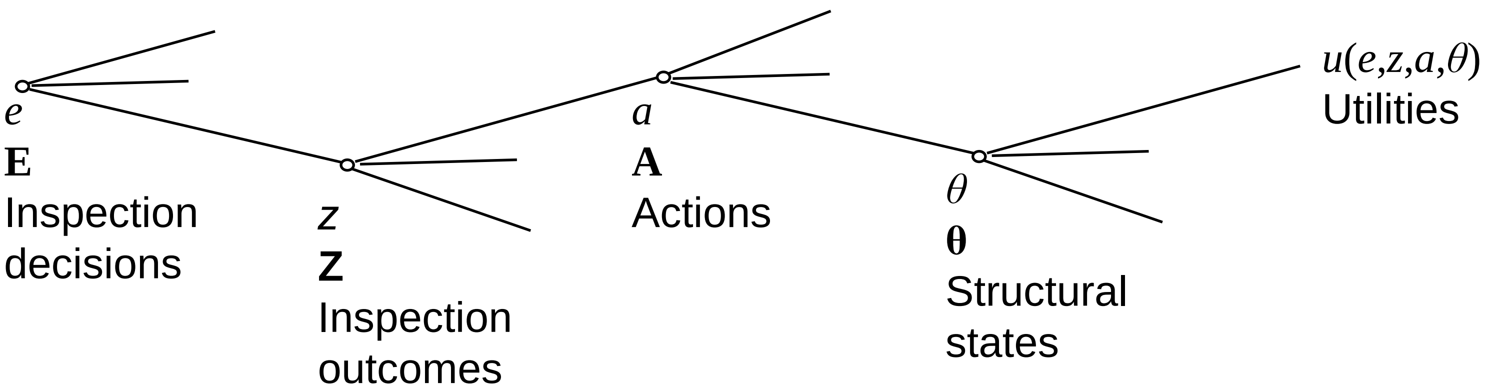
$$a^* = \arg\max_a E''_{\theta|z}\left[u(a,\theta)\right]$$

where the expectation operation with respect to posterior probabilities

$$E''_{\theta|z}\left[u(a_i,\theta)\right] = \sum_{j=1}^{m} u(a_i,\theta_j) P''\left[\theta_j \mid z_k\right]$$

Decisions based on “unknown” information: pre-posterior analysis

Decision tree



Objective: to select optimal strategy (e,a) to maximize the expected utility

Two forms of analysis: extensive form and normal form

Decisions based on "unknown" information: pre-posterior analysis

Extensive form

Analysis is carried out 'backward' – from the right hand end of the decision tree to its starting point

- Determine maximum expected utilities for possible (e,z) - similar to posterior analysis

$$u^*(e,z) = \max_a E''_{\theta|z}\left[u(e,z,a,\theta)\right]$$

- Determine expected utilities for all possible experiments e

  Information needed: probabilities of experiment outcomes $P[z|e]$

$$E_{z|e}\left[u^*(e_i,z)\right] = \sum_{j=1}^{l} u^*(e_i,z_j)P\left[z_j \mid e_i\right]$$

- Find the maximum expected utility and the optimal experiment

$$u^* = \max_e E_{z|e}\left[\max_a E''_{\theta|z}\left[u(e,z,a,\theta)\right]\right]$$

$$e^* = \arg\max_e E_{z|e}\left[\arg\max_a E''_{\theta|z}\left[u(e,z,a,\theta)\right]\right]$$

Decisions based on “unknown” information: pre-posterior analysis

Normal form

Analysis is carried out ‘forward’ – from the starting point of the decision tree to its right end

- Determine decision rule – assign optimal action a to each possible outcome z of experiments $\mathbf{E}$

$$a = d(e,z)$$

- Find expected utilities for all possible combinations (e,d)

  Information needed: $P'[\theta]$ and $P[z|\theta,e]$

$$u(e,d) = E'_{\theta}\left[E_{z|\theta,e}\left[u(e,z,d,\theta)\right]\right]$$

- Find the maximum expected utility and the optimal combination (e,d)

$$u^{*} = \max_{e}\max_{d} E'_{\theta}\left[E_{z|\theta,e}\left[u(e,z,d,\theta)\right]\right]$$

Extensive form and normal form of analysis should lead to the same result

Value of Information (VoI)

For an outcome z the difference between $u^*(e,z)$ and the maximum utility based only on prior information represents the value of the information z

The expected VoI, aslo Expected Value of Sample Information (EVSI)

$$u_0^* = \max_a E'_{\theta}\left[u(a,\theta)\right]$$ - from prior analysis

$$u_1^* = \max_e E_{z|e}\left[\max_a E''_{\theta|z}\left[u(e,z,a,\theta)\right]\right]$$ - from pre-posterior analysis

$$VoI = u_1^* - u_0^*$$

- Conditional Value of Sample Information (CVSI)
- Expected Value of Perfect Information (EVPI)
- Conditional Value of Perfect Information (CVPI)

Thank you for your attention!